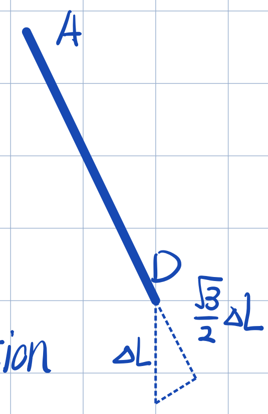
P1: (a) assume the displacement at point D is ΔL

then the elongation of truss element

BD is ΔL (2')



the elongation of truss element

AD is $\frac{\sqrt{3}}{2}\Delta L$ (3')

(note: the displacement at transverse direction is rigid body motion)

Similarly, the elongation of truss element CD is $\frac{\sqrt{3}}{2}\Delta L$

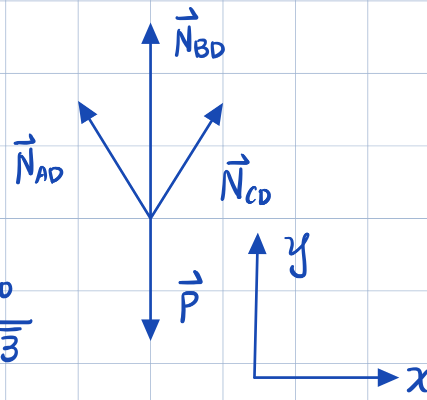
$$\Rightarrow \quad \epsilon_{BD} = \frac{\Delta L}{L}, \quad \epsilon_{AD} = \frac{\frac{\sqrt{3}}{2}\Delta L}{2/\sqrt{3}L} = \frac{3}{4}\cdot\frac{\Delta L}{L} \quad \epsilon_{CD} = \frac{\frac{\sqrt{3}}{2}\Delta L}{2/\sqrt{3}L} = \frac{3}{4}\frac{\Delta L}{L}$$

therefore, the internal axial forces are

$$|\vec{N}_{BD}| = \frac{EA\Delta L}{L}, \quad |\vec{N}_{AD}| = \frac{3}{4}\cdot\frac{EA\Delta L}{L}, \quad |\vec{N}_{CD}| = \frac{3}{4}\frac{EA\cdot\Delta L}{L}$$

i.e. $|\vec{N}_{AD}| = \frac{3}{4}|\vec{N}_{BD}|$, $|\vec{N}_{CD}| = \frac{3}{4}|\vec{N}_{BD}|$ (5')

$\sum F_y = 0 \Rightarrow$ (5')



$$-P + N_{BD} + \frac{3}{4} N_{BD} \cdot \frac{\sqrt{3}}{2} + \frac{3}{4} N_{BD} \frac{\sqrt{3}}{2} = 0$$

$$\left(1 + \frac{3\sqrt{3}}{4}\right) N_{BD} = P \qquad N_{BD} = \frac{4P}{4+3\sqrt{3}} = \frac{4 \times 1000}{4+3\sqrt{3}}$$

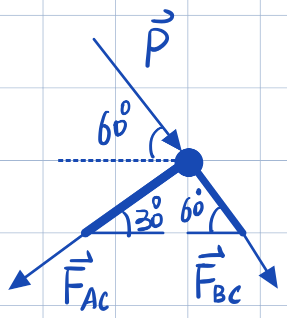
$$\Rightarrow \boxed{N_{BD} = 434.96 \text{ N (T)} \quad N_{AD} = 326.22 \text{ (T)} \quad N_{CD} = 326.22 \text{ (T)}}$$ (5')

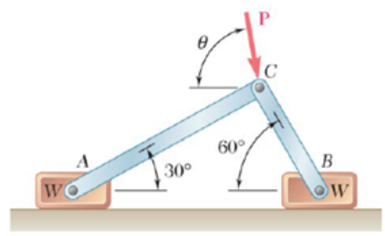
(b) for truss element BD, if $\sigma_{BD} = \sigma_{all} = 150 \text{ MPa}$

$$A_{BD} = \frac{N_{BD}}{\sigma_{BD}} = \frac{434.96}{150} = 2.90 \text{ mm}^2$$ (5')

$$\boxed{d_{BD} = \sqrt{\frac{4A_{BD}}{\pi}} = \sqrt{\frac{4 \times 2.90}{\pi}} = 1.92 \text{ mm}}$$ (5')

P2: take joint C as a free body

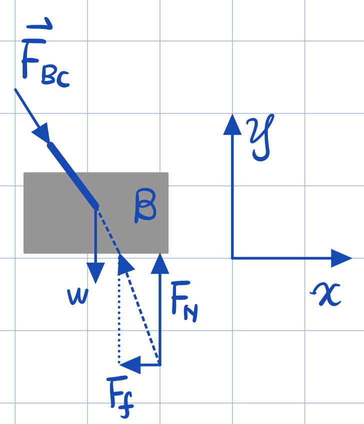




Since $\vec{P}$ is parallel with $\vec{F}_{BC}$ and perpendicular to $\vec{F}_{AC}$, it can be easily obtained:

$F_{BC} = P\,(C) \qquad F_{AC} = 0$ (5')

take bock B as a free body



use $\Sigma F_y = 0$ (5')

$$-F_{BC}\sin(60^\circ) - W + F_N = 0, \quad F_N = \frac{\sqrt{3}}{2}P + W$$

use $\Sigma F_x = 0$ and $F_f = F_N \cdot 0.3$ (5')

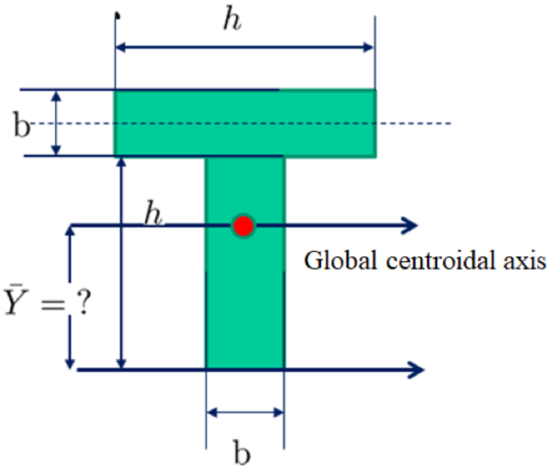
$$F_{BC}\cdot\cos(60^\circ) - F_f = 0, \quad 0.3\left(\frac{\sqrt{3}}{2}P + W\right) = \frac{1}{2}P,$$

$$\Rightarrow \boxed{P = \frac{6}{10-3\sqrt{3}}W = 1.250W}$$ (5')

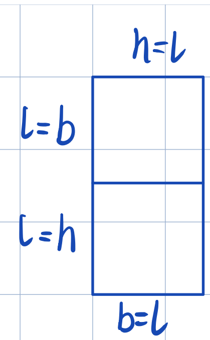
P3

(a)

index	A_i	$\bar{y}_i$	$A_i \bar{y}_i$
1	hb	$\frac{h}{2}$	$\frac{1}{2}h^2 b$
2	bh	$h+\frac{b}{2}$	$bh(h+\frac{b}{2})$
sum	$2hb$		$\frac{1}{2}b^2h+\frac{3}{2}h^2b$ (5')



$$\boxed{\bar{Y} = \frac{\Sigma A_i \bar{y}_i}{\Sigma A_i} = \frac{\frac{1}{2}b^2h+\frac{3}{2}h^2b}{2hb} = \frac{1}{4}b+\frac{3}{4}h}$$ (5')



double check: if $b=h=l$ $\bar{Y}=\frac{1}{4}l+\frac{3}{4}l=l$ ✓

(b)

index	$I_{x,i}$	$\bar{y}_i$	$(\bar{Y}-y_i)^2 A_i$	$I_{x,i}+(\bar{Y}-y_i)^2A_i$
1	$\frac{bh^3}{12}$	$\frac{h}{2}$	$\frac{1}{16}(b+h)^2bh$	$\frac{1}{16}(b+h)^2bh+\frac{bh^3}{12}$ (5')
2	$\frac{hb^3}{12}$	$h+\frac{b}{2}$	$\frac{1}{16}(b+h)^2bh$	$\frac{1}{16}(b+h)^2bh+\frac{hb^3}{12}$ (5')
sum				$\frac{1}{8}(b+h)^2bh+\frac{(b^2+h^2)}{12}bh$

$$\boxed{I_x = \left[\frac{1}{8}(b+h)^2+\frac{1}{12}(b^2+h^2)\right]bh}$$ (5')

double check: if $b=h=l$ $I_x=[\frac{1}{8}\times4l^2+\frac{1}{12}\times2l^2]l^2=\frac{2}{3}l^4$ ✓

P4:

A: because stress matrix must be symmetric

correct answers are $\boxed{(b), (d)}$

B: (a) X elongation is in the unit of [L], while

strains are in the unit of [1]

$\boxed{(b) \checkmark}$ By definition

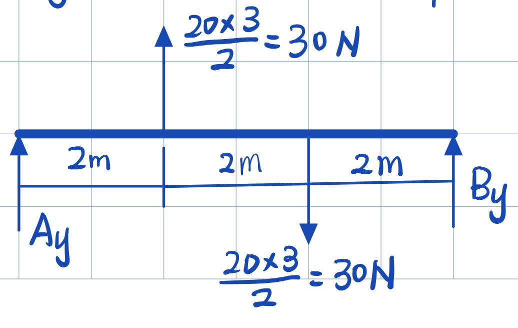
(c) X Shear strain can only characterize the

change of shape

(d) X the unit of relative displacement is [L]

while the unit of strain is [1]

C. The system can be equated as

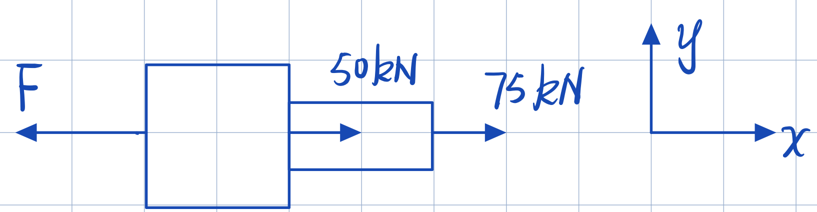


take moment at point B,

$$-A_y \times 6 + 30 \times 2 - 30 \times 4 = 0$$

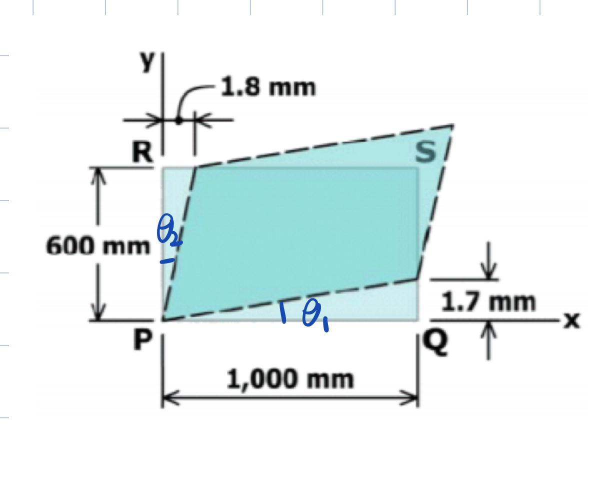
$$6A_y = -60, \quad \boxed{A_y = -10\ N\downarrow\ (a)}$$

D draw FBD as



$$-F + 50 + 75 = 0 \quad \boxed{F = 125\ kN, \text{ choose (c)}}$$

E



because

$$\theta_1 \approx \tan\theta_1 = 1.7 \times 10^{-3}$$

$$\theta_2 \approx \tan\theta_2 = 3 \times 10^{-3}$$

$$\Delta\theta = \theta_1 + \theta_2 = 4.7 \times 10^{-3}$$

$$\boxed{\gamma_{xy} = 4.7 \times 10^{-3}, \text{ choose (c)}}$$