**University of California, Berkeley**
**Department of Civil and Environmental Engineering**
**Structural Engineering, Mechanics, and Materials**

# CE120: Structural Engineering

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# CE120: Structural Engineering

## Outline

1. Introduction to Structural Engineering
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3. Analysis of Truss Structures
4. Analysis of Cable Structures
5. Principle of Virtual Displacements for Determination of Forces and Influence Lines
6. Principle of Virtual Forces for Determination of Displacements
7. Use of Computer Software for Structural Analysis
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12. Design Methods
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# 1. Introduction to Structural Engineering

## 1.1 STRUCTURAL ENGINEERING

*Structural engineering* is the field of engineering covering the design and construction of structural systems to provide functionality as specified by the project owner. Examples of functional requirements for a structure include: Defining space in a building for occupants, equipment, or business operations; providing a road surface for a bridge; impounding water in a reservoir for a dam; or resisting operational loads for an automobile or aircraft.

Structural engineering requires the application of specialized civil engineering knowledge, training, and experience to evaluate, analyze, design, specify, detail, and observe the construction of force-resisting elements of structures. Such expertise includes consideration of strength, stability, deflection, stiffness, ductility, potential modes of failure, and other characteristics that affect the behavior of a structure (Structural Engineers Association of Northern California).

## 1.2 LOADS ON STRUCTURES

*Structures* exist to resist loads. *Loads* are the result of interactions between the structure and its environment. *Dead loads* are the loads produced by gravity acting on the structure itself and on items that are permanently fixed to the structure. *Live loads* are the loads produced by gravity acting on items that are in or on the structure but are not fixed in magnitude or position. Examples include occupants, equipment, or vehicles. *Snow*, *ice*, and *water* pressure, are another form of gravity loads, which are generally treated separately from live loads because of the difference in their origin and loading duration. Wind produces pressure on the surface of a structure, inducing *wind loads*. When the ground moves during an earthquake, *earthquake loads* are caused by the acceleration of the building mass as it vibrates in response to the ground movement. Other loads can be induced by volume change due to shrinkage or change in temperature.

The performance of a structural system is important to occupants/users, owners, and society as a whole. For most structures, the primary performance requirement is *safety*. A safe structure is one that has an acceptably low probability of collapse over the life of the structure, and a low probability of producing other life-threatening falling hazards. A second performance requirement is *serviceability*, that is, the capability to resist routine loads and other effects without disturbing occupants or requiring repair. Requirements of serviceability may involve limitations on displacements, floor vibration, or crack width. Other performance considerations can include *durability*. Modern designs may also consider aspects of *sustainability*.

## 1.3 STRUCTURAL SYSTEMS

There is a wide variety of structural systems from which the structural engineer can choose. The choice will depend on many factors, possibly including the functional and performance requirements, the required height or span length, the local materials and construction practices, and the creativity of the design team. Structural systems are continuously evolving as new ideas, performance objectives, and materials enable and promote innovation. Before the advent of modern analysis methods, evolution in structural engineering was largely by trial and error, with limits in structural systems being exposed by structural failures. Today, computer simulation is increasingly effective in identifying optimal systems that provide required safety and serviceability with low risk of failure.

Commonly used structures can be classified into categories based on the type of primary stress resisted by members of the structures under design loads. The main categories are:

*Tensile structures*: Structures in which members are subjected to pure tension under design loads are known as tensile structures. Tensile structures tend to be very efficient because tensile stress is distributed uniformly over the cross-sectional area of members without concern for instability. A *cable* is an example of tensile structure.

*Compressive structures*: Structures in which members are subjected to pure compression under design loads are known as compressive structures. Compressive structures tend to be efficient because compressive stress is distributed uniformly over the cross-sectional area. Susceptibility to buckling may reduce efficiency of compressive structures, either because of requirements to reduce stress by increasing cross sections or, alternatively, to provide additional members to brace the compression member and thereby increase stability. An *arch* is an example of a compressive structure.

*Trusses*: Trusses are composed of straight members connected at their ends to form a stable configuration. The connections ideally are pins, but in actual structures they might be rigid connections.

*Frames*: Frames are structures that resist loads through combinations of shear and moment, with or without axial forces. Common frame members include slabs, beams, columns, and walls. (Sometimes walls are considered separately from frames.) Frames may be less efficient than other structures because the members resist loads through bending action, which generally makes less efficient use of the material capacities. However, members can be optimized to improve efficiency, as in, for example, I-beams that are configured efficiently resist bending and shearing stresses. Frames have the functional advantage of providing rectilinear open spaces in which the members are aligned with floors and walls.

*(Structural) Walls*: Slender structural walls resist lateral and vertical loads through combinations of shear, moment, and axial forces. Low-rise walls, and infill panels in structural frames, are a special case in which the wall resists lateral forces primarily in shear, with little flexural action.

Additionally, structural systems can be made up of combinations of different types of structural members. For example, a suspension bridge uses cables (tensile structures), towers (primarily compressive structures, but sometimes trusses), and bridge decks (frame members). Some examples are shown in Figure 1.1

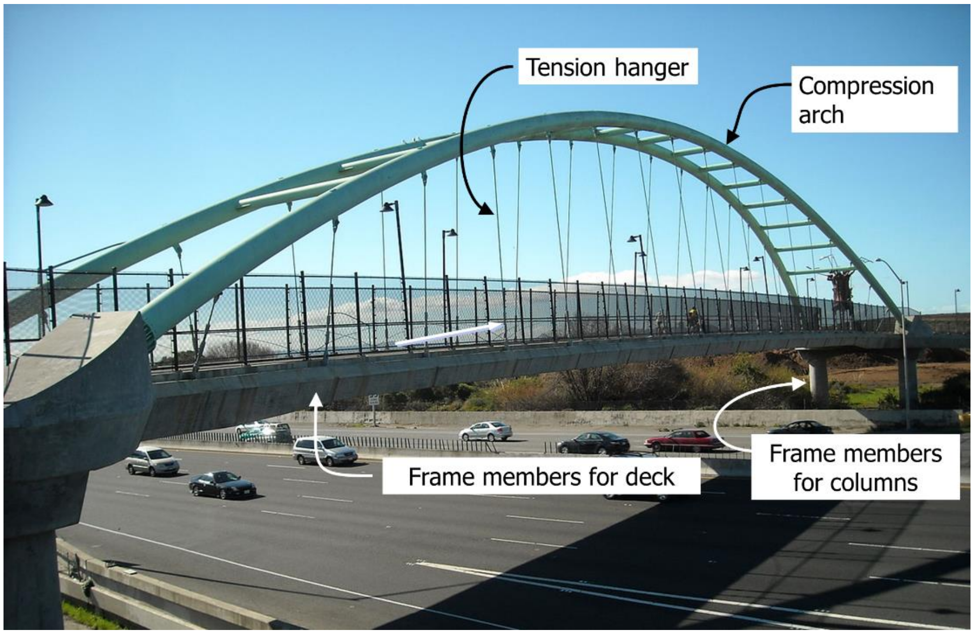



**Figure 1.1 Berkeley I-80 pedestrian bridge. Photo by Daniel Ramirez from Honolulu, USA - Uploaded by Kurpfalzbilder.de. Licensed under CC BY 2.0 via Wikimedia Commons - http://commons.wikimedia.org/wiki/File:Berkeley_I-80_bridge_02.jpg#mediaviewer/File:Berkeley_I-80_bridge_02.jpg**

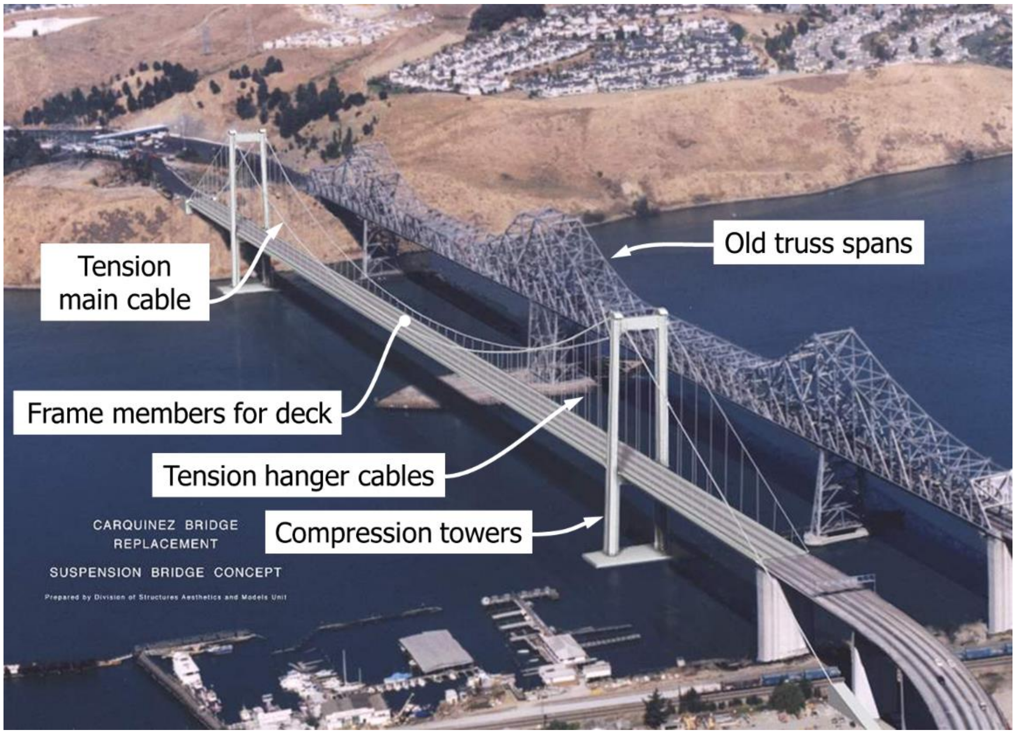



**Figure 1.2 Carquinez Bridge. Photo from [http://www.ketchum.org/carquinez.html](http://www.ketchum.org/carquinez.html)**

## 1.3 STRUCTURAL ENGINEERING WORK

### 1.3.1 Structural design and structural analysis

For any facility design, the structural engineer generally will work as part of a team engaged in the design of the facility. The structural engineer's tasks are oriented toward developing a structural system that fits within the functional space of the facility and that provides an efficient load path for both vertical and lateral loads. The design and analysis tasks include the following:

- *Identify a concept for a structural system* that will be capable of efficiently providing a load path. For the pedestrian bridge of Figure 1.1, the concept is an arch spanning across a freeway, with tension hangers that support the suspended walkway. For the suspension bridge of Figure 1.2, the arch (compression) is replaced with a cable (tension), which requires two support towers. The remainder of the system with hangers and bridge deck is similar to the pedestrian walkway. For the building of Figure 1.3, the concept is a beam-column frame that resists vertical and lateral forces.
- *Estimate the loads*. Once the general concept is developed, preliminary member sizes can be estimated using experience or, where experience is lacking, by making an educated guess. Given the preliminary member sizes, design loads can be estimated. As the loads depend on the member sizes, some iteration may be required.
- *Analyze the structure*. Given a structural idealization and design loads, the structure can be analyzed to determine the structure reactions and the member internal forces and moments. The analysis may also determine deflections of individual members or of the entire structure.
- *Develop final member/structure proportions*. Now that member internal forces and moments are determined by analysis, the members and their connections to one another can be designed. This step is usually driven by considerations of safety. However, serviceability must also be considered.
- *Specify the design*. The design intent must be conveyed via design and construction documents. The *design documents* contain the calculations used to demonstrate safety and serviceability of the structure. The *construction documents* contain information on how to build the structure, and include documents such as detailed specifications for materials and components, and detailed structural drawings that convey unambiguously the required dimensions, member sizes, member connections, and any other required details.

The process outlined above involves both elements of structural design and structural analysis. *Structural analysis* (the third bullet) involves the determination of the reactions, internal actions, and deformations/deflections of the structure under the design loads. *Structural design* is a much broader endeavor, involving development of a structural concept, determination of loads, structural analysis, proportioning of the elements and their connections, and specifying the design. In this regard, structural analysis is an essential tool in the broader endeavor of structural design.

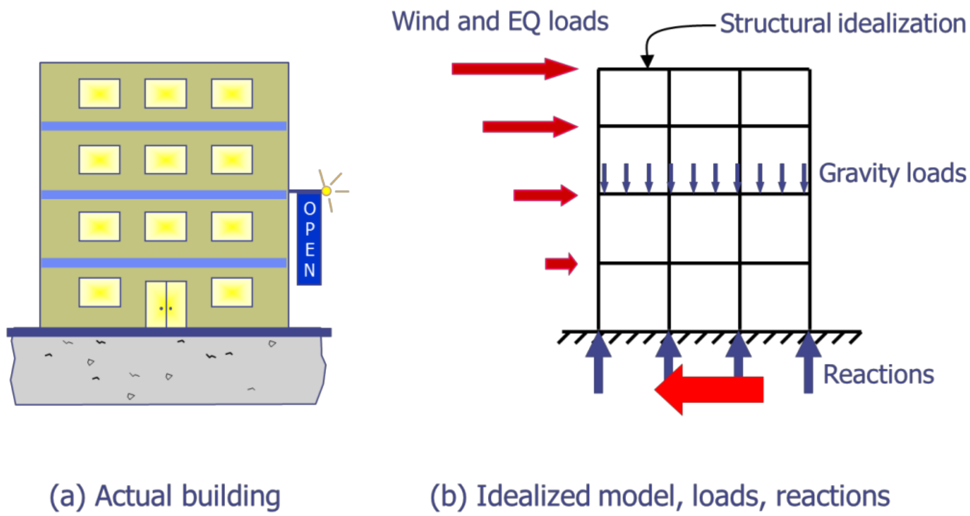



**Figure 1.3 Idealized model, loads and reactions for a building concept.**

### 1.3.2 Structural engineering design phases

Structural design generally does not involve the development of a grand equation that defines the structural system given a set of owner requirements. Rather, structural design is a process that is iterative, developing gradually from the owner requirements to the final design. Structural design may develop in tiered phases, as follows:

- *Conceptual or Schematic Design*: In this phase, the structural engineer establishes a primary structural system, including alternative schemes where appropriate, considering materials, systems, and budgets; describes the primary structural system, detailing significant primary structural elements and materials; explains in writing to the client the proposed new structural system and the alternatives, including the short and long term advantages and disadvantages; and recommends the preferred structural system. Note that the engineer might explore multiple design schemes including alternative structural systems or alternative materials. The designs are carried through in sufficient detail to be able to make a recommendation on a preferred system. By the end of this stage, the design may be sufficiently developed to be considered a *preliminary design*.
- *Design Development*: In the design development phase, the selected preliminary design is developed in sufficient detail to produce the final design and construction documents. During this phase, the engineer will carry out detailed structural analysis and calculations to consider the performance requirements for safety and serviceability, considering such elements as structural strength, deflections, vibrations, lateral drift, concrete and masonry crack control, and foundation settlement.
- *Contract Documents Stage*: In this phase, the engineer will document the structural calculations that support the design, prepare structural design drawings, and prepare materials and other specifications.
- *Construction Stage*: In this phase, the engineer will attend construction meetings and review submittals from the contractor to confirm conformance with the design intent or, where required, to make adjustments to the design to facilitate proper implementation of

the design intent. In some cases, the structural engineer will provide inspection services to verify that the structure was constructed in accordance with the intent.

# 2. Analysis of Beams and Frames

## 2.1. INTRODUCTION

Beams and frames are structural members that resist loads through shear and bending moment, with or without axial forces. Beams can be made of wood, steel, or reinforced concrete. Frames are usually either steel or reinforced concrete. This chapter introduces methods of analysis of statically determinate beams and frames. In later chapters we will see how to design structural systems using beams and frames.

## 2.2. IDEALIZATIONS

For the purpose of conducting structural analysis, structural engineers usually idealize the geometry of the structural framing, its supports, and its loads. The following conventions are common.

### 2.2.1. Structural members

Structural members commonly are represented by *line members* located at the member centerlines. See Figure 2.1. The lines have structural properties associated with the axial, shear, bending, and other important structural properties of the original member.

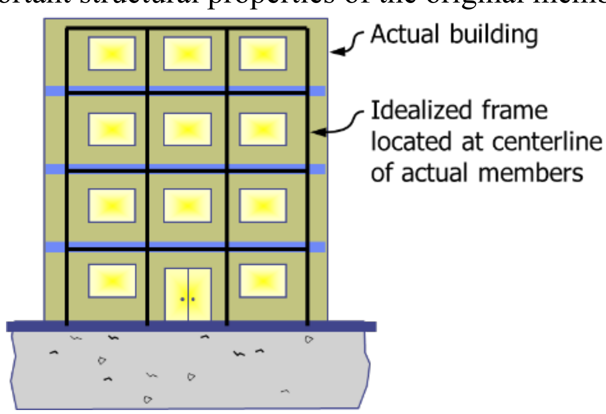



**Figure 2.1 Idealization of structural members by line members.**

### 2.2.2. Structural supports

The most common structural supports are idealized as being *fixed*, *pinned*, or *roller* supports. Some unusual support conditions, such as a sliding support that is free to translate in one direction, but restrained against translation in other directions and restrained against rotation, can also be represented. Figure 2.2 illustrates some commonly used symbols to represent the support conditions and the types of reactions that these can resist.

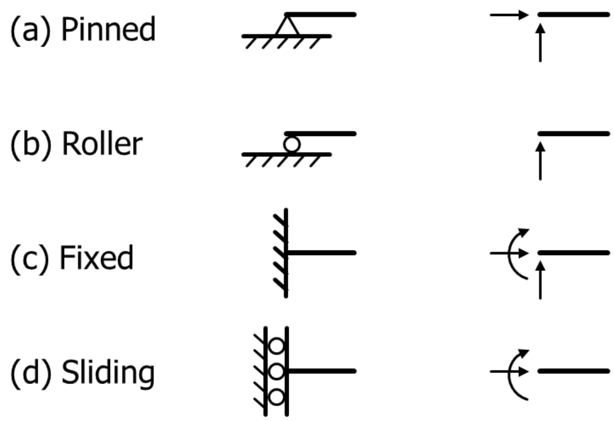
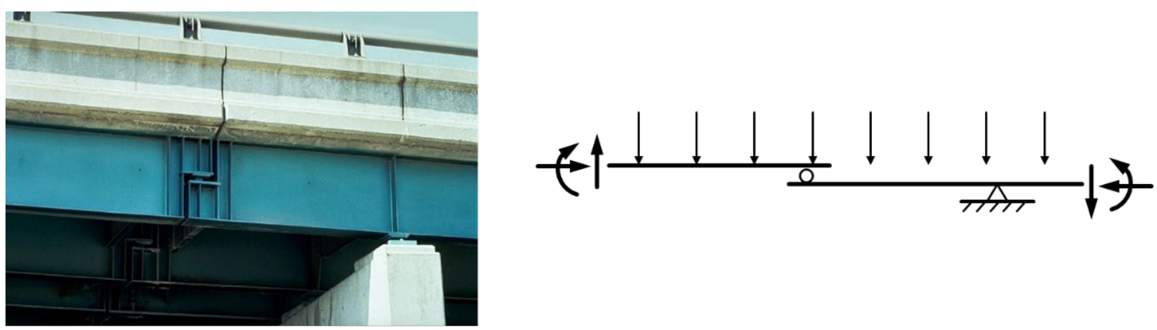



**Figure 2.2 Some support representations and the reactions they can resist.**

The idealizations of Figure 2.2 are used to idealize real boundary conditions in actual structures. See example in Figure 2.3.



(a) Bridge structure (b) Idealization

**Figure 2.3 Bridge structure and its idealization.**

### 2.2.3. Loads

Loads are usually point loads (actually concentrated loads distributed over a very short distance) or distributed loads. Distributed loads can be uniform from one end to the other, as occurs when people are distributed uniformly throughout a room, or can vary from one end to the other, as occurs with snow drifts. Inclined surfaces can have distributed vertical loads, for example, the gravity load associated with roof shingles. Inclined surfaces also can have distributed loads acting normal to the surface, as occurs with wind loads. See Figure 2.4.

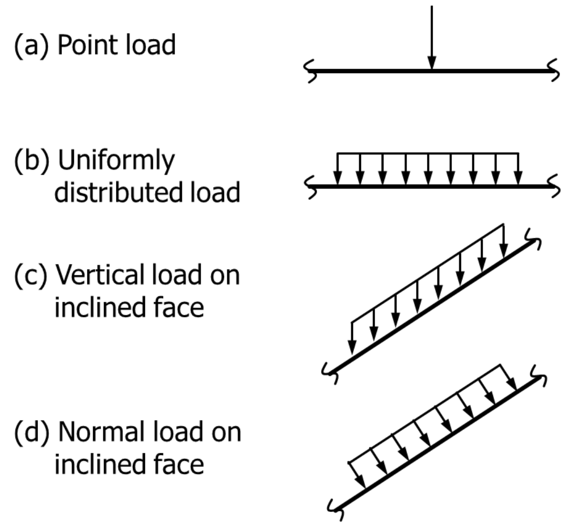



**Figure 2.4 Load idealizations.**

## 2.3. EQUILIBRIUM

First we will examine an individual element and then later examine behavior of the whole system. A beam provides a simple starting point.

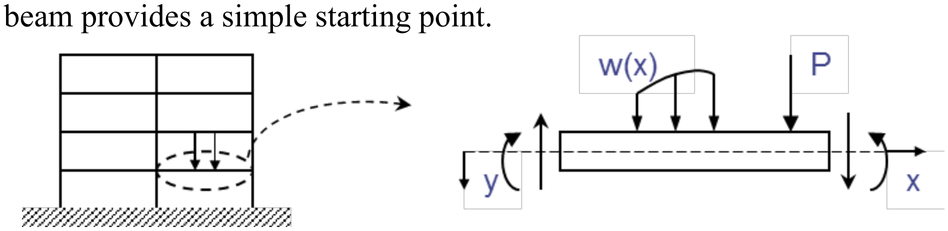



**Figure 2.5 Free body diagram of beam.**

Consider the free body diagram of a beam (Figure 2.5). Considering only actions within the plane of the paper, the three equilibrium requirements are:

$$\sum F_Y = 0 \quad \sum M_O = 0 \quad and \quad \sum F_X = 0 \tag{2.1}$$

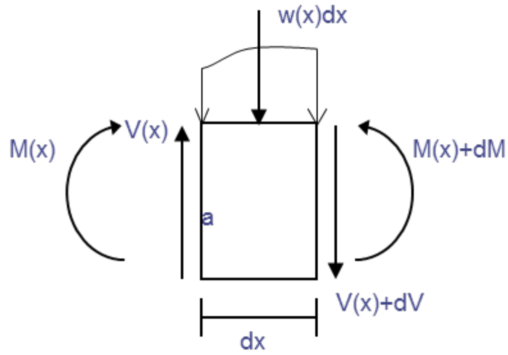



**Figure 2.6 Free body diagram of a differential length of a beam.**

We can write the equilibrium equations for a differential length of a beam (Figure 2.6), resulting in the following expressions.

$$\sum F_Y = 0$$

$$-V + w(x)dx + V + dV = 0$$

$$\frac{dV}{dx} = -w(x) \longrightarrow V = -\int w(x)dx$$

$$\sum M_a = 0 \qquad \textit{Clockwise positive}$$

$$M + w(x)\frac{dx^2}{2} + (V + dV)dx - M - dM = 0$$

Neglecting higher order terms, $dx^2$, dVdx

$$\frac{dM}{dx} = V \longrightarrow M = \int V(x)dx$$

The integral form of the equilibrium equations has a useful interpretation for construction of shear and moment diagrams, as will be illustrated later.

## 2.4. STATIC DETERMINACY

Equilibrium must always be satisfied. The number of equilibrium equations is related to the free-body diagrams for the structure. For a two-dimensional FBD, there are three independent equilibrium equations. A three-dimensional FBD has six independent equilibrium equations.

Although the equilibrium equations must be satisfied, they may not be sufficient to determine all the forces in a structure. In such cases the statics equations are indeterminate and the structure is called *statically indeterminate*.

Consider the case where a structure is cut into m free body diagrams. There are 3 equations per free body diagram, and, therefore, 3*m* equations of equilibrium. Suppose also that there are *r* unknown forces acting at the boundaries of the m free body diagrams. The system can only be solved by equations of equilibrium if r = 3m. Put differently, the *degree of static indeterminacy* can be expressed as

$$n = r - 3m \tag{2.2}$$

where $n$ = degree of static indeterminacy
$r$ = number of unknown forces
$m$ = number of free body diagrams or parts of the structure.

| Cases of Static Indeterminacy | |
|---|---|
| $n = 0$ | The structure is *statically determinate* and can be solved by equations of equilibrium alone. |
| $n > 0$ | The structure is *statically indeterminate*, and cannot be solved by equations of equilibrium alone. |
| $n < 0$ | The structure cannot satisfy some equilibrium conditions, that is, it is *unstable*. |

Figure 2.7 illustrates three examples checking the degree of static indeterminacy for beams. Figure 2.8 illustrates examples for frames.

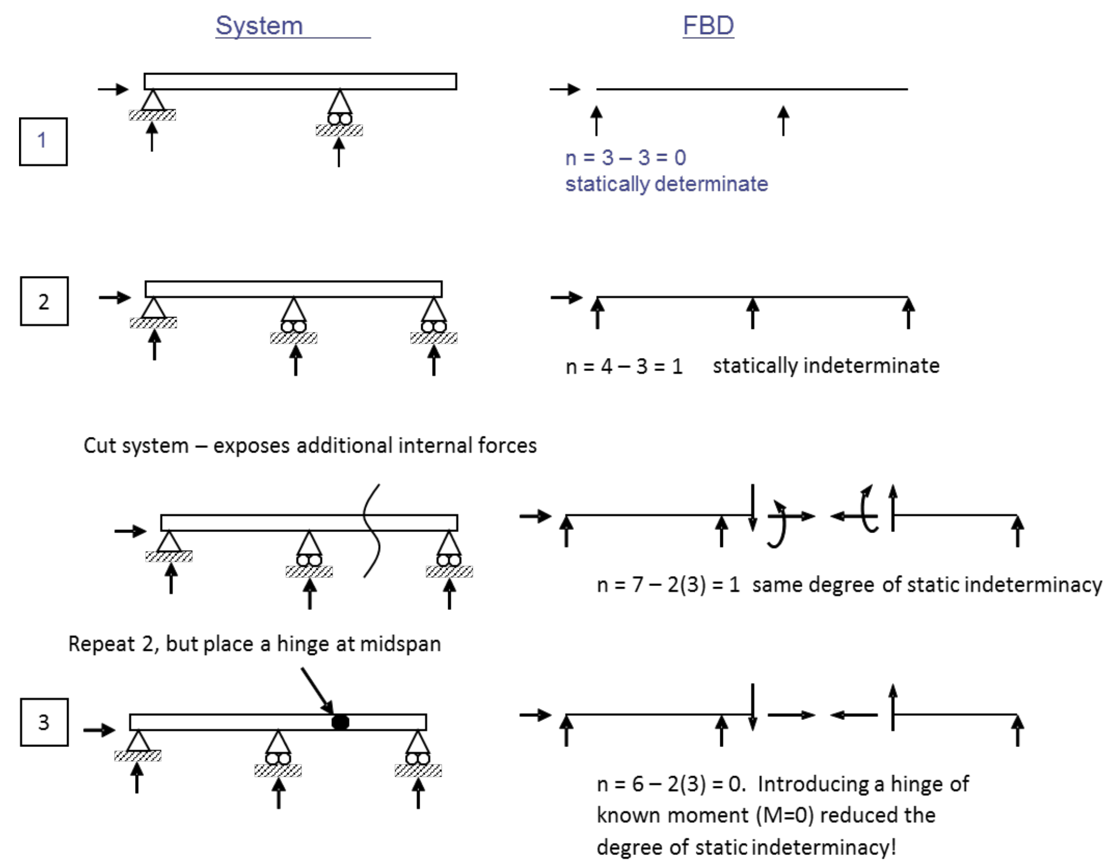



**Figure 2.7 Examples for degree of static indeterminacy for beams.**

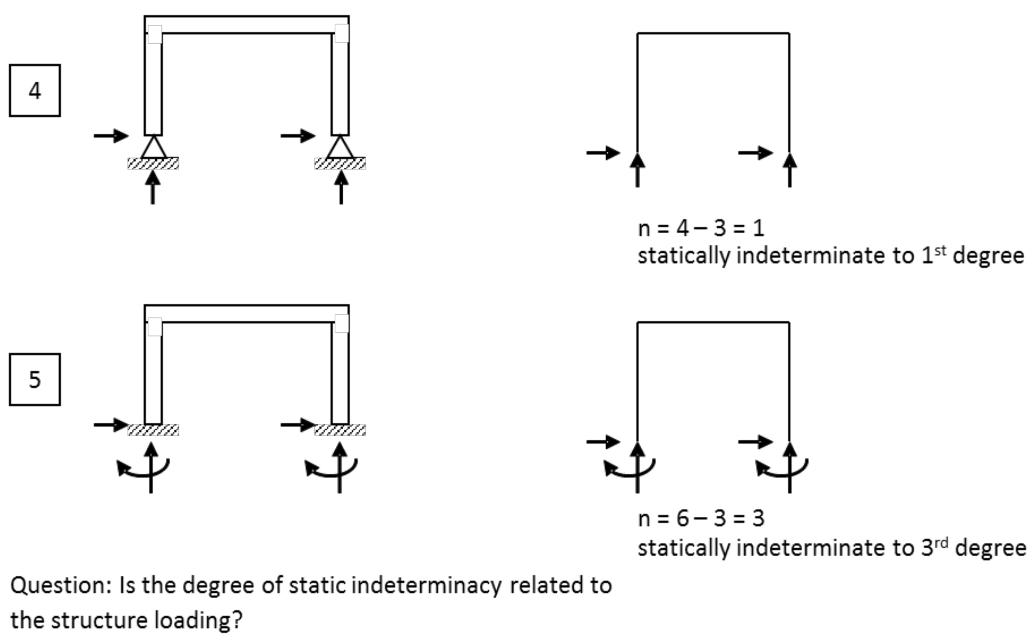



**Figure 2.8 Examples for degree of static indeterminacy for frames.**

## 2.5. STABILITY

To be stable a structure must have a configuration and set of external supports that allow equilibrium to be satisfied under all imaginable loading combinations. (Stability of slender members, such as Euler buckling of columns, is a different issue not to be confused with stability as defined here.) The following paragraphs consider some common examples.

Figure 2.9a considers a simply supported beam. In this example, $n = r - 3m = 0$. Therefore, the beam is statically determinate and stable.

Figure 2.9b illustrates a special case. Although $n = r - 3m = 0$, the structure is unstable because equilibrium of forces in the horizontal direction cannot be satisfied. In general, if the reactions are all collinear, the beam is unstable. As a more general condition, if all of the reactions have projections that intersect a single point, moment equilibrium about that point cannot be satisfied and the beam is unstable. For the case in Figure 2.9b, the point is located an infinite distance above (or below) the beam.

Figure 2.9c appears to be stable if checked as a single free body diagram. However, if we cut through the hinge, exposing two free body diagrams, we find that $n < 0$, that is, the beam is unstable. In general, if there is a hinge or other unusual condition along the beam span, cut the beam into two distinct free body diagrams and make the check on that group of free body diagrams.

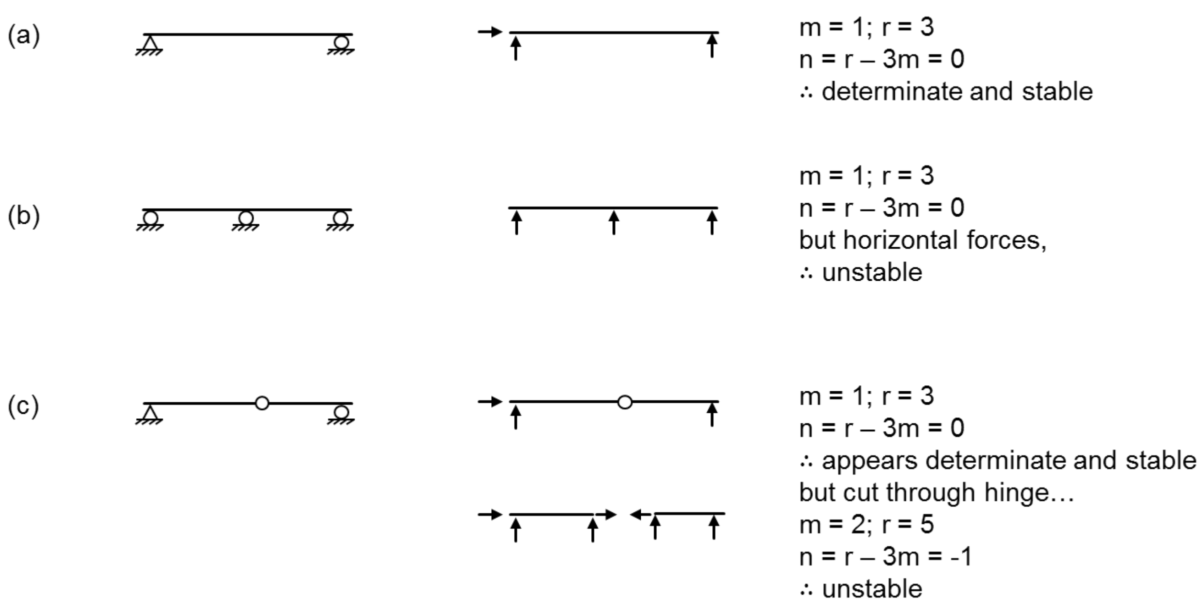



**Figure 2.9 Stability checks for beams.**

Stability of frames with multiple hinges can be difficult to assess by equations alone. Sometimes, inspection and intuition are required. See Figure 2.10.

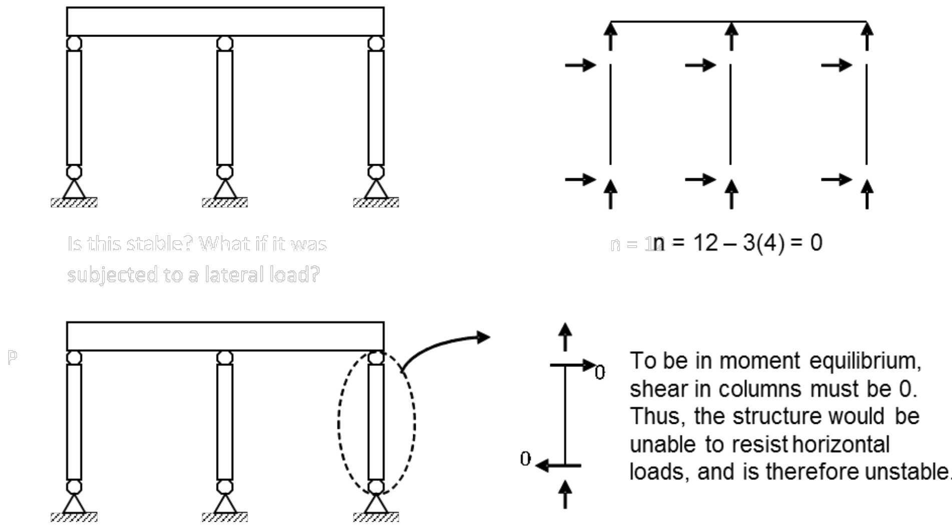



**Figure 2.10 Unstable frame.**

Importantly, stability is a property of a structure, and is independent of the loading. A structure that can resist a narrowly defined loading, but that will be unable to resist a more general loading, is defined as being unstable.

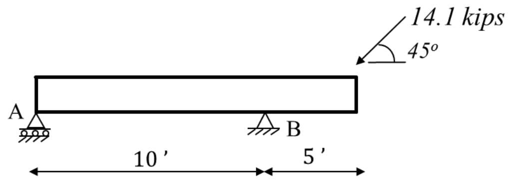
## 2.5. ANALYSIS FOR REACTIONS

For beams and frames that are stable and statically determinate, the equations of equilibrium can be used to establish the reactions and any internal member forces. Here we focus on calculation of the reactions. The general procedure is:

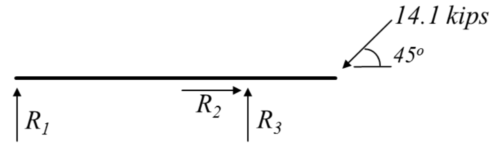
- Draw a free body diagram, showing all external forces and replacing reaction points with the unknown reactions.
- Check that the free body diagram is stable and determinate.
- Apply equations of equilibrium to solve for the reactions.

The approach is illustrated in the following examples.

Example 1




1)




2) stable, determinate

3) $\overset{+}{\rightarrow}\Sigma F_x = 0 \quad \rightarrow R_3 - 14.1*\cos 45^o = 0$

$\rightarrow \underline{R_3 = 10\text{ kips}}$

$\overset{+}{\curvearrowleft}\Sigma M_A = 0 \quad \rightarrow -R_2(10) + (14.1*\sin 45^o)(15) = 0$

$\rightarrow \underline{R_2 = 15\text{ kips}}$

$\overset{+}{\curvearrowleft}\Sigma M_B = 0 \quad \rightarrow R_1(10) + (14.1*\sin 45^o)(5) = 0$

$\rightarrow \underline{R_1 = -5\text{ kips}}$

check $+\!\uparrow\Sigma F_y = 0 \rightarrow R_1 + R_3 - 14.1*\sin 45^o = 0$

Solution

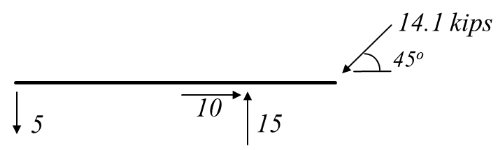
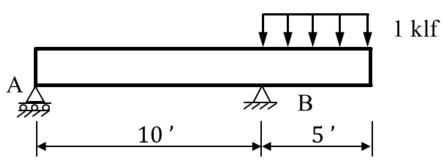



**Figure 2.11 Propped cantilever with concentrated force.**

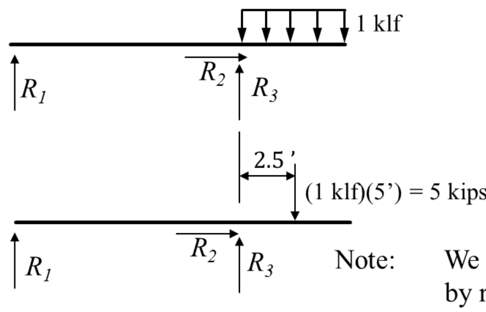
In Figure 2.11, note how the positive direction for summing forces and moments is noted. For example, for summing forces in the x direction, the arrow from left to right indicates that all forces are taken positive in that direction. By showing the positive direction in the summation sign, it is easier to keep track of the sign on all the forces. Of course, positive could be defined in the opposite direction as well. Regardless, selecting a direction and showing it consistently is a big help to keeping the signs straight and getting the correct answer.

The next example (Figure 2.12) considers the same structure, but this time with a distributed load along segment BC. <u>Because the uniformly distributed load is internal to the free body diagram</u>, for the purpose of solving the equilibrium equations it is acceptable to replace the distributed load by its resultant acting at the centroid of the distributed load.

Example 2




*Free Body Diagram*




Note: We can replace uniform load by resultant because it is internal to the FBD.

$\curvearrowright + \Sigma M_A = 0 \quad \rightarrow -R_3(10') + (5k)(12.5') = 0$

$\rightarrow \underline{R_2 = 6.25 \text{ kips}}$

$\curvearrowright + \Sigma M_B = 0 \quad \rightarrow R_1(10') + (5k)(2.5') = 0$

$\rightarrow \underline{R_1 = -1.25 \text{ kips}}$

check $+\uparrow \Sigma F_y = 0$

$\overset{+}{\rightarrow} \Sigma F_x = 0 \rightarrow \underline{R_2 = 0}$

**Figure 2.12 Propped cantilever with distributed loads.**

The next example considers a beam with an internal hinge. For such structures, the usual approach is to cut the structure into multiple free body diagrams (FBDs) by cutting through the internal hinges. Remember, each cut creates a new FBD with three new equations, but exposes only two additional unknown forces, so by cutting at the hinge the number of unknowns is reduced by 1. Once the structure is separated into multiple FBDs, equilibrium equations are applied to one or more of the FBDs to solve for the reactions and the hinge forces. Usually it is possible to solve for all of the forces on one of the FBDs first, and then move on to the remaining FBDs to solve the forces there. In the example shown in Figure 2.13, the structure is separated into segments abd and cde. By inspection, it is possible to solve for the unknown forces acting cde using first moments about point c and then moments about point e. Having solved cde, we next move on with the known hinge force of 0.5 and solve abc by summing moments about a and then about b. Note that this structure will require a tie down at point a for this loading.

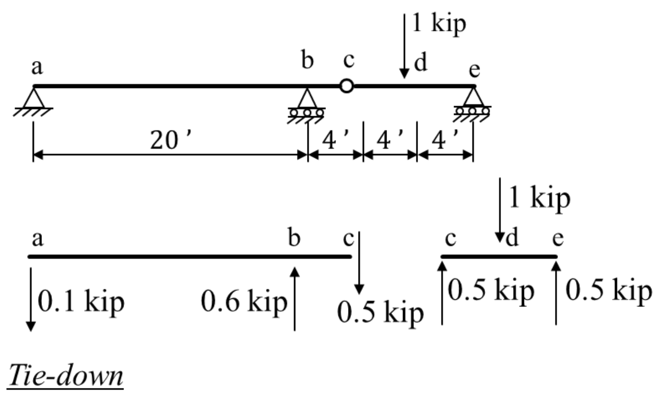



**Figure 2.13 Beam with internal hinge.**

## 2.6. INTERNAL FORCES

Once the reactions have been determined, it is possible to determine the internal forces. For two-dimensional structures, the important forces are shears, moments, and axial forces. Two approaches are commonly considered by structural engineers. One of the approaches is to cut the structure at multiple locations, calculate the internal forces, and then calculate the internal force diagrams from these section cuts. An alternative approach is to use the differential relations between load, shear, and moment derived in Section 2.3. The following text illustrates these approaches.

### 2.6.1. Shear and moment diagrams by sectional cuts

We return to the example of Figure 2.12, with the assignment to calculate and plot the variations of shear and moment along the beam span, commonly known as the *shear and moment diagrams*.

The solution for the reactions is shown in Figure 2.12, and we copy those solutions into Figure 2.14. To write the algebraic expressions for the shear and moment diagrams, we begin at a convenient location, in this case point A at the left end of the beam, and draw a FBD extending a distance x to the right of that point. Summing forces and moments, we can write the algebraic expressions for the shear and moment along span AB. The discontinuity in loading at point B requires that we draw a new FBD, this time extending from A through B and extending a distance *x* beyond B. (Notice that we are redefining the variable *x*.) Again summing forces and moments, we obtain expressions for the shear and moment along span BC.

The shear and moment diagrams are plotted at the bottom of Figure 2.14. Notice that the sign convention for the shear and moment diagrams are always shown next to the diagrams. The convention used in this reader is shown in the figure. Notice that, in the solution of the shears V and moments M for each FBD, we have consistently drawn V and M in the positive directions. By so doing, the algebraic expressions have the correct signs for plotting in the shear and moment diagrams. It can be instructive to also show the orientation of curvature in the moment diagrams. Although this is redundant with the sign convention shown to the left of the diagram, it is convenient to be able to sketch the direction of curvature, as we will use this later in sketching deflected shapes.

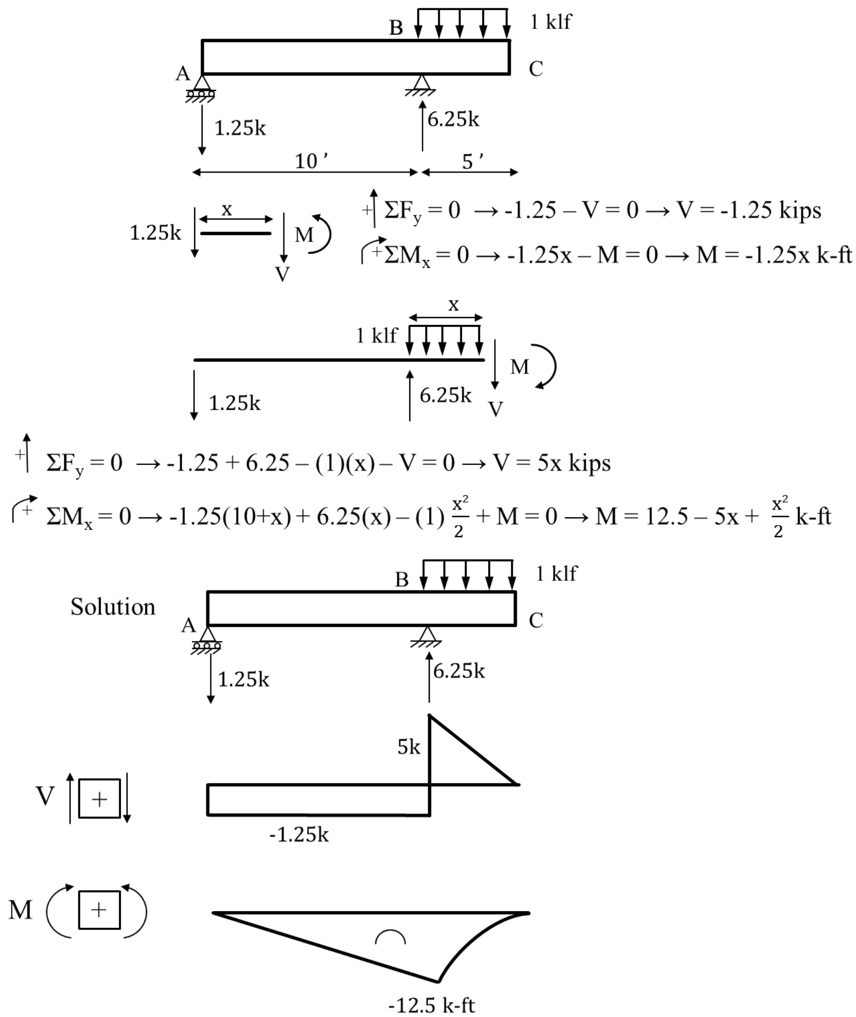



**Figure 2.14 Shear and moment diagrams by section cuts.**

## 2.6.2. Shear and moment diagrams using relations between load, shear, and moment

Section 2.3 derived differential relations between load w, shear V, and moment M. We repeat those relations here.

$$\frac{dV}{dx} = -w(x) \tag{2.3}$$

$$\frac{dM}{dx} = V(x) \tag{2.4}$$

It is important to understand the physical meaning of these two equations. Specifically, Eqs. (2.3) and (2.4) state the following:

- The slope of the shear diagram at any point is equal to the negative of the externally applied distributed load at that point; and
- The slope of the moment diagram at any point is equal to the shear at that point. Consequently, the moment is maximum (zero slope) where the shear is zero.

If we integrate both sides of Eqs. (2.3) and (2.4) as was done in Section 2.3, we can also state the following:

- The change in shear between any two points along a member is equal to the negative of the area of the loading between those two points; and
- The change in moment between any two points a long a member is equal to the area of the shear diagram between those two points.

These observations provide us very important tools for drawing shear and moment diagrams without having to take multiple section cuts as was done in the example of Figure 2.14. To illustrate the procedure, we repeat the example of Figure 2.14 in Figure 2.15, with the following main points:

- *Shear diagram*: Using the adopted sign convention, the shear starts at -1.25k. If there is any question about the direction of the shear diagram, draw a FBD, as shown to the right of the shear diagram. To the right of point A, the shear is acting upward, which is defined as negative shear. Shear is the integral of the loading. Therefore, from A to B there is no change in the shear. At point B, the concentrated reaction causes a 6.25k change in the shear, resulting in 5k shear force just to the right of point B. See the FBD of point B to demonstrate the sign of the shear force. Along BC, the load is 1 klf, so the shear diagram has to have a negative slope of -1 kip per foot. Alternatively, we can note that the change in shear along BC is the negative of the area of the loading diagram, or -1 klf x 5 ft = -5k. Because the load is constant along BC, the slope of the shear diagram is constant along BC. Note that the shear reaches 0k at point C, as it must at the free end.
- *Moment diagram*: Along AB, the shear is constant, therefore the slope of the moment diagram is constant. The total change in moment along AB is the area under the shear diagram, resulting in moment equal to -12.5 k-ft at point B. Along BC, the change in moment diagram again must be equal to the area of the shear diagram along BC, resulting in zero moment at point C, as required at the free end. Note that the shear is greatest near point B and gradually reduces to zero at point C. Therefore, the slope of the moment diagram must be steepest at point B and must gradually reduce to zero at point C. It is important to be able to draw the slope of the moment diagram correctly along the span.

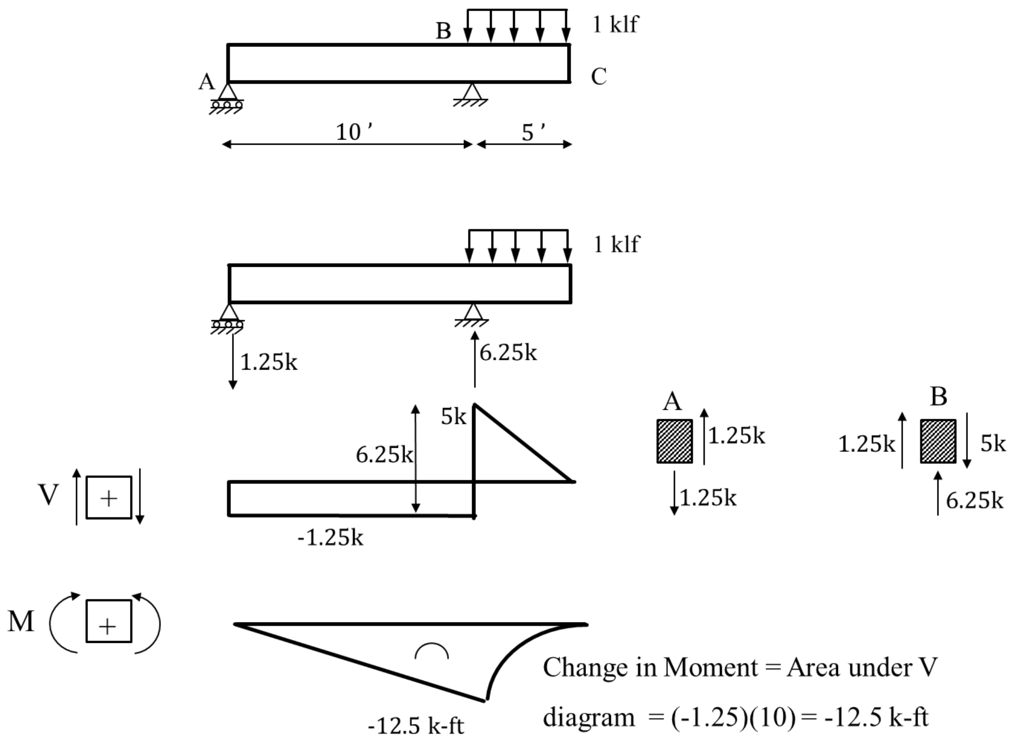



**Figure 2.15 Shear and moment diagrams using relations between load, shear, and moment.**

## 2.7. DEFLECTED SHAPES

Serviceability requirements usually require that deflections be limited to allowable values that are specified as part of the building code or as part of building-specific performance requirements. Therefore, structural engineers need to be able to calculate deflections and verify that calculated values do not exceed the allowable values. We will consider methods for calculating deflections, and methods for determining allowable values, later in this reader.

In the present section, we are interested only in being able to sketch deflected shapes, without attempting to calculate actual deflection values. This skill is important for several reasons:

- It gives the engineer a sense of the direction of displacements. This can be important for checking analysis results obtained from computer analyses.
- It gives a sense of the direction of curvature, so that locations of concrete beam cracking and requirement placement of reinforcement can be identified.
- It identifies approximate locations of points of inflection; this can be useful for approximate analysis of statically indeterminate structures.

### 2.7.1. Curvature in beams and columns

In beams and columns of usual proportions, deflections are primarily due to flexural curvature. Effects of shear and axial deformations can usually be ignored without serious impact on the final answer (although exceptions can be found). Here we consider only flexural curvature.

Curvature is defined as the rate of change of angle along the length of a flexural member. Referring to Figure 2.16, curvature is defined as

$$\kappa = \frac{d\theta}{dx} \tag{2.5}$$

For linear-elastic response, we can relate curvature *K*, moment *M*, and flexural rigidity *EI* by

$$\kappa = \frac{d\theta}{dx} = \frac{M}{EI} \tag{2.6}$$

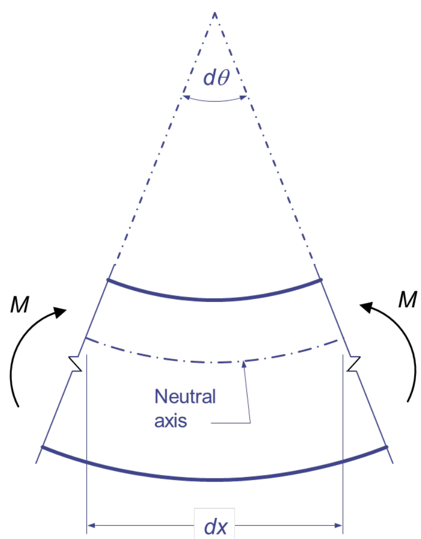



**Figure 2.16 Flexural curvature of a beam.**

### 2.7.2. Rules for sketching deflected shapes

The following rules are applied to sketching deflected shapes:

1. Ignore axial and shear deformations (actually, assume them to be zero).
2. The sense of curvature must be consistent with the sign of the moment.
3. The boundary conditions must be satisfied.
4. Rigid joints between members should not change angle in the deflected position.

Rule 3 warrants additional discussion. As shown in Figure 2.17, a member framing into a fixed support is not permitted to displace or rotate at that end. A member framing into a pinned support is permitted to rotate at the support, but not permitted to displace in any direction. A member framing into a roller support is permitted to rotate and translate parallel to the roller direction, but it is not permitted to displace perpendicular to the roller direction.

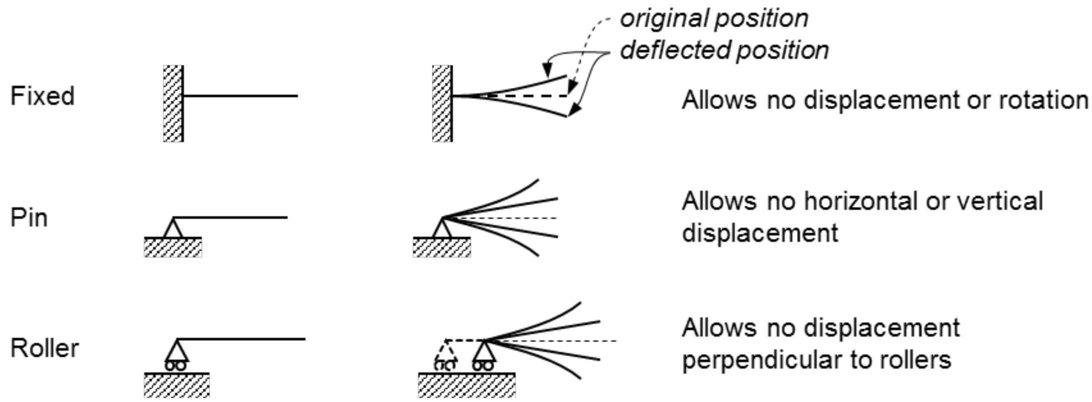



**Figure 2.17 Requirements for satisfying boundary conditions.**

### 2.7.3. Examples of deflected shape sketches

Some examples of sketching deflected shapes follow.

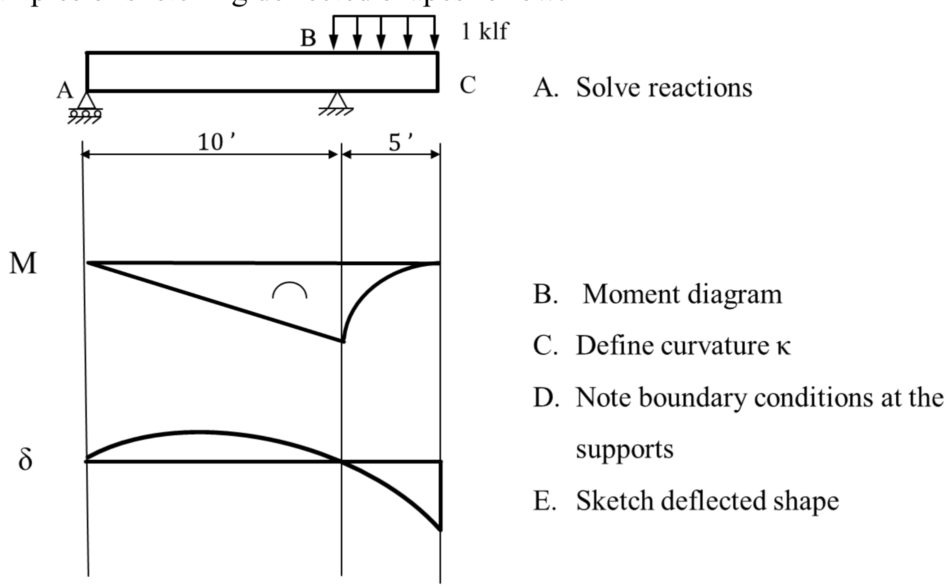



**Figure 2.18 Deflected shape for beam of Figure 2.15.**

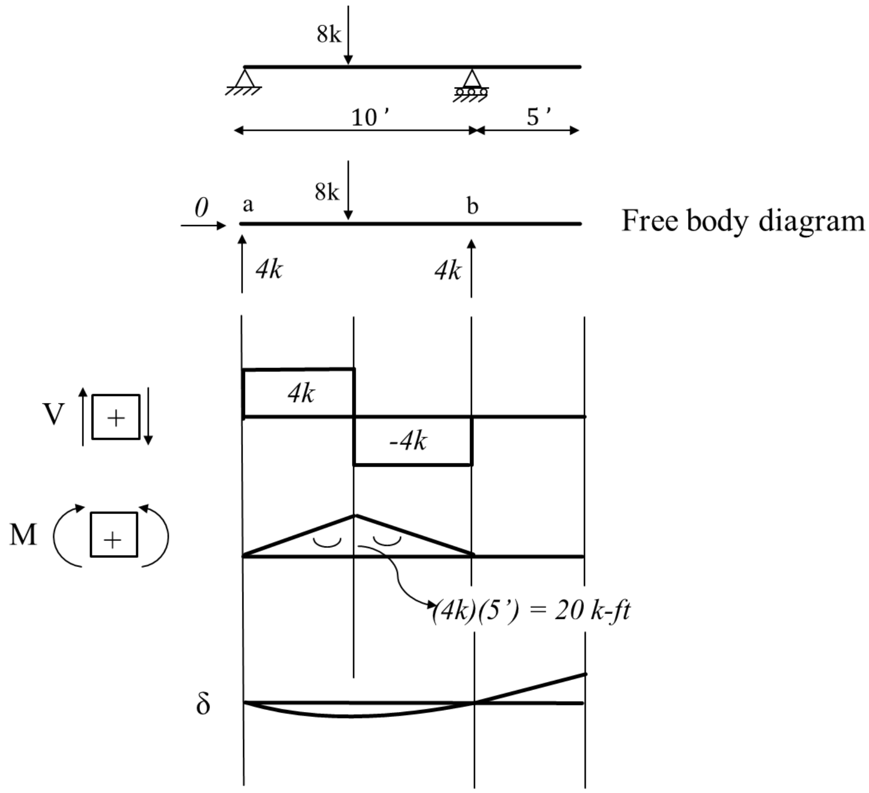



**Figure 2.19 Deflected shape for beam. Note the cantilever segment has no moment and, therefore, no curvature (it remains straight).**

## 2.8. FRAMES

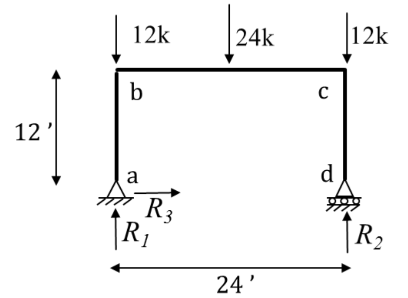
Frames are assemblies of beams and columns. Analysis of frames follows the same procedures used for beams, but the details are somewhat more complicated because of the two-dimensional nature of frames versus the single dimension of beams.

Figure 2.20 illustrates the analysis of a frame under vertical loads, including determination of reactions; axial force, shear, and moment diagrams; and deflected shape. The following points are worth consideration:

- *Determinacy and stability*: The checks here are the same as were used for beams.
- *Reactions*: Similar to procedures for beams.
- *Internal forces*: Note that the shear in the beam causes axial force in the supporting column. Thus, in the "exploded diagram" of the system (third diagram from the top), the 12k beam shear and the 12k external load combine to produce 24k axial compression in the column.
- *P-V-M diagrams*: Be certain to show the convention for axial force. Here we take tension as positive axial force. The shear convention, also shown, is consistent for beams and columns, regardless their inclination. The moment convention, however, is ambiguous for beams and columns, depending on the direction the moment convention indicator is rotated. It can be a good idea to sketch the curvature, as is done here.
- *Deflected shape*: Sketching the deflected shape of a frame can take some practice and trial and error iterations. Important aspects of this deflected shape are 1) the horizontal deflections of the beam-column joints are equal, 2) the vertical deflection of the beam-column joints is zero, 3) the 90-degree angles at the beam-column joints are maintained

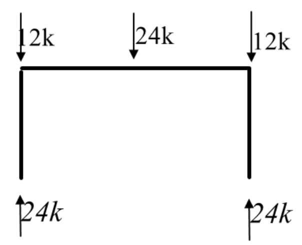
in the deformed shape, 4) the columns are straight because there is no moment for this applied loading, and 5) the two support column ends remain in contact with the "ground."

Example




m=1 , r=3, n=0
Determinate & Stable

$\overset{+}{\rightarrow}\Sigma F_x = 0 \quad \rightarrow R_3 = 0$

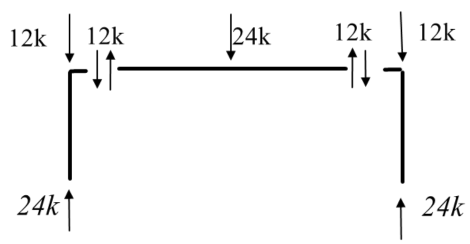


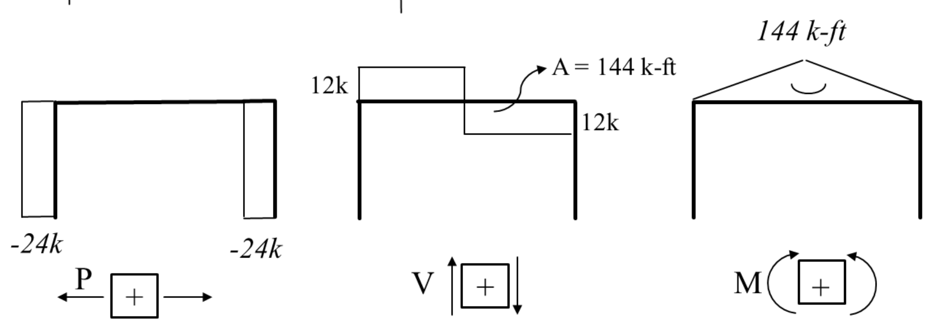

$\curvearrowright^{+}\Sigma M_a = 0 \quad \rightarrow (24)(12) + (12)(24) - (R_2)(24) = 0$

$\rightarrow \underline{R_2 = 24 \text{ kips}}$

$\curvearrowright^{+}\Sigma M_d = 0 \quad \rightarrow \underline{R_1 = 24 \text{ kips}}$

check $+\uparrow \Sigma F_y = 0$



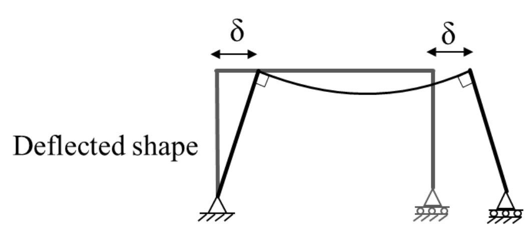




**Figure 2.20 Frame under vertical loads.**

Figure 2.21 solves the same frame under lateral loads.

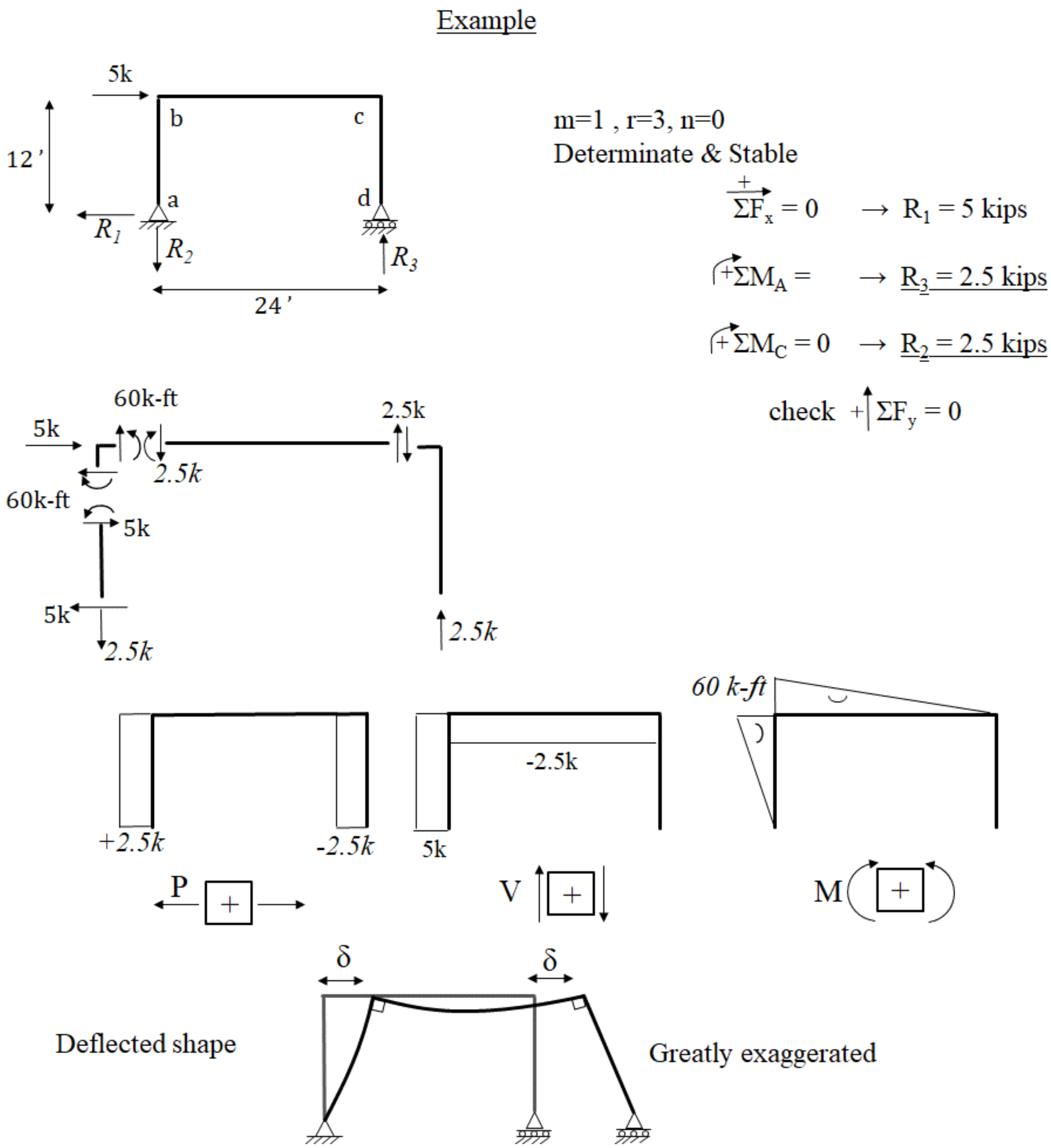



**Figure 2.21 Frame under lateral loads.**

## 2.9. SUPERPOSITION OF EFFECTS

The *principle of superposition* states that the effects of individual loadings can be added to obtain the combined effect for combined loadings. The principle of superposition can be used to calculate reactions, internal forces and moments, and deflections. Figure 2.22 illustrates the use of the principle for the determination of beam moments under the action of a concentrated load and a uniformly distribute load. Figure 2.23 illustrates the solution of a frame under combined vertical and lateral loading by superimposing results from the individual loads (see Figure 2.20 and Figure 2.21).

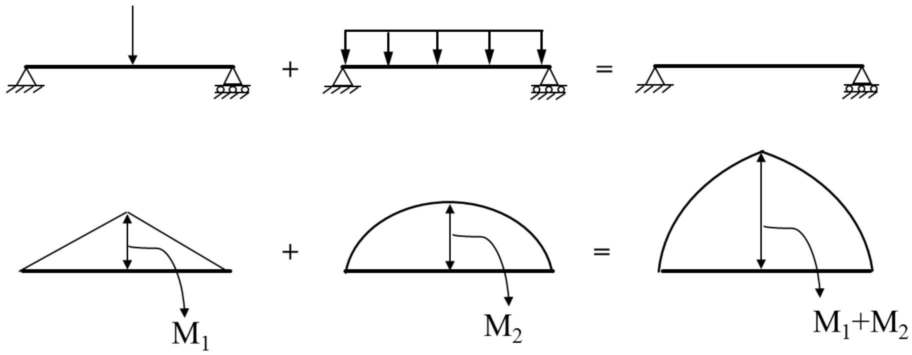



**Figure 2.22 Illustration of the principle of superposition for determination of moments.**

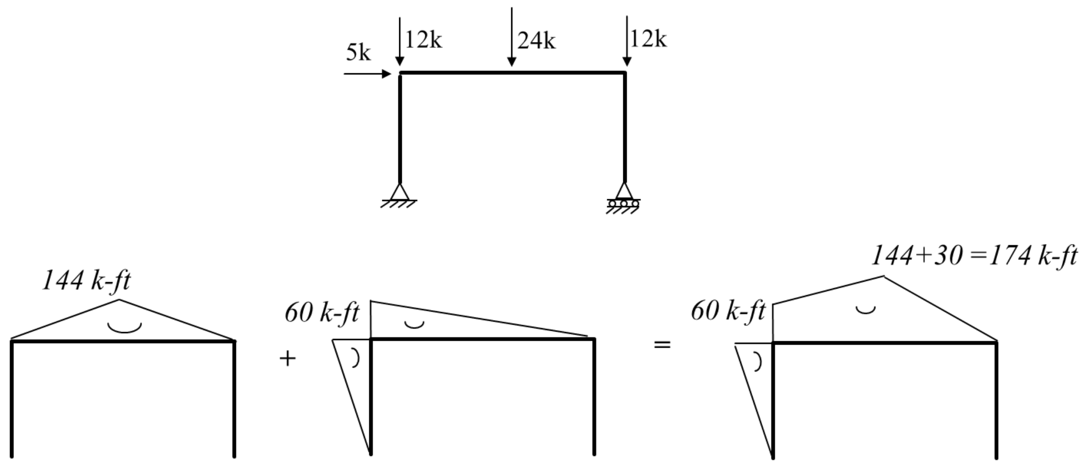



**Figure 2.23 Solution of moment diagrams for combined vertical and lateral loading of a frame.**

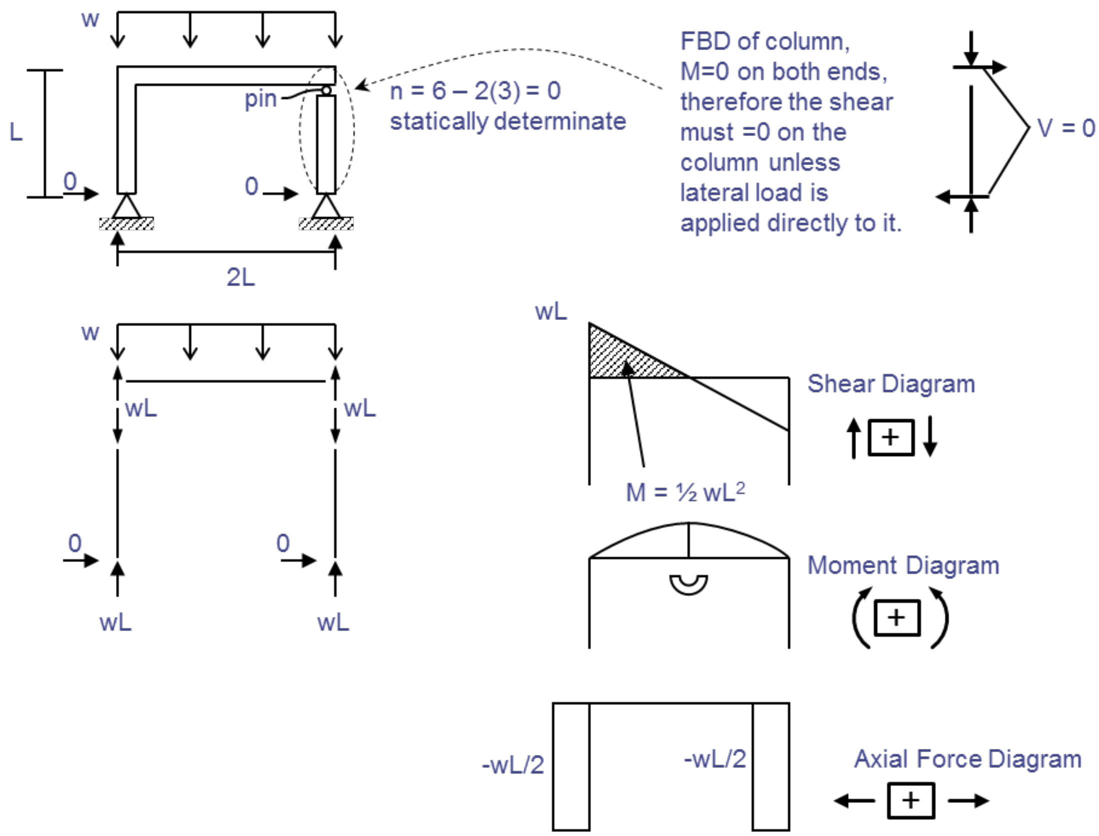
There are two important conditions for which the principle of superposition does not hold:

1) It does not hold for structures whose geometry changes significantly under imposed loads; and
2) It does not hold for structures whose materials respond in the nonlinear range under either the individual or the combined loadings.

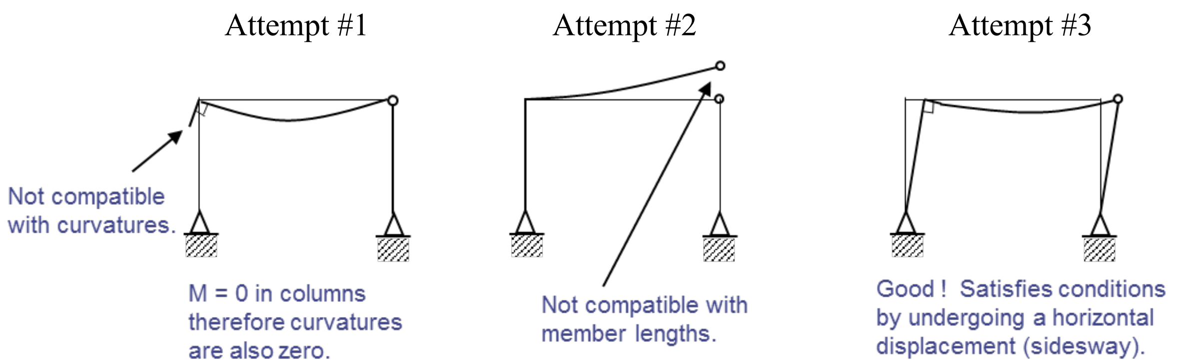
The first condition is almost always satisfied for properly designed beams and frames, but it is generally not satisfied for cable structures whose geometry depends on the loading. The second condition strictly is not satisfied for design earthquake loadings because structures may well yield under such loads. Regardless, it is common to use the principle of superposition even for earthquake loadings even if the required condition of linearity is not satisfied.

## 2.10. FRAME EXAMPLES

**Example 1**: Find the reactions, internal force diagrams, and displaced shape of the frame shown.

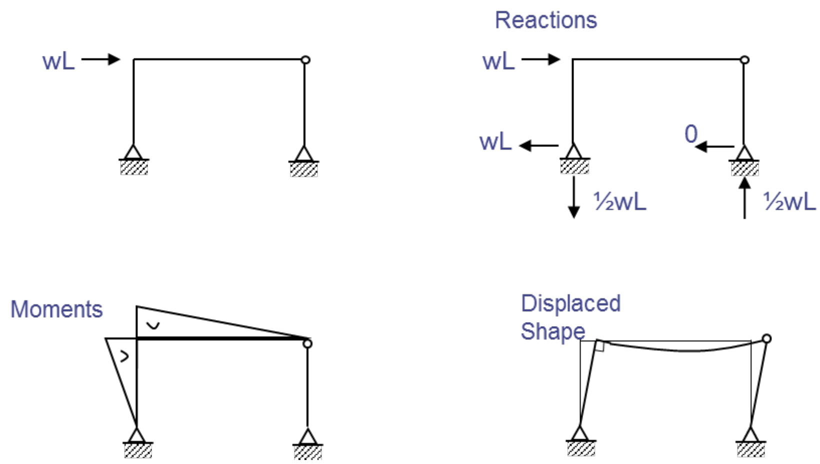



Determination of the displaced shape may require some iterations, as it may not be immediately obvious how to make the shape compatible with the moment diagram and the boundary conditions. In Attempt #1, the curvature of the beam is imposed. It is immediately seen that the left column is pointing away from the lower left support, which is incorrect. In Attempt #2, the left column is held in place and the beam curvature is imposed, resulting in uplift of the right end of the beam relative to the right-hand column, which is incorrect. In Attempt #3, we rotate the structure to the right to bring the upper right-hand pins back into contact. Checking, the members are on their supports, the columns are straight as required because there is no moment in them, and the rigid corner in the upper left retains its right angle. This is the correct displaced shape.

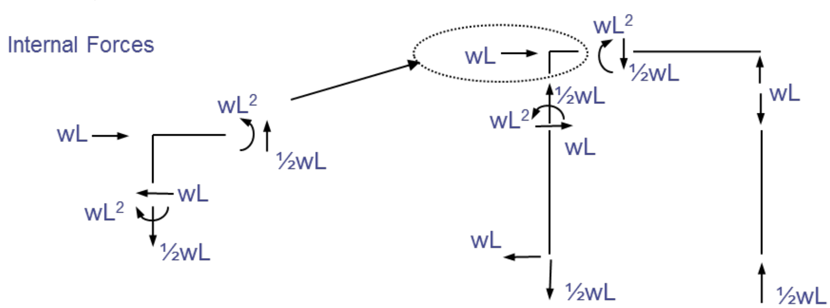


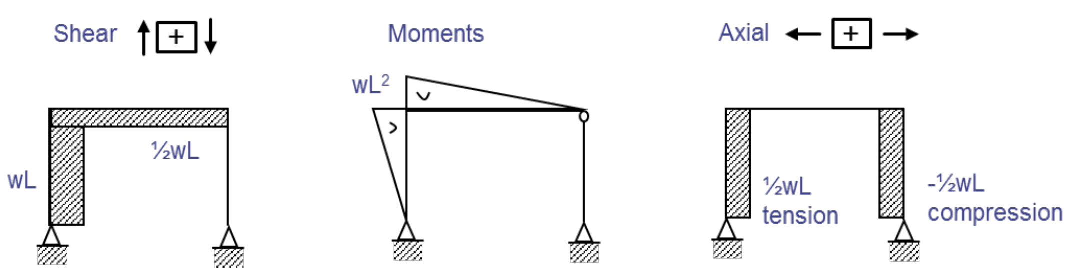

**Example 2**: Reconsider the frame of example 1, but with lateral force wL applied as shown. Determine the reactions, internal forces, and displaced shape.

The reactions are obtained by sequentially summing moments about the lower left support, about the lower right support, recognizing that the shear in the right-hand column is zero, and summing forces in the horizontal direction to obtain the left-hand horizontal reaction.



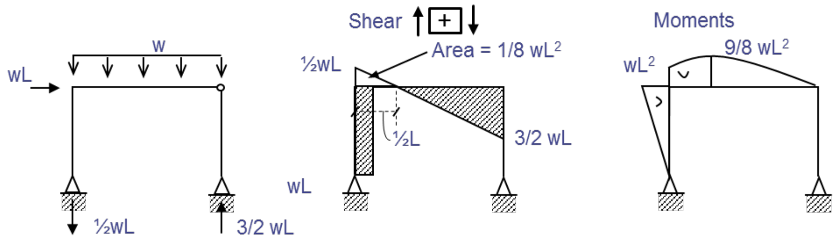

Calculate moments, shears and axial loads.



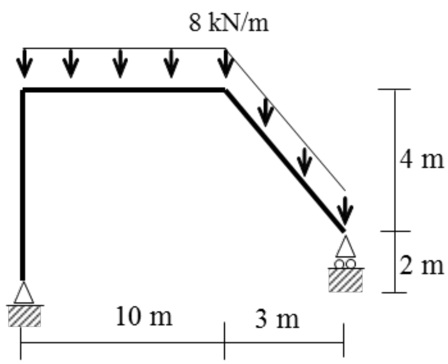
**Example 3**: Combine Gravity and Lateral Loads

Superpose results of Examples 1 and 2.




**Example 4**: Frame with inclined member

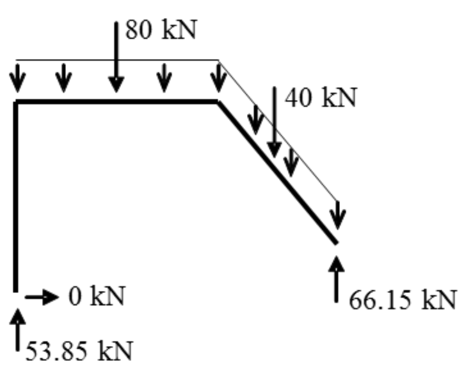
Structure and loading



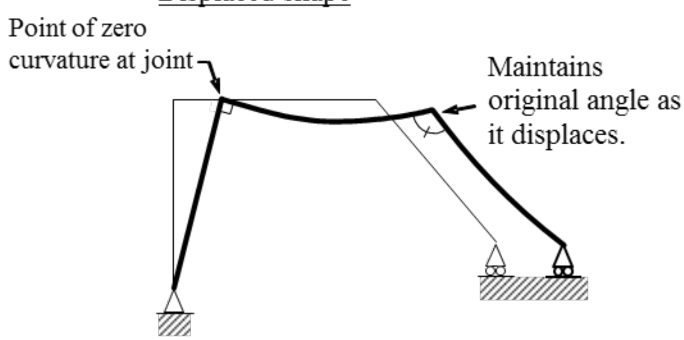

Steps to Analyze Frame

1. Determine reactions
2. Determine displaced shape
3. Determine internal forces
   A. Moment (On first pass, consider values at ends and midspan of members, not maxima/minima)
   B. Shear
   C. Axial

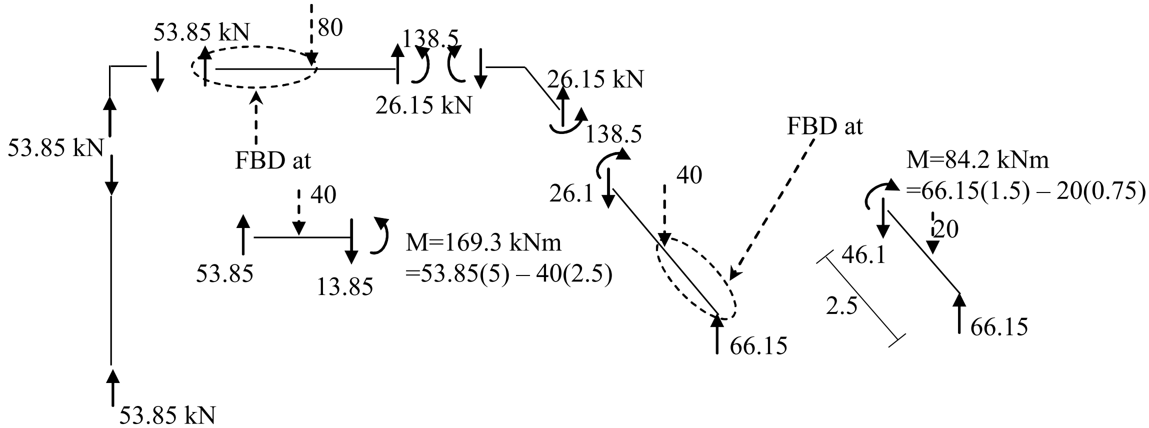
Reactions



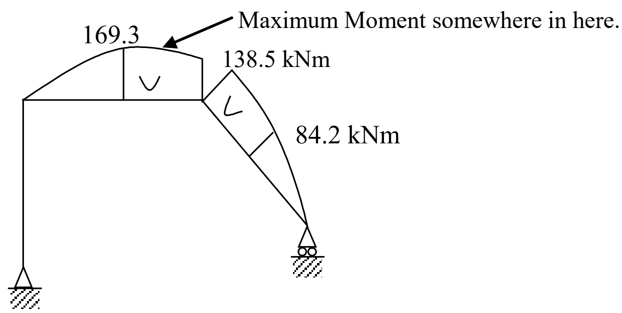

Displaced shape



## Draw Moment Diagram



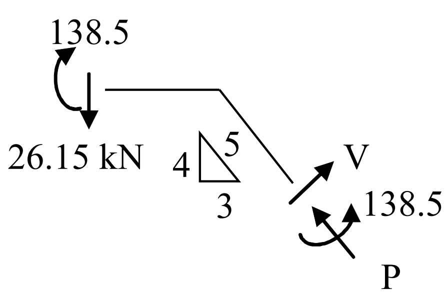

Moment Diagram (Check and complete!)



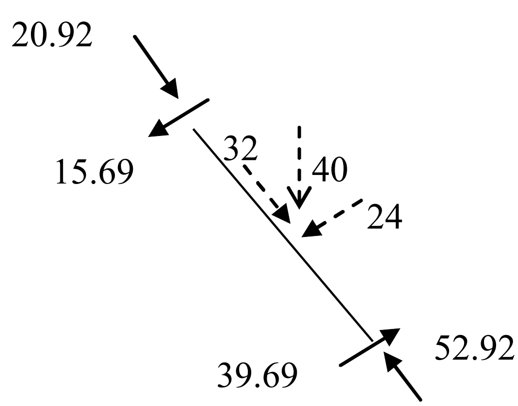

## Determine Shear and Axial Diagrams (Resolve forces at angle.)

$P = \frac{4}{5}(26.15\ kN) = 20.92\,kN$ $\qquad$ $V = \frac{3}{5}(26.15\,kN) = 15.69\,kN$

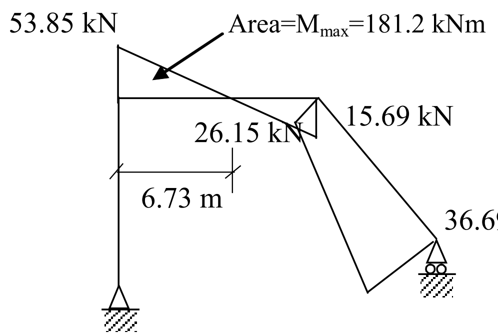
Joint Equilibrium



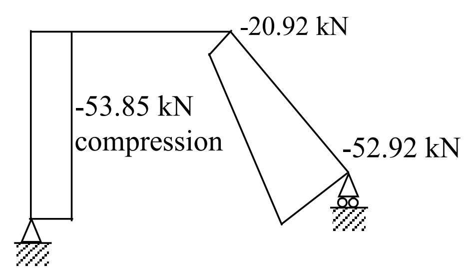

Inclined Member (Axial and Shear)



Shear Diagram

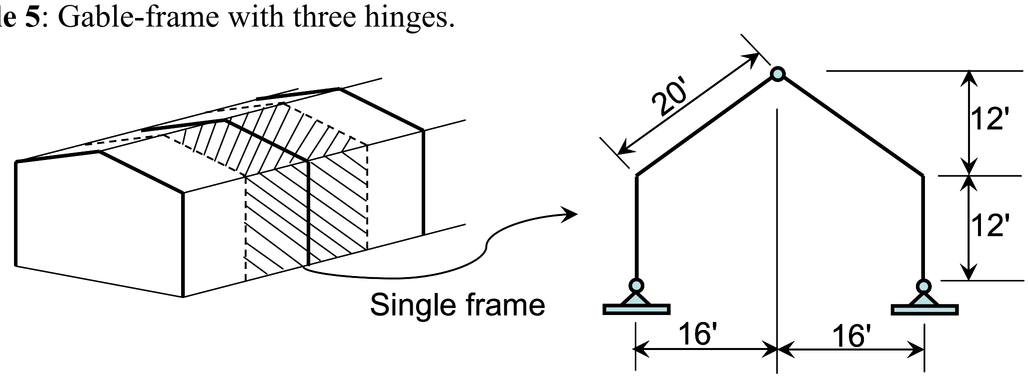



Axial Force Diagram



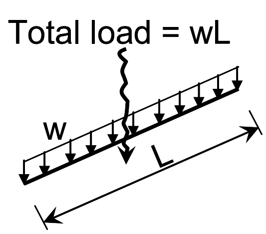

Note that in the inclined member the shear does not reduce to zero. Therefore, the maximum moment does not occur along the length but instead is at the top end of the member (138.5 kNm). The shear diagram has a negative slope so our moment diagram shape is correct.

**Example 5**: Gable-frame with three hinges.

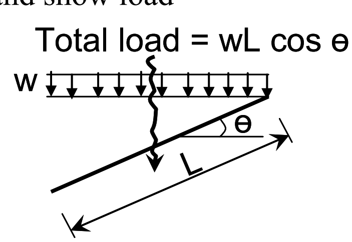



Types of distributed load on inclined member:

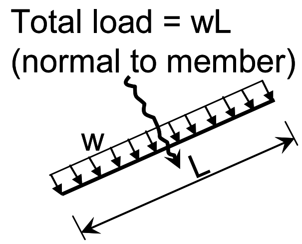
1. Load per unit length, e.g. dead load



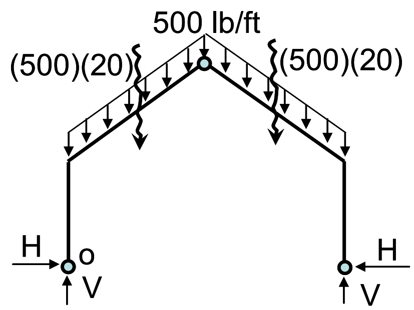

2. Projected load, e.g. live and snow load



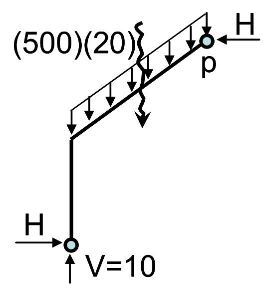
3. Normal load, e.g. wind load



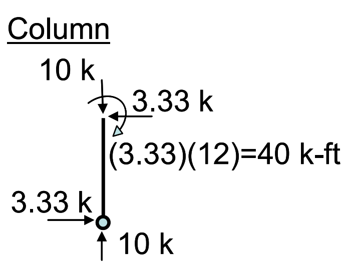

**Dead load analysis**

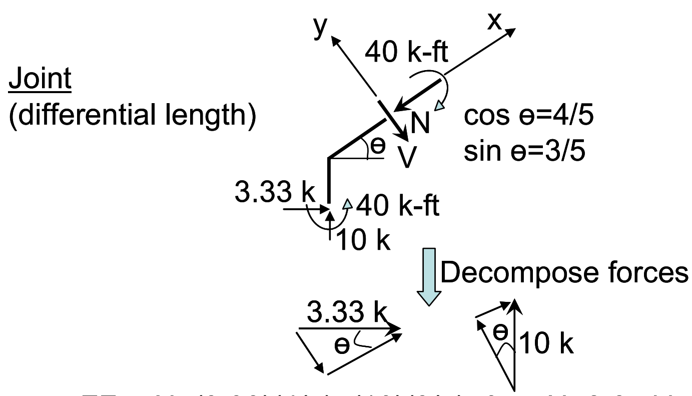



$\circlearrowleft + \Sigma M_o = (0.5)(20)(8) + (0.5)(20)(24) - V(32) = 0$
$\rightarrow V = 10$ kips

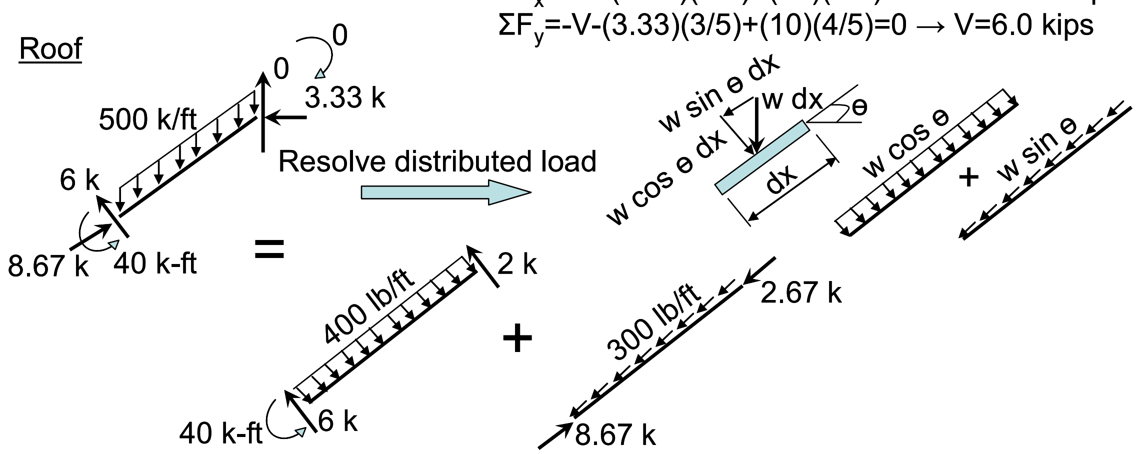



$\circlearrowleft + \Sigma M_p = -(0.5)(20)(8) + 10(16) - H(24) = 0$
$\rightarrow H = 3.33$ kips

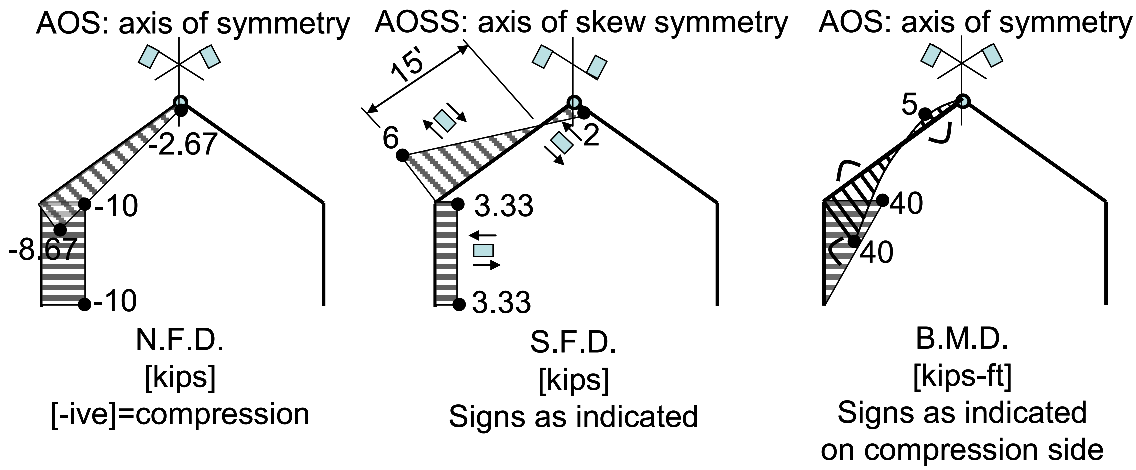


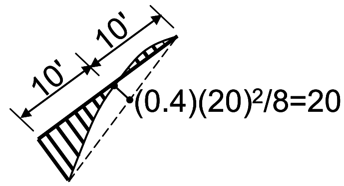

$\Sigma F_x = -N + (3.33)(4/5) + (10)(3/5) = 0 \rightarrow N = 8.67$ kips
$\Sigma F_y = -V - (3.33)(3/5) + (10)(4/5) = 0 \rightarrow V = 6.0$ kips



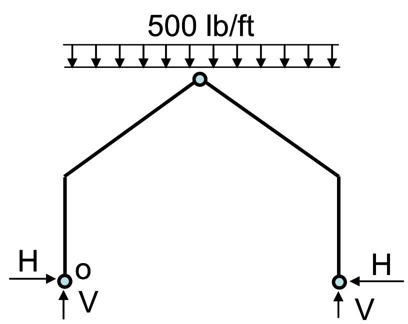
Internal Force Diagrams

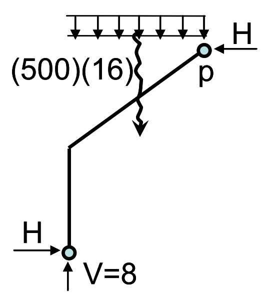



Note:



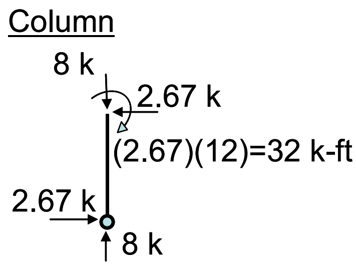

**Live Load Analysis**

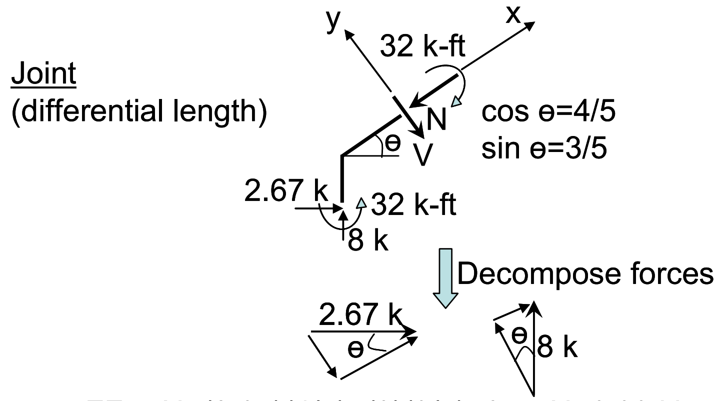


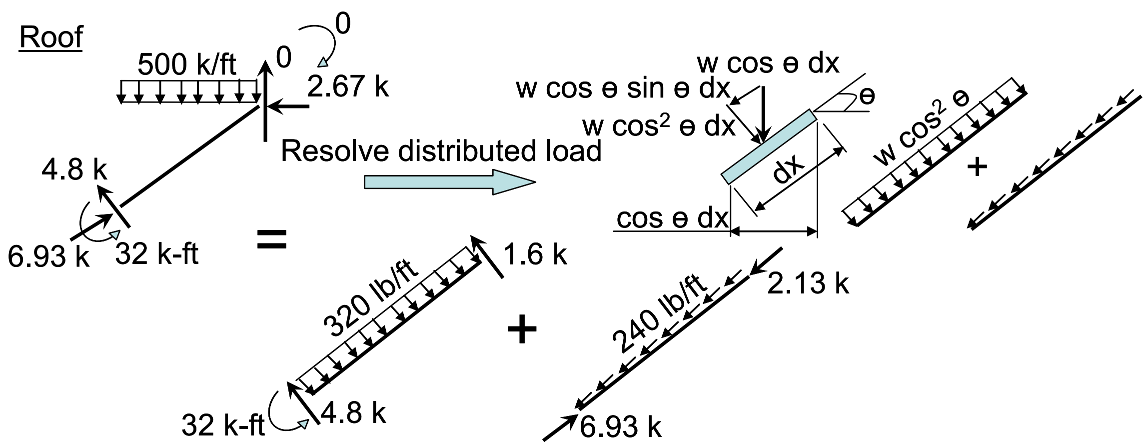

$\circlearrowleft + \Sigma M_o = (0.5)(32)(16) - V(32) = 0$
$\rightarrow V = 8$ kips

$\circlearrowleft + \Sigma M_p = -(0.5)(16)(8) + 8(16) - H(24) = 0$
$\rightarrow H = 2.67$ kips

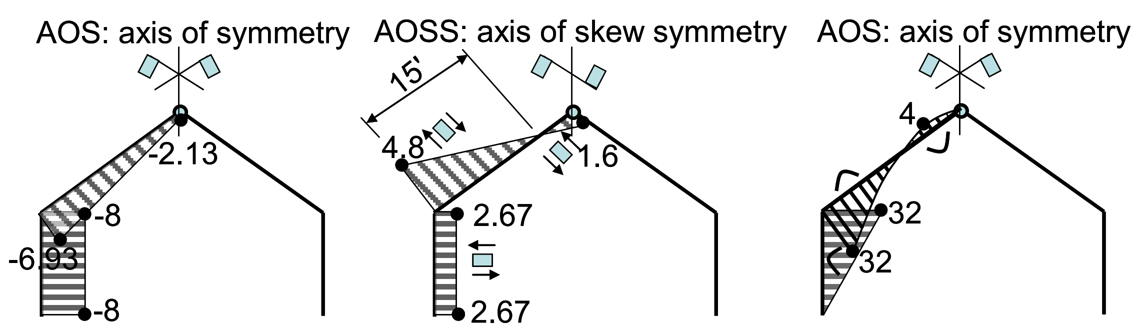



$\Sigma F_x$=-N+(2.67)(4/5)+(8)(3/5)=0 → N=6.93 kips
$\Sigma F_y$=-V-(2.67)(3/5)+(8)(4/5)=0 → V=4.8 kips




Internal Force Diagrams

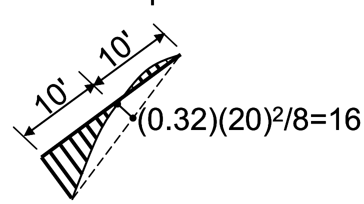



N.F.D.
[kips]
[-ive]=compression

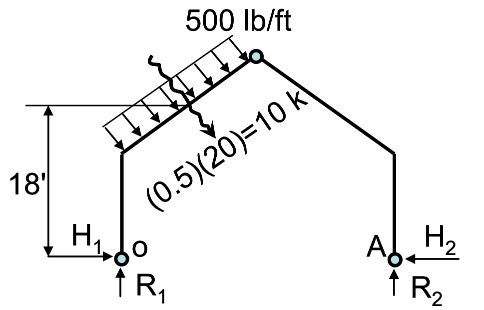
S.F.D.
[kips]
Signs as indicated

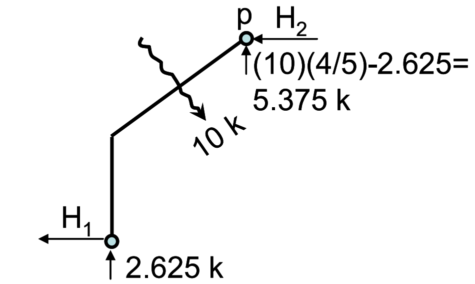
B.M.D.
[kips-ft]
Signs as indicated
on compression side

Note:



## Wind Load Analysis



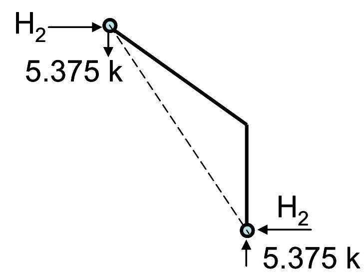




$+\circlearrowleft \Sigma M_p=(10)(10)-2.625(16)-H_1(24)=0$
$\rightarrow H_1=2.417$ kips

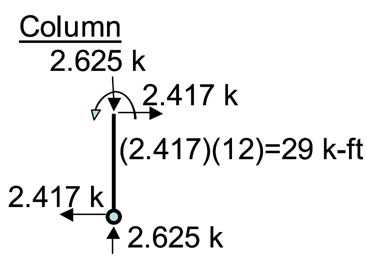
$+\circlearrowleft \Sigma M_o=(10)(4/5)(8)+(10)(3/5)(18)-R_2(32)=0$
$\rightarrow R_2=5.375$ kips

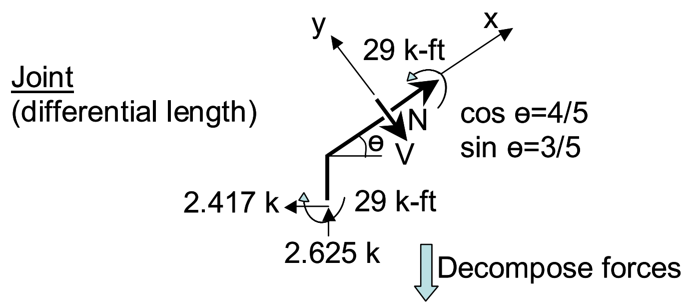
$+\circlearrowleft \Sigma M_A=(10)(4/5)(24)-(10)(3/5)(18)-R_1(32)=0$
$\rightarrow R_1=2.625$ kips

Check: $-(10)(4/5)+5.375+2.625=0.0 \rightarrow$ OK

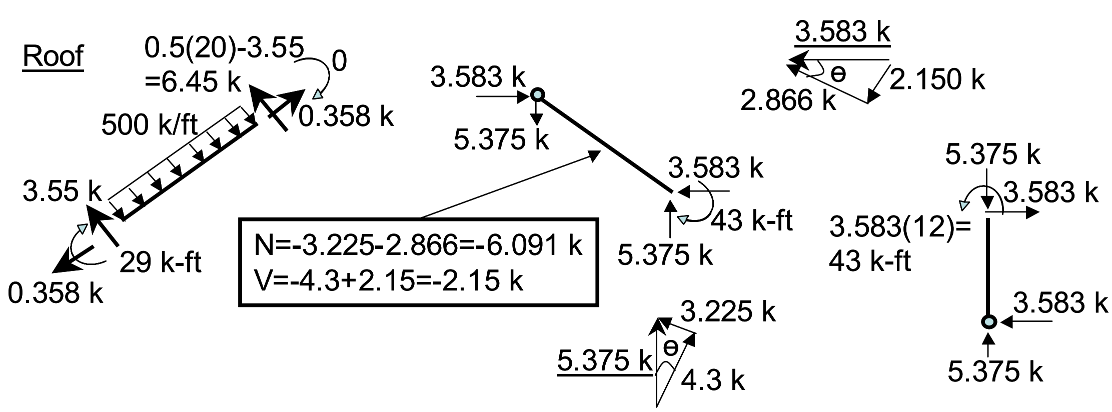



$H_2:5.375=16:24$
$\rightarrow H_2=(5.375)(16/24)=3.583$ kips

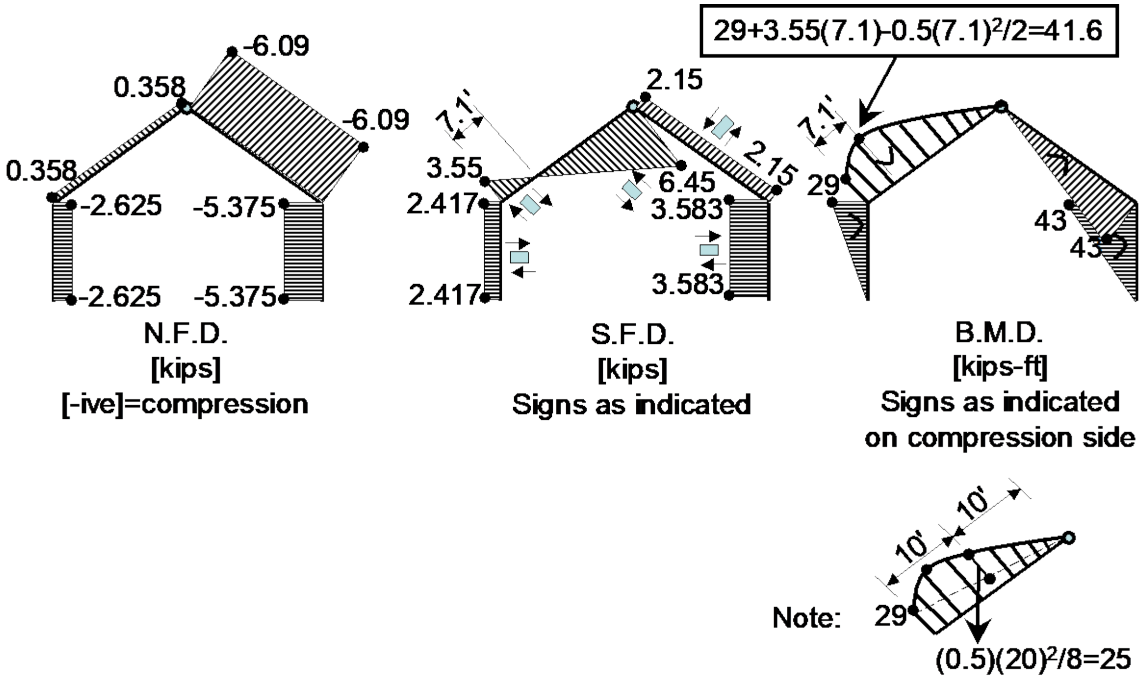


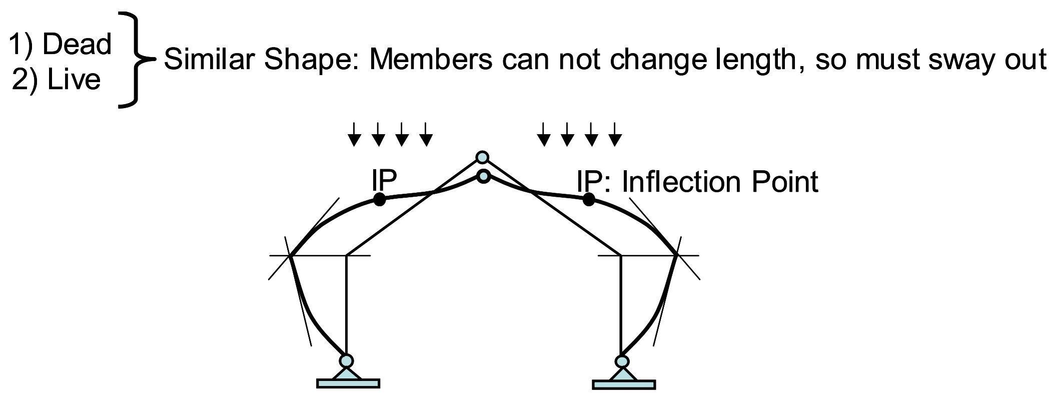

$\Sigma F_x=N-(2.417)(4/5)+(2.625)(3/5)=0 \rightarrow N=0.358$ kips
$\Sigma F_y=-V+(2.417)(3/5)+(2.625)(4/5)=0 \rightarrow V=3.55$ kips



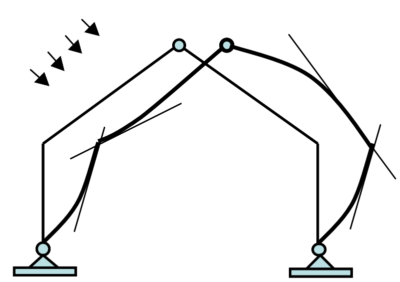
Internal Force Diagrams




Displaced Shape of Gable Frame




3) Wind Load



# 3. Analysis of Truss Structures

## 3.1. TRUSS STRUCTURES

Truss structures are frameworks of pin-connected members. In a *simple truss*, the truss members are arranged in triangular patterns. The simplest of such truss structures comprise three members with three joints and two supports (Figure 3.1a). Simple trusses can be constructed from this simplest truss by adding one joint and two members (Figure 3.1b). Other truss types can be assembled by combining simple trusses, as in the compound truss, or using other complex truss configurations. Our main interest here is simple trusses, as these are the most common in current practice.

Trusses can be made of wood or steel. Reinforced concrete trusses are unusual.

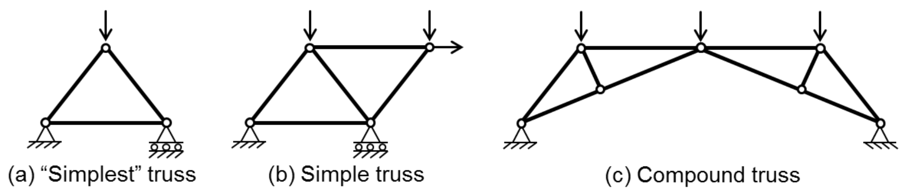
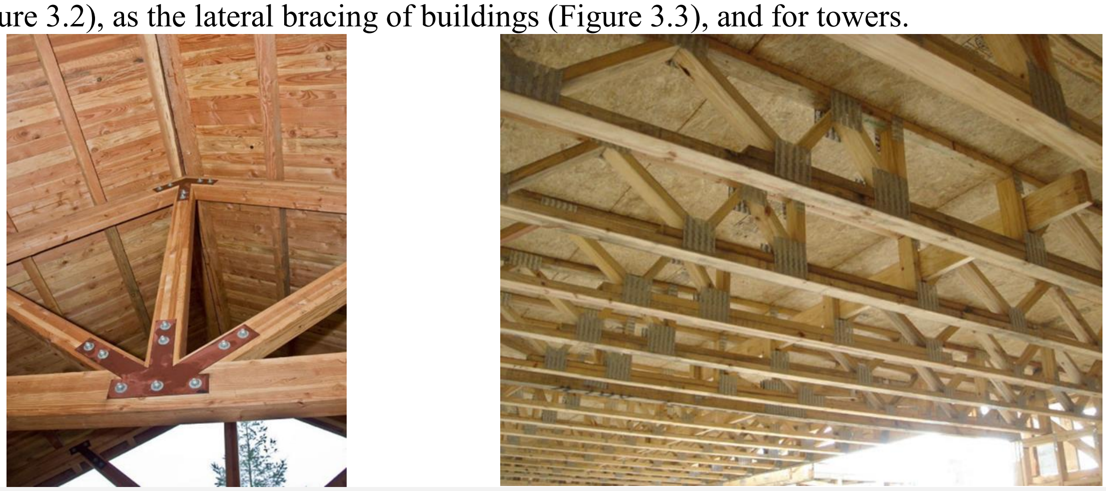



**Figure 3.1 Some idealized trusses.**

Trusses were very common for bridges in the past, but are less common now. Today they are more commonly used in residential construction, both for roof structures and prefabricated floor joists (Figure 3.2), as the lateral bracing of buildings (Figure 3.3), and for towers.



(a) (b)

**Figure 3.2 Wood trusses. (a) roof truss. (b) floor truss-joist.**

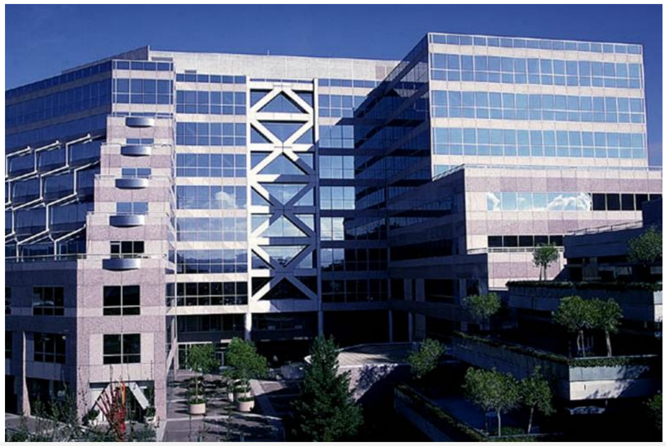


**Figure 3.3 Steel braced frame (actually a truss) as lateral-force-resisting system in building.**

In an idealized truss, the members are connected by pins that do not transmit moment. In many real trusses, however, the members are connected by hardware that enables moment transfer (Figure 3.4). Most trusses do not develop significant deformations, such that the rotations and resulting moments at these connections are relatively small. In practice, such connections are commonly treated as if they are ideally pinned, without significant errors in the resulting member forces.

(a)

(b)

**Figure 3.4 Lift bridge, Sacramento River Delta. (a) Overall view. (b) Typical connection.**

Ideally, loads are only applied at the joints of trusses. With this assumed loading, and the assumption of pinned connections, truss members only resist axial forces, either tensile or compressive. In residential construction, however, it is not unusual for floor and roof trusses to support some distributed roof or floor loads between the joints, resulting in bending moments in the members that should be considered in design.

This reader often considers structures as two-dimensional objects (it is a convenience when discussing this subject on a 2-dimensional sheet of paper or computer screen). However, actual structures exist in three dimensions. Figure 3.5 depicts a bridge truss structure in three dimensions. The type of truss depicted on either side of the deck is known as a Pratt truss (its diagonals slope downward toward the center). (See http://en.wikipedia.org/wiki/Truss_bridge for a summary of different bridge truss types.) The floor beams of the deck are located such that they transmit their reactions at the joints along the bottom chord of the truss. Under this loading, the top chord of each truss is in compression. The compressive force will cause the pin-ended compression chord to buckle unless it is braced by another truss that connects the top chords of the two trusses. The trusses also require some lateral bracing to prevent the entire truss from flopping over on its side. Sway bracing is provided to support the trusses against this failure mode.

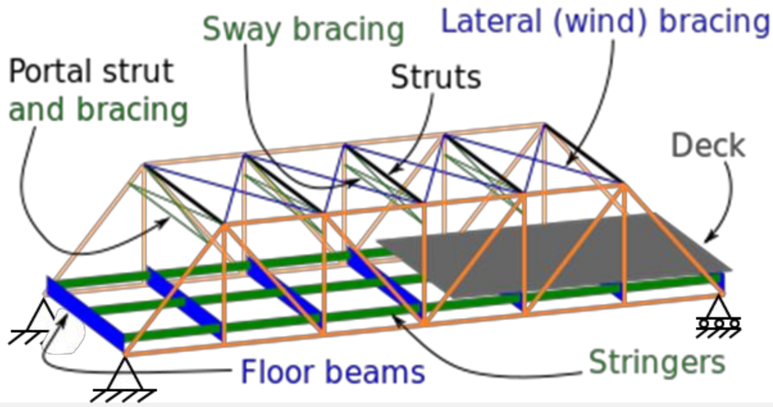



**Figure 3.5 Parts of a truss bridge structure. (Science and Industry, Members of a Truss Bridge by Benj. F. La Rue, Home Study Magazine, Published by the Colliery Engineer Company, Vol 3, No. 2, March 1898, pages 67-68.)**

## 3.2. STATIC DETERMINACY

For each joint in a truss, two equations can be written, specifically $\sum F_x = 0$ and $\sum F_y = 0$. Thus, the number of equations is equal to 2j, where j = the number of joints. For each member there is one unknown force, plus there are unknown forces for each reaction. Defining r = number of members plus number of unknown reactions, we can write that the degree of static indeterminacy is

$$n = r - 2j \tag{3.1}$$

where n = degree of static indeterminacy
r = number of unknown forces = number of members plus number unknown reactions
j = number of joints in the truss.

## 3.3. STABILITY

A truss structure having n < 0 [see Eq. (3.1)] is classified as being unstable. Similar to beams and frames, however, one must also check whether the reactions are co-linear (such that the truss cannot resist forces perpendicular to the co-linear direction), whether all of the reactions point to a single point (in which case the truss cannot resist moment about that point), or whether there is an internal instability. Figure 3.6 shows five examples, with the following key points

a) This truss structure has 5 joints, 7 members, and three unknown reactions. It is stable and determinate.
b) This truss structure is indeterminate to the 1st degree.
c) Even though n = 0, this truss structure is unstable because the reactions are all co-linear. In this case, the truss cannot resist horizontal forces.

d) In this structure, the projection of all of the reactions coincides at point o. Thus, this structure cannot resist moment about point o, and is therefore unstable.
e) Although n = 0, this truss is unstable because panel bcfg lacks a diagonal member and will form a mechanism. The reason this is unstable even though n = 0 is because panel cdgh has an extra diagonal. That panel actually is indeterminate, but that is not relevant given that the structure as a whole is unstable.

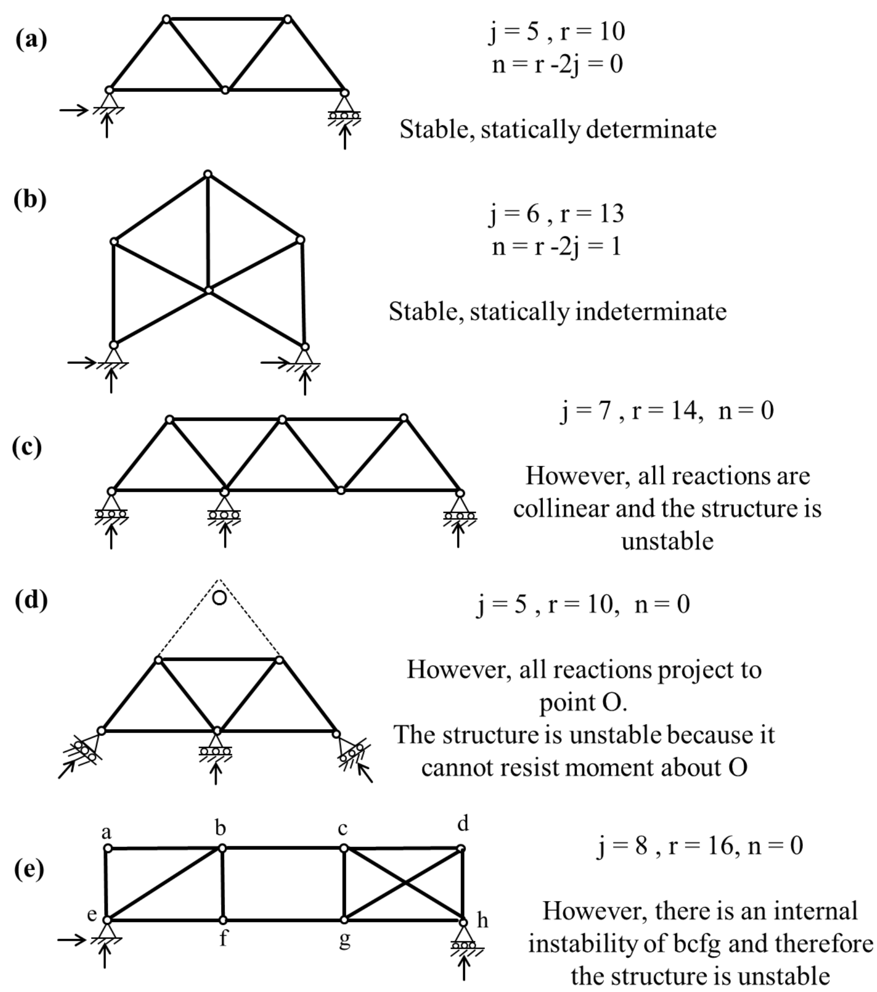



**Figure 3.6 Stable and unstable truss examples.**

## 3.4. TRUSS ANALYSIS

There are two traditional methods of analysis for statically determinate trusses, the *method of joints* and the *method of sections*. These are illustrated in the following sections.

### 3.4.1. Method of joints

The method of joints is based on the requirement that all joint must be in equilibrium. The method of joints can be an efficient method when it is required to find the forces in all members of a truss. The procedure generally proceeds along the following lines:

1. Find the reactions.
2. Proceed to find the member forces working joint by joint.
3. Check that the joints are in equilibrium with the previously calculated reactions.

Figure 3.7 presents an example.

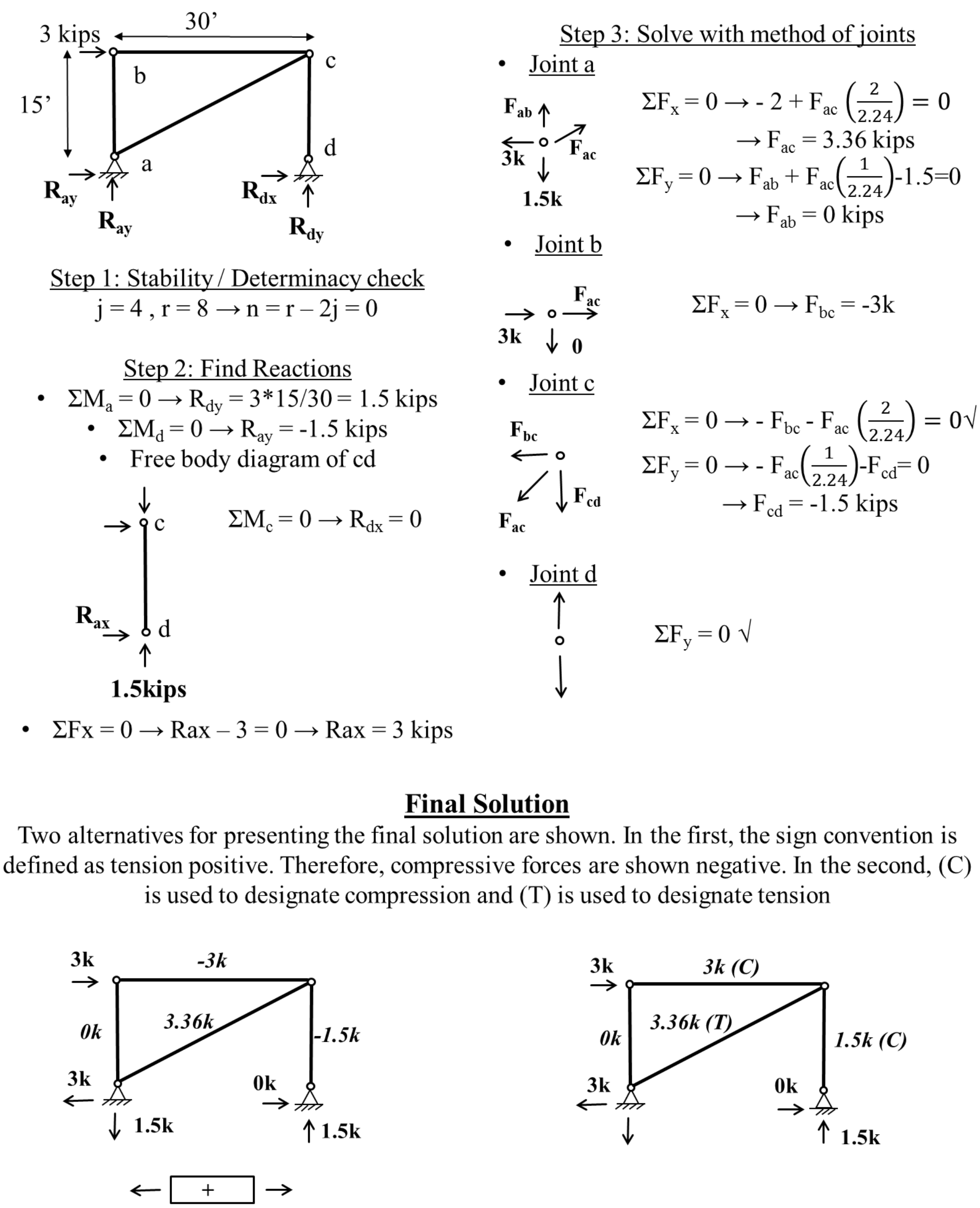



**Figure 3.7 Truss structure solution by method of joints.**

### 3.4.2. Method of sections

The method of sections is based on the requirement that all free body diagrams (FBDs) must be in equilibrium. By strategically selecting FBDs, it may be possible to isolate unknown forces in individual members and solve them directly. The method of sections can be an efficient method when it is required to find the forces in all members of a truss, and it can be especially efficient if only a few forces are required. The procedure generally proceeds along the following lines:

1. Find the reactions. Note that sometimes it is not necessary to find the reactions in order to find the member forces.
2. Create FBDs to isolate member forces and use equilibrium to solve for them.

Figure 3.8 and Figure 3.9 present examples.

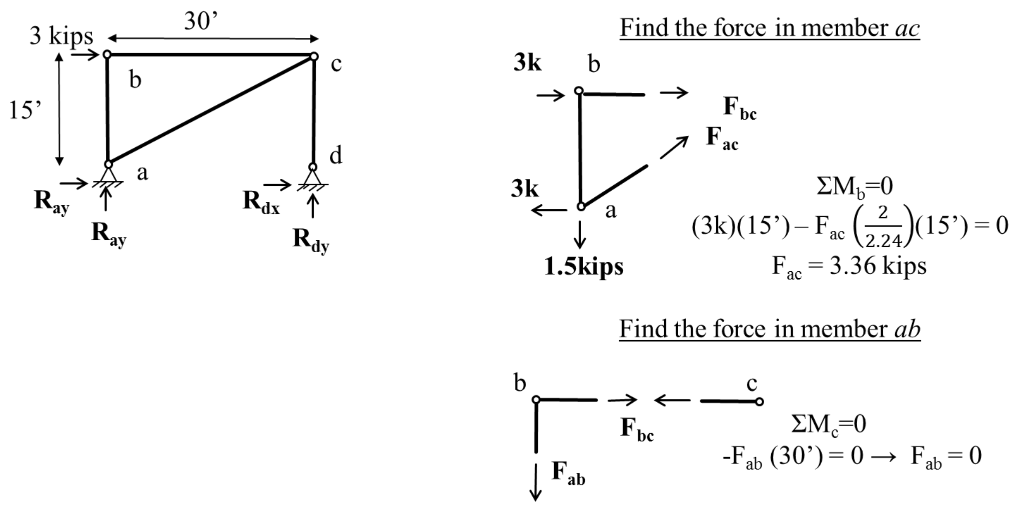
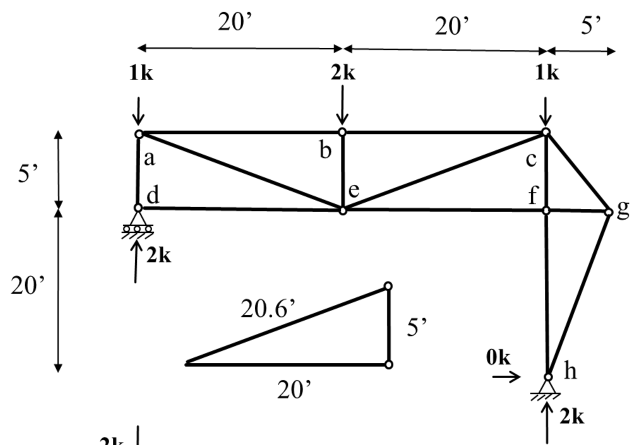
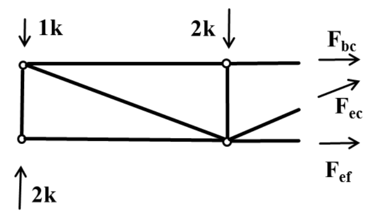



**Figure 3.8 Truss structure solution by method of sections. Example 1**




<u>Find the force in member *ac*</u>

$$\Sigma F_y = 0$$

$$2k - 1k - 2k + F_{ec}\left(\frac{5}{20.6}\right) = 0$$

$$F_{ec} = 4.12 \text{ kips}$$

<u>Find the force in member *bc*</u>

$$\Sigma M_e = 0$$

$$(2\text{-}1k)(20') + F_{bc}(5') = 0$$

$$\rightarrow F_{bc} = -4k \text{ (compression)}$$

**Figure 3.9 Truss structure solution by method of sections. Example 2.**

# 4. Analysis of Cable Structures

## 4.1. CABLE STRUCTURES

Cables are used in many engineered structures. They are the main load carrying elements in suspension bridges and cable-stayed bridges (Figure 4.1). They are used as elements of tensile membranes, which are a structural form for long-span roofs. In addition, they are used for permanent guys on structures such as derricks and radio towers and for temporary guys during erection. Cables in permanent structures are usually made of steel, and usually comprise smaller steel wires bound together to form a larger cable. Cables (or rope) can also be made of natural or synthetic fibers.

**Figure 4.1 Suspension bridge (Golden Gate Bridge) and cable-stayed bridge (Millau Viaduct, France.**

**Figure 4.2 Tensile membrane structure (Denver International Airport).**

## 4.2. BEHAVIOR AND EQUILIBRIUM OF CABLES

Cables resist forces in pure axial tension only. Thus, a cable of negligible weight and supporting a single concentrated load must take the form of two line segments (Figure 4.3). If the load moves along the cable, the cable must change shape such that it remains in tension without shear or moment (Figure 4.3). We assume that the cable is sufficiently flexible to change shape without developing moment. This is a unique characteristic of cables that we do not find for beams, frames, or trusses.

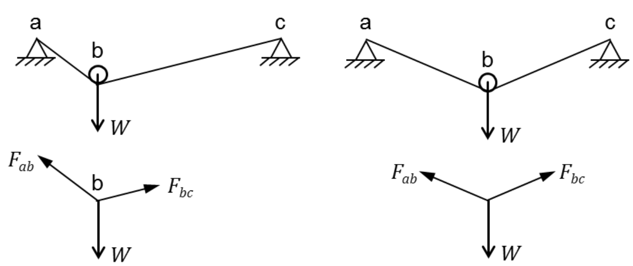



**Figure 4.3 Changing shape of a cable as a function of the position of applied loads.**

Another observation we can make about cables is that, at the point of external load application, the vertical forces in the cable must sum to balance the applied load. Thus, in Figure 4.3, the sum of the vertical components of $F_{ab}$ and $F_{bc}$ must be equal to $W$. Also, if the applied forces are only vertical, which is a common case, then the horizontal components of $F_{ab}$ and $F_{bc}$ in Figure 4.3 must be equal and opposite. In general, a horizontally spanning cable supporting vertical loads will develop a component of cable tension acting in the horizontal direction and, consequently, will require horizontal reactions to support the cable tension. The shallower the sag in the cable, the greater the horizontal reaction.

In a traditional suspension bridge (e.g., the Golden Gate Bridge), the horizontal component of tension in the main cable is resisted by an external anchorage (Figure 4.4a). In this type of bridge, the towers and anchorages are constructed first, then cable is spun between anchorage blocks, and finally the deck segments are lifted into place. In a self-anchoring suspension bridge (e.g., the Eastern replacement span of the San Francisco-Oakland Bay Bridge), the horizontal component of tension in the main cable is resisted by compression in the bridge deck (Figure 4.4b). This requires that the deck be constructed on falsework (which is later removed) before the cable is spun. A cable-stayed bridge (Figure 4.4c) supports loads by cables that extend directly from the tower, with the horizontal load resisted by the deck in compression. Construction begins with the main towers, and then segments of the deck are individually constructed and supported by cables, starting at the tower and building outward without falsework.

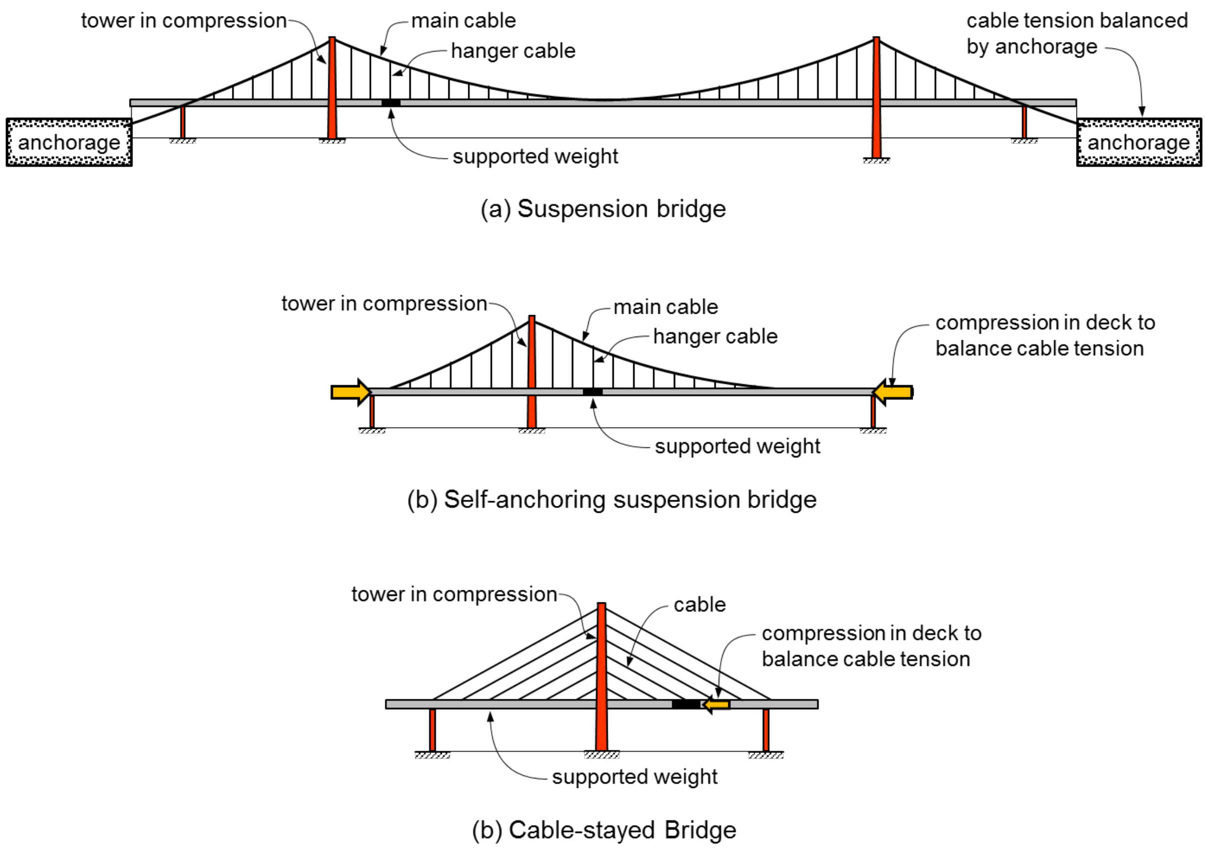


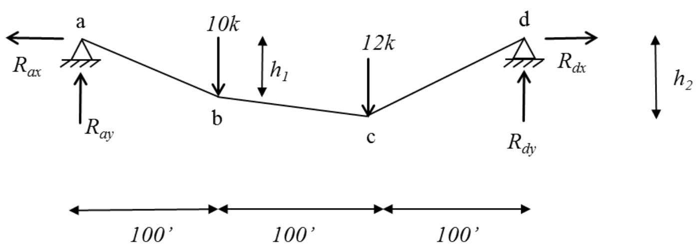

**Figure 4.4 Different types of cable-supported bridges.**

## 4.3. ANALYSIS OF CABLES UNDER CONCENTRATED LOADS

When a cable of negligible weight supports concentrated loads, it will form into a shape comprising multiple line segments. For example, consider the cable of Figure 4.5. For this cable, there are nine unknowns, these being four reactions at the supports, three cable forces, and the elevations $h_1$ and $h_2$ at the loading points. We can write two equations of force equilibrium at each of points a, b, c, and d, giving us eight equations, or one short of the number of unknowns. To complete the problem, we will need to know something about the geometry of the cable. If we know the length of the cable, we can use the Pythagorean theorem to relate the total length of the cable to each of the segment lengths, written in terms of the horizontal positions of each of the points. This type of problem is somewhat onerous to solve. A more tractable solution is obtained if the elevation of the cable at one of the loading points is known instead of the cable length. This latter approach is assumed for all of the problems in this reader. Once the cable is solved, locating all of the points, the length can be determined readily.

Figure 4.5 shows the complete solution of the cable reactions, forces, and length for the case where $h_1$ = 15 ft. The solution is completed as follows:

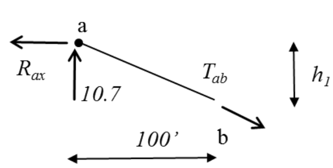
1. Moments and forces are summed on the FBD of the entire structure, producing $R_{dy}$ and $R_{ay}$.
2. To solve for the horizontal reaction at a, we create a FBD of segment ab and sum moments about point b. The cable tension $T_{ab}$ is then equal to the vector formed by $R_{ax}$ and $R_{ay}$. Because the tension force must be aligned with the cable, we can also solve $T_{ab}$ in terms of the geometry of the cable, from which we can see how the tension force varies with the sag $h_1$.
3. To solve the cable force $T_{bc}$, create a FBD of segment abc and solve for the force. Note that the height $h_2$ is determined by equilibrium.
4. Check that the cable for $T_{bc}$ is in equilibrium with the reactions at d. OK.
5. Show final solution on a sketch of the cable. The cable length is obtained using the Pythagorean theorem.




4 reactions, 3 Member forces, 2 elevations → 9 unknowns
2 equations per node (a,b,c,d) → 8 equations

Need to know another parameter ($h_1$, $h_2$ or cable length) → Suppose $h_1$ = 15':

- $\Sigma M_a = 0 \rightarrow R_{dy}$ = 11.3 kips | $\Sigma M_d = 0 \rightarrow R_{ay}$ = 10.7 kips | $\Sigma Fx = 0 \rightarrow R_{ax} = R_{dx}$



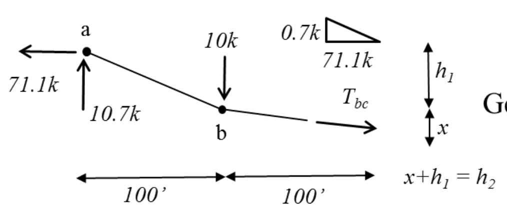

- $\Sigma M_b = 0 \rightarrow R_{ax} = \left(\frac{(10.7k)(100')}{15'}\right) = 71.1$ *kips*

  $\rightarrow T_{ab} = \sqrt{10.7^2 + 71.1^2} = 71.9$ *kips*

- $\Sigma F_y = 0 \rightarrow$ vertical component in cable must be

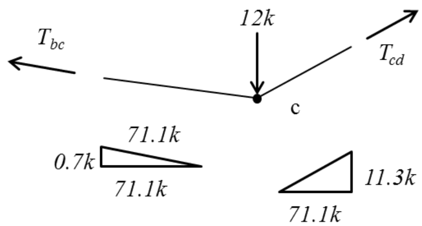
  *0.7 kips* ($T_{bc,y} = 0.7$ *kips*)

  $\rightarrow T_{bc} = \sqrt{10.7^2 + 71.1^2} = 71.9$ *kips*



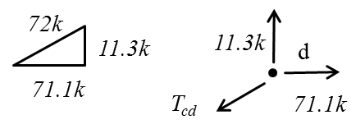

Geometry : $x/100 = 0.7/71.1 \rightarrow x = 0.98' \rightarrow h = 15.98'$

$T_{bc} = \sqrt{0.7^2 + 71.1^2} = 71.1$ kips

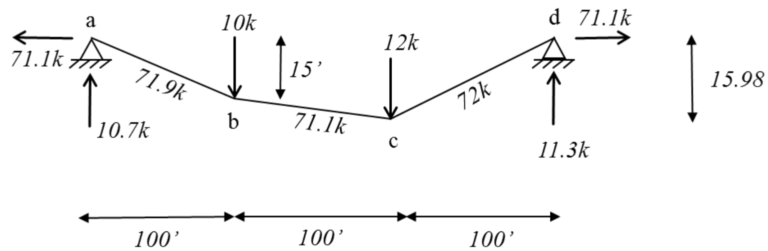



- $\Sigma F_y = 0 \rightarrow T_{cd,y} = 11.3\ kips$

$\rightarrow T_{cd} = \sqrt{71.1^2 + 11.3^2} = 72\ kips$




***Summary***



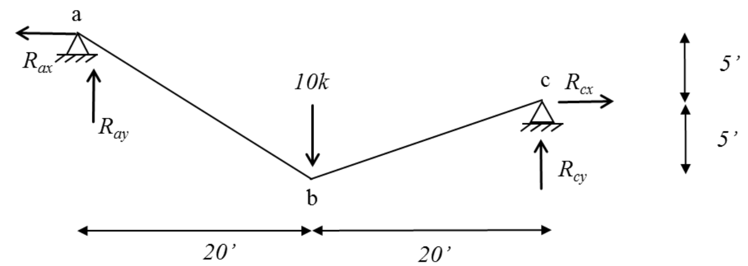

Cable length : $l = l_{ab} + l_{bc} + l_{cd} = \sqrt{15^2 + 100^2} + \sqrt{0.98^2 + 100^2} + \sqrt{15.98^2 + 100^2} = 302.4'$

**Figure 4.5 Example 1: Cable supporting concentrated forces.**

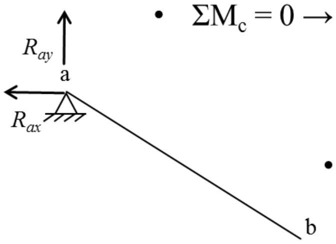
Some key points are:

1. The horizontal component of the cable force is constant provided all externally applied forces are vertical.
2. Decreasing the sag increases the cable tension and the horizontal reactions.
3. The shape (for example, $h_2$) is determined by equilibrium.

The next example considers a cable with supports at different elevations (Figure 4.6). This variation complicates the solution mildly, as the vertical reactions cannot be solved directly but must be solved using simultaneous equations. Overall, however, the approach is the same as for Example 1.




- $\Sigma M_a = 0 \rightarrow (10k)(20') - R_{cy}(40') - R_{cx}(5') = 0$ (1)
- $\Sigma M_c = 0 \rightarrow R_{ay}(40') - R_{\alpha x}(5') - (10k)(20') = 0$ (2)




- $\Sigma M_b = 0 \rightarrow R_{\alpha y}(20') - R_{\alpha x}(10') = 0$
  $\rightarrow R_{\alpha x} = 2\,R_{\alpha y}$ (3)
- Substituting (3) into (2) $\rightarrow R_{\alpha y}(40') - 2R_{\alpha y}(5') - 200 = 0$
  $\rightarrow R_{\alpha y} = \frac{200}{30} = 6.67\ kips$
- $\Sigma F_x = 0 \rightarrow\ R_{\alpha x} = R_{cx} = 13.3\ kips$
- (1) $\rightarrow R_{cy} = 3.33\ kips$
- Check $\Sigma Fy = 0 \rightarrow 6.67k + 3.33k - 10k = 0$

Cable length : $l = l_{ab} + l_{bc} = \sqrt{20^2 + 10^2} + \sqrt{20^2 + 5^2} = 42.97'$

- $T_{ab} = \sqrt{R_{\alpha x}{}^2 + R_{\alpha y}{}^2} = 14.9\ kips$
- $T_{bc} = \sqrt{R_{cx}{}^2 + R_{cy}{}^2} = 13.7\ kips$

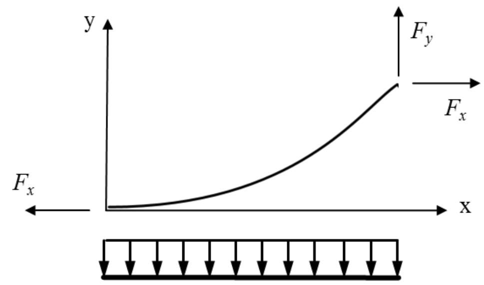
**Figure 4.6 Example 2: Cable supporting vertical forces.**

## 4.4. ANALYSIS OF CABLES UNDER UNIFORM HORIZONTALLY DISTRIBUTED LOADS

Cables provide an effective means of supporting uniformly distributed loads over long spans, as is done in suspension bridges. We first need to establish the shape of a cable under uniform horizontally distributed load. Consider the cable shown in Figure 4.7, with x, y = (0, 0) at point of zero slope. Summing moments about point "o", we arrive at the equation for the shape of the cable as

$$y = \frac{w}{2F_h}x^2 \tag{4.1}$$

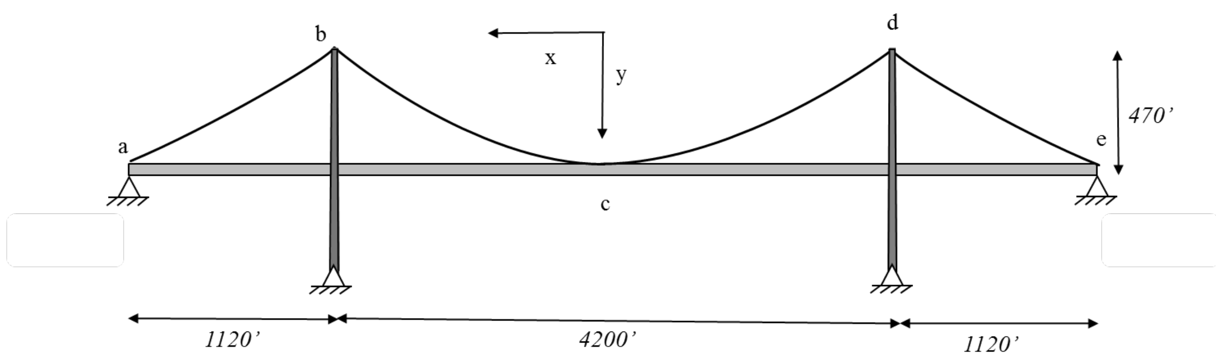
Thus, a cable under uniform horizontally distributed load is in the shape of a parabola.




$$\Sigma M_O = 0 \rightarrow F_x \cdot y - \frac{w \cdot x^2}{2} = 0 \rightarrow y = \frac{w \cdot x^2}{2\,Fx}$$

**Figure 4.7 Cable under uniform horizontally distributed load.**

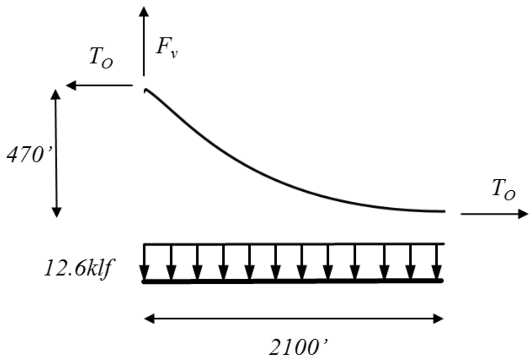
We can use statics to analyze complicated suspension bridge structures. In the following pages, we conduct an approximate analysis of the Golden Gate Bridge. (The main approximation is ignoring the weight of the cable.) Figure 4.8 shows the geometry and loading. The bridge deck is found to weigh 25.2 klf, or 12.6 klf per cable. The cable itself weighs 3.4 klf. Although not negligible in the final design, for a preliminary analysis we will ignore the weight of the cable.




Cable 3ft in diameter $\rightarrow A = 1000\ in^2$, $w = 3.4\ klf$
*Deck / Cable = 12.6 klf*

**Figure 4.8 Geometry and loading of the Golden Gate Bridge.**

We first analysis for the cable forces in the center span. Summing forces in the vertical direction produces the vertical force in the cable just to the right of the tower at point b. At midspan, the cable must be horizontal, so the only unknown is the horizontal force $T_o$. Summing moments about point b produces the force $T_o$ at mid-span. Because there are no horizontal external forces applied to the bridge, the horizontal force in the cable just to the right of the tower at point b must also be $T_o$.

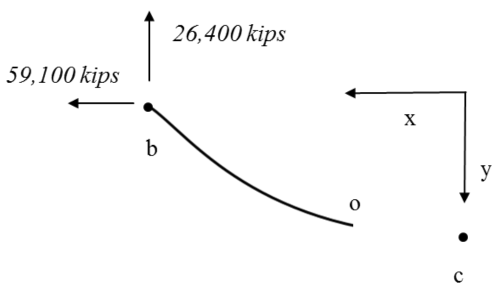



$$\Sigma F_y = 0 \rightarrow$$
$$F_v - (12.6klf) \cdot (2100') \rightarrow$$
$$F_v = 26{,}400\ kips$$

$$\Sigma M_b = 0 \rightarrow$$
$$(12.6) \cdot \frac{(2100)^2}{2} - To(470') = 0 \rightarrow$$
$$T_O = 59{,}100\ kips$$

**Figure 4.9 Cable forces in the center span of the Golden Gate Bridge.**

We next solve for the geometry of the center span. This is done by summing moments about point "o" along the span, as shown in Figure 4.10. The shape is a parabola.

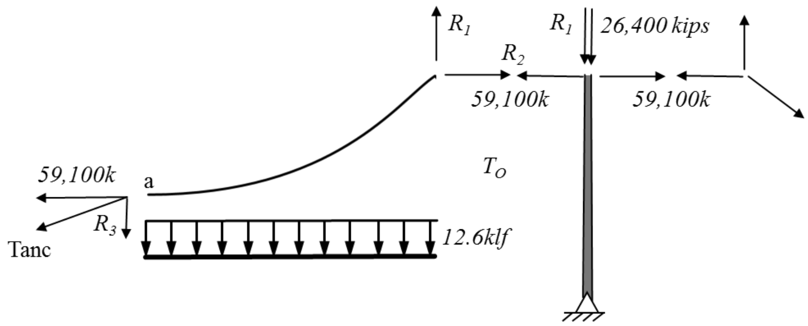



$$\Sigma M_o = 0 \rightarrow$$
$$(26{,}400k) \cdot (2100' \cdot x) - (59{,}100k) \cdot y - (12.6) \cdot \frac{(2100'-x)^2}{2} = 0$$

$$y = 470'\left[1 - \left(\frac{x}{2100'}\right)^2\right]$$

**Figure 4.10 Geometry of the center span of the Golden Gate Bridge.**

We next solve for the forces in the side span, as shown in Figure 4.11. We do not want the tower to resist forces along the longitudinal axis of the bridge, so the system should be designed such that the horizontal component of the cable force just to the left of point b is also $T_o$ = 59,000 kips, thereby balancing the horizontal force to the right of point b. Summing moments about point a, we calculate the vertical force in the cable to the left of point b. The cable tension to the left of point b is obtained as the vector sum of the horizontal and vertical forces. At the anchorage, the horizontal component of the cable force remains 59,100 kips. We can find the vertical force at point a by summing forces in the vertical direction. The cable tension at the anchorage is also determined.




$$\Sigma M_a = 0 \rightarrow$$
$$(-1120') \cdot (R_1) + (59{,}100k) \cdot (470') + (12.6klf) \cdot \left(\frac{(1120')^2}{2}\right) = 0 \quad \rightarrow \quad R_1 = 31{,}860 \text{ kips}$$

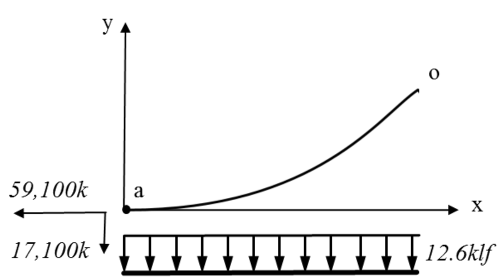
$$T_b = \sqrt{31{,}860^2 + 59{,}100^2} = 67{,}100 \text{ kips}$$

$$\Sigma F_y = 0 \rightarrow R_3 - (12.6klf) \cdot (1120') = 0 \rightarrow R3 = 17{,}700 \text{ kips}$$

$$T_{anc} = \sqrt{17{,}700^2 + 59{,}100^2} = 61{,}700 \text{ kips}$$

**Figure 4.11 Forces in the side span of the Golden Gate Bridge.**

The geometry of the side span similarly is determined by equilibrium (Figure 4.12). We can differentiate the equation for the geometry to obtain the slope, which tells us the ratio of the vertical to horizontal cable forces at any point.




$$\Sigma M_o = 0 \rightarrow$$
$$(59{,}100k) \cdot (y) - (17{,}700k) \cdot x - (12.6klf) \cdot \frac{x^2}{2} = 0$$

$$y = \frac{x}{3.33\,ft} + \frac{x^2}{9380\,ft^2} \rightarrow$$
$$\frac{dy}{dx} = \frac{1}{3.33} + \frac{x}{4690}$$

Check at point a: $\frac{dy}{dx} = \frac{1}{3.33} = \frac{17{,}700}{59{,}100}$

Check at point b: $\frac{dy}{dx} = 0.539 = \frac{31{,}860}{59{,}100}$

**Figure 4.12 Geometry of the side span of the Golden Gate Bridge.**

Now that we have solved for the cable forces, we can check the cable stresses. The maximum cable force is $T_{max}$ = 67,100 kips. Given the cable area of A = 1000 in.$^2$, the cable stress is $\sigma =$

$67{,}100/1000 = 67$ ksi. The allowable stress for the steel used in the Golden Gate Bridge is 96 ksi, so the design is acceptable, even including the cable self-weight.

Tower forces are solved in Figure 4.13. We would need to know the weight w of the tower to complete its analysis and design.

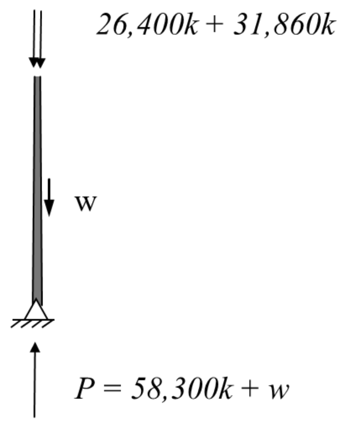



**Figure 4.13 Tower forces in the Golden Gate Bridge.**

Finally, we examine the anchorage forces (Figure 4.14). The cable force is 61,700 kips. This includes a component in the vertical direction. Thus, the net vertical force acting on the bottom surface of the anchorage block is the weight W of the block minus the vertical component of cable tension. We would want to consult with a geotechnical engineer to determine resistance of the anchorage block. The resistance might involve friction along the base of the block, or a combination of friction and passive bearing pressure along the front face.

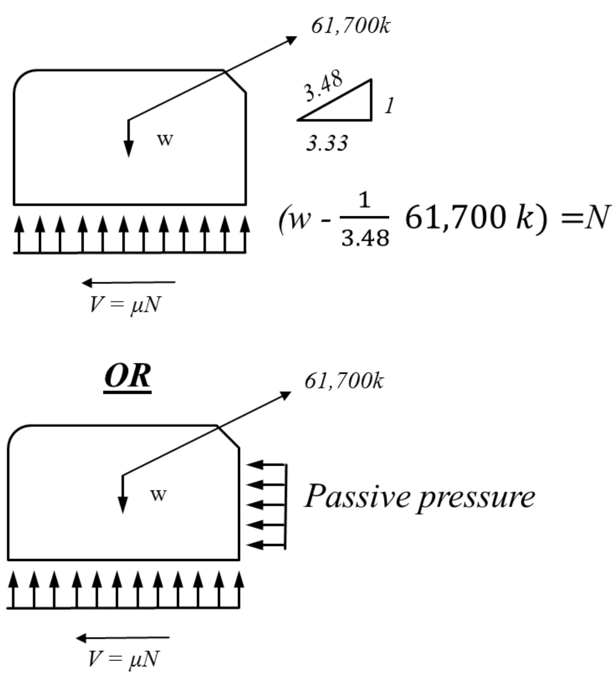


**Figure 4.14 Anchorage forces for the Golden Gate Bridge.**

We have found that a cable assumes a parabolic shape under the action of a uniform horizontally distributed load. A cable subjected to its own weight and free of any other loads will take the form of a *catenary*. For most structural applications, however, the ratio of the cable sag to span is small, in which case the catenary shape is very close to a parabola.

# 5. Principle of Virtual Displacements for Determination of Forces and Influence Lines

## 5.1. INTRODUCTION

This chapter introduces the principle of virtual displacements as a new method for the determination of reactions and internal forces and moments in structures. This new method can streamline the necessary calculations. It is also useful for construction of influence lines, which themselves will serve as a powerful tools for determining load patterns that produce maximum design effects.

## 5.2. WORK AND VIRTUAL WORK

To begin, we define *work* as the vector product of force and the displacement through which it moves (Figure 5.1). We denote work by the variable $W$. For a force moving through a differential displacement, the differential work is defined as $dW = \bar{P} \cdot d\bar{\Delta}$. Integrating, we obtain the work as $W = \int_0^{\Delta} \bar{P} \cdot d\bar{\Delta}$. If $\bar{P}$ is a constant, then the work is simply $W = \bar{P} \cdot \bar{\Delta}$.

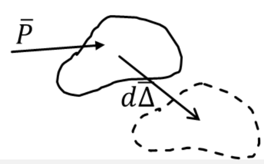



**Figure 5.1 Force moving through a differential displacement.**

Work can be either *real work* or *virtual work*. *Real work* is the work done by a real force (or moment) and the conjugate[1] displacement (or rotation) produced by that force. For example, consider a cantilever with horizontal force $Q_0$ at the free end (Figure 5.2). The reactions required for equilibrium are horizontal force H and moment $M_0$ at the base. Load $Q_0$ induces the displacements shown. Because the top displacement $\delta$ is conjugate to $Q_0$, that is, it is at the same location and in the same direction as $Q_0$, the real work done by $Q_0$ is the product $Q_0\delta$. We note that the reactions H and $M_0$ at the base perform no work, because there is no displacement of the structure that is conjugate to either H or $M_0$. Put more simply, horizontal force H at the base of the structure undergoes zero horizontal displacement and therefore produces no real work; likewise for $M_0$.

[1] As used here, conjugate means at the same location and in the same direction.

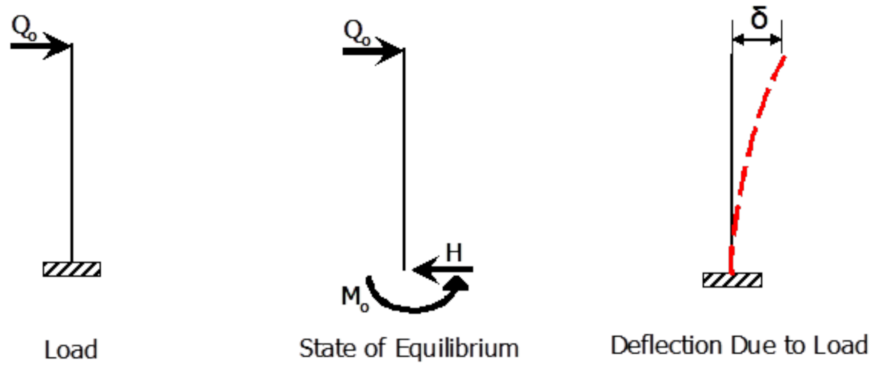



**Figure 5.2 Equilibrium and deflections of a cantilever loaded laterally at its free end.**

In structural engineering we are mainly interested in real forces and real displacements. However, we will use virtual forces and virtual displacements as constructs to calculate real displacements and real forces (we'll see how later). A *virtual force* is a force that is imagined to act on a structure, but it does not correspond to any real states of equilibrium arising from the real loads on the structure. Using the example of Figure 5.2, subjected to a real force $Q_0$ and experiencing real displacement δ, we could imagine a virtual moment m applied at the free end (Figure 5.3). Obviously, this virtual moment has nothing to do with the real equilibrium or the real displacements. (You may be wondering why we would do this. As we will see Chapter 6, we would apply virtual moment m as shown in order to calculate the rotation of the free end in the direction of m for a beam loaded by the real load $Q_0$.)

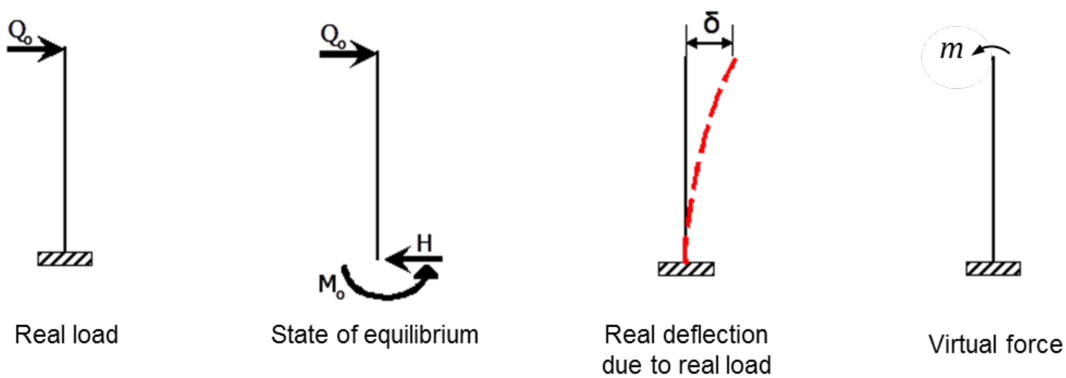



**Figure 5.3 Introduction of a virtual force to a cantilever.**

Similarly, we could introduce a virtual displacement to the cantilever of Figure 5.2, as shown in Figure 5.4. Obviously this virtual displacement field has nothing to do with the real displacements. (We will see later in this chapter that imposing a virtual rotation at point a as shown can be used to determine the moment at point a.)

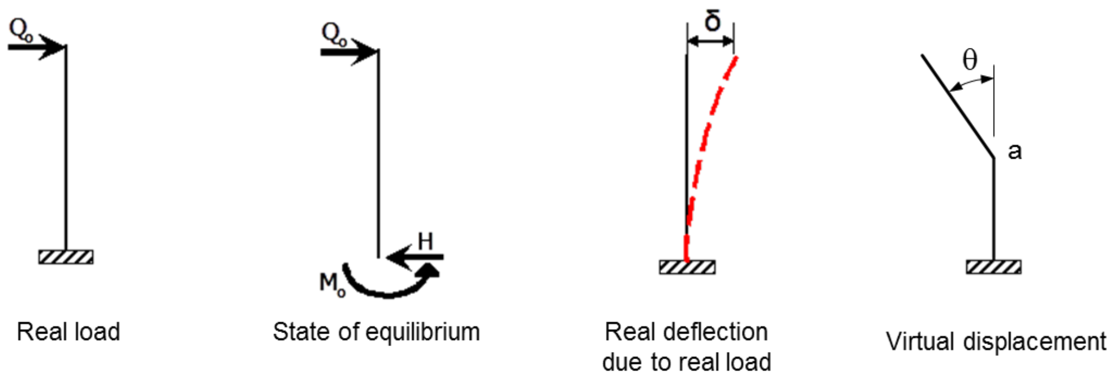



**Figure 5.4 Introduction of a virtual displacement to a cantilever.**

*Virtual work* is work that is produced by either virtual forces or virtual displacements. It is work that exists "in effect" or is "imagined," but is not real work. We denote virtual work by the variable $\delta W$. Virtual work can be one of either (a) or (b):

a) Virtual work produced by real forces moving through virtual (or imagined) displacements. We will use this for the purpose of finding the real forces (reactions, or internal forces and moments);
b) Virtual work produced by virtual (or imagined) forces moving through real displacements. We will use this for the purpose of finding the real displacements.

In this chapter, we only consider virtual work produced by real forces moving through virtual displacements. We will consider the case of virtual forces moving through real displacements in the next chapter.

## 5.3. PRINCIPLE OF VIRTUAL DISPLACEMENTS

To derive the principle of virtual displacements, we first consider the case of a virtual displacement and then of a virtual rotation, as follows:

1) Consider a rigid body subjected to a virtual (or imagined) displacement vector $\overline{\Delta}$. The virtual work is $\delta W = \sum \overline{P_i} \cdot \overline{\Delta} = (\sum \overline{P_i}) \cdot \overline{\Delta}$. If a rigid body is in equilibrium, we know that $\sum \overline{P_i} = \sum \overline{F} = 0$. Therefore, the virtual work done is also zero, that is $\delta W = 0$.
2) Next, consider a rigid body subjected to a virtual (or imagined) rotation vector $\overline{\theta}$ about an arbitrary point "o." Defining the distance $d_i$ from point "o" to the applied loads $P_i$, the virtual work is $\delta W = \sum \overline{P_i} \cdot d_i \cdot \overline{\theta} = (\sum \overline{P_i} d_i) \cdot \overline{\theta} = (\sum \overline{M}) \cdot \overline{\theta}$. If a rigid body is in equilibrium, we know that $\sum \overline{M} = 0$. Therefore, the virtual work done is also zero, that is $\delta W = 0$.

Because any displacement of a rigid body can be made up of a translation plus a rotation, we have $\delta W = 0$ for any virtual displacement. We can thus state the principle of virtual displacements as follows:

*Principle of Virtual Displacements*: For any rigid body in equilibrium, the work done by the forces acting on that rigid body as they move through any compatible set of virtual displacements is zero.

To use the principle of virtual displacements, the following rules generally apply:

1) The virtual displacement should be a rigid body displacement, that is, one that does not deform the structure. Such deformations do additional work that would need to be considered in the solution. It is acceptable, however, for rotations to occur at any in-span hinges. Such hinges are locations of zero moment; hence, no additional work is done by the hinge.
2) The virtual displacement should expose a single reaction or internal force whose value is being sought.
3) In calculating virtual work, include the work done by all forces acting on the rigid body, including:
    a) External forces, including self-weight;
    b) the reactions (if the structure has been separated from its supports);
    c) and any internal forces if such internal forces have been exposed by cutting through the structure to create a free body diagram (FBD).

*Virtual work done by a concentrated load P* moving through a virtual displacement $\delta$ in the same direction as $P$ is $\delta W = P\delta$. If P and $\delta$ are in opposite directions, the work is $-P\delta$. If $P$ is at an angle relative to $\delta$, the work is done by the component of $P$ parallel to $\delta$. For distributed loads (Figure 5.5), we can integrate virtual work along beam segment ab to obtain the virtual work as

$$\delta W = \int_0^L (w_x dx) \cdot \delta x \tag{5.1}$$

*For uniformly distributed load w oriented perpendicular to the beam,* the virtual work is

$$\delta W = \int_0^L (w_x dx) \cdot \delta x = \int_0^L w(\delta x dx) = w \int_0^L dx \delta x = w \times A_\delta \tag{5.2}$$

in which $A_\delta$ = area under the virtual displacement diagram along the distributed loading. Alternatively, it can be shown that the virtual work for uniformly distributed load perpendicular to a beam is

$$\delta W = \int_0^L (w_x dx) \cdot \delta x = R \times \delta_R \tag{5.3}$$

in which $R$ = the resultant of the distributed load and $\delta_R$ = the virtual displacement of the resultant $R$.

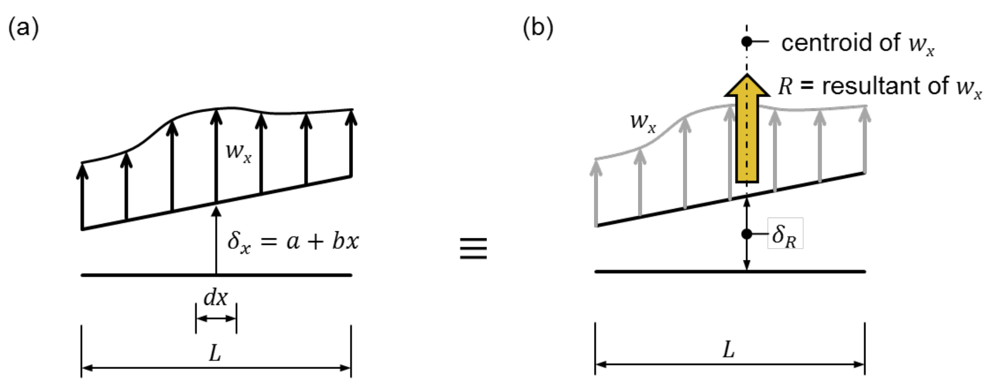



**Figure 5.5 Work done by distributed load.**

## 5.4. EXAMPLES USING THE PRINCIPLE OF VIRTUAL DISPLACEMENTS

The following examples illustrate how we can use the principle of virtual displacements.

**Example 1**: For the structure shown, find $R_1$.

Here we require a virtual displacement field that avoids bending the beam and that produces work in the unknown reaction $R_1$ but does not produce any work by other unknown forces. The correct displacement field is shown.

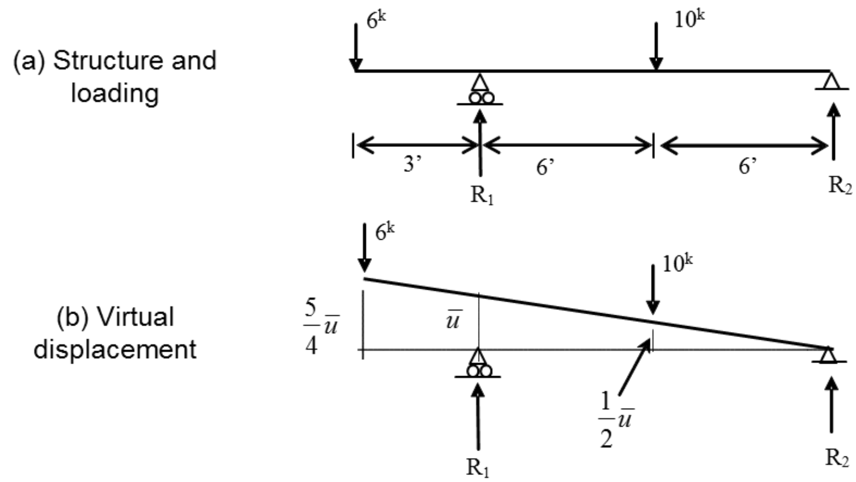



**Figure 5.6 Example 1.**

The virtual work equation is written as follows:

$$\delta W = \left(-6 \times \frac{5}{4}\bar{u}\right) + R_1 \times \bar{u} + \left(-10 \times \frac{1}{2}\bar{u}\right) = 0$$

$$\therefore R_1 = 12.5\ kips$$

We can use a similar procedure to solve for $R_2$ = 3.5 kips. Check these results with the results you would obtain using equations of equilibrium. The answers should be identical.

**Example 2**: Determine $M_A$.

The solution is obtained by cutting the beam at A to form two FBDs. Imposing a unit virtual displacement at point A produces rotations of each of the FBDs by amount 1/6. Thus, the moment on each of the FBDs, shown according to the positive sign convention used in this reader, produces work equal to $M_A/6$. Note that shears $V_A$ are exposed at A, but one of them does virtual work 1 x VA whereas the other does virtual work -1 x $V_A$, so no net work is done by the shear.

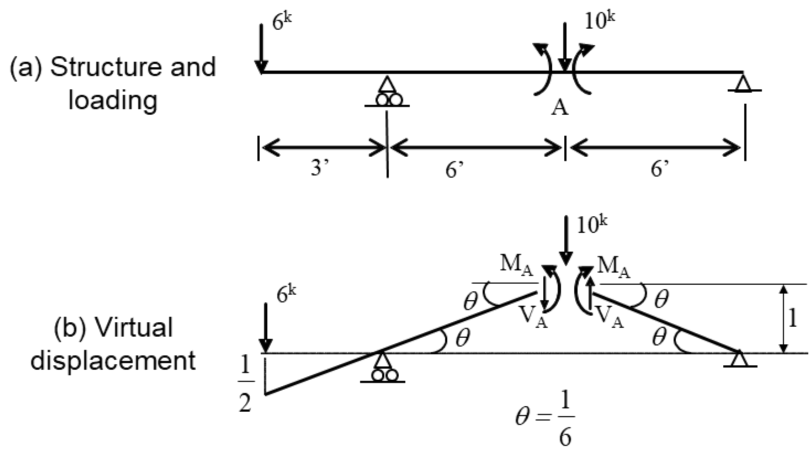



**Figure 5.7 Example 2.**

$$\delta W = \left(M_A \times \frac{1}{6}\right) + \left(M_A \times \frac{1}{6}\right) + (-10 \times 1) + \left(6 \times \frac{1}{2}\right) = 0$$

$$\therefore M_A = 21 \text{ k-ft}$$

**Example 3**: Determine shear $V_A$ just to the left of the 10-kip load.

The solution is obtained by cutting the beam at A to form two FBDs, and imposing a unit displacement such that the rotation of the left-hand FBD of the beam is equal and opposite that of the right-hand FBD. By having equal and opposite rotations, the virtual work done by the moments $M_A$ cancels, and the only unknown quantity doing virtual work is the unknown shear we are trying to calculate.

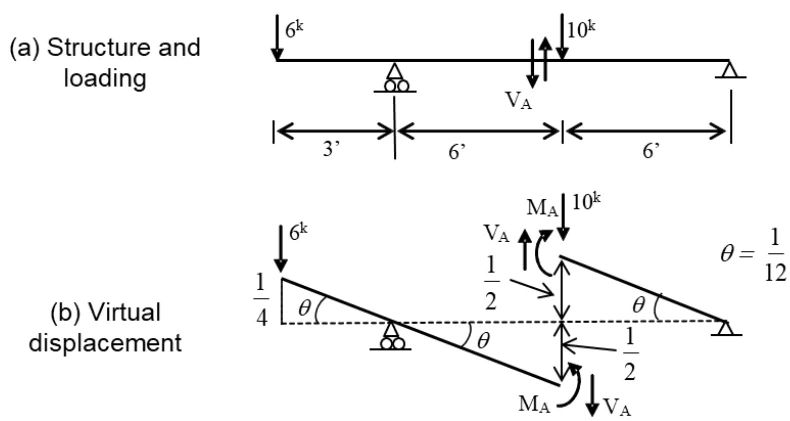



**Figure 5.8 Example 3.**

$$\delta W = (V_A \times 1) - \left(6 \times \frac{1}{4}\right) - \left(10 \times \frac{1}{2}\right) = 0$$

$$\therefore V_A = 6.5 \text{ kips just to the left of the 10-kip load.}$$

**Example 4**: Find the moment $M_A$ over the interior roller support?

See solution below.

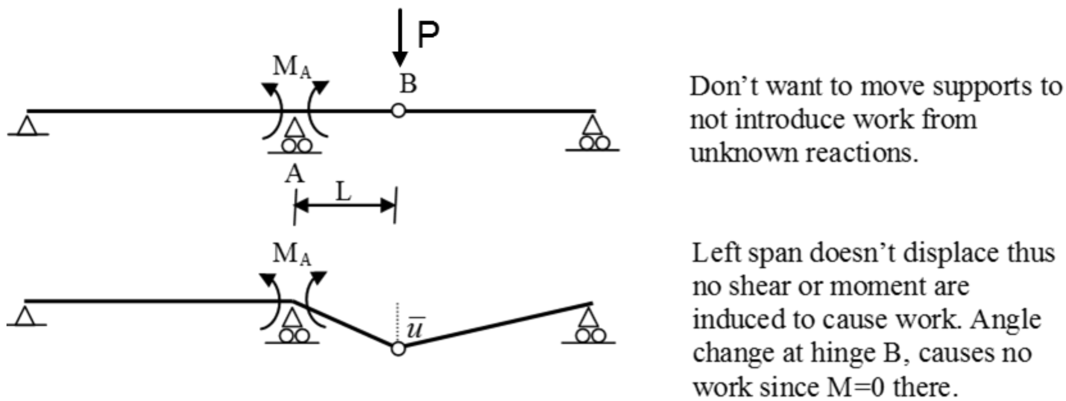



**Figure 5.9 Example 4.**

Rotation at A is $\bar{\theta}$ and $\bar{\theta} = \dfrac{\bar{u}}{L}$

$$M_A \times \bar{\theta} + P \times \bar{u} = 0 \quad So \quad M_A = -PL$$

**Example 5**: Find the shear just to the right of the interior roller support?

See solution below.

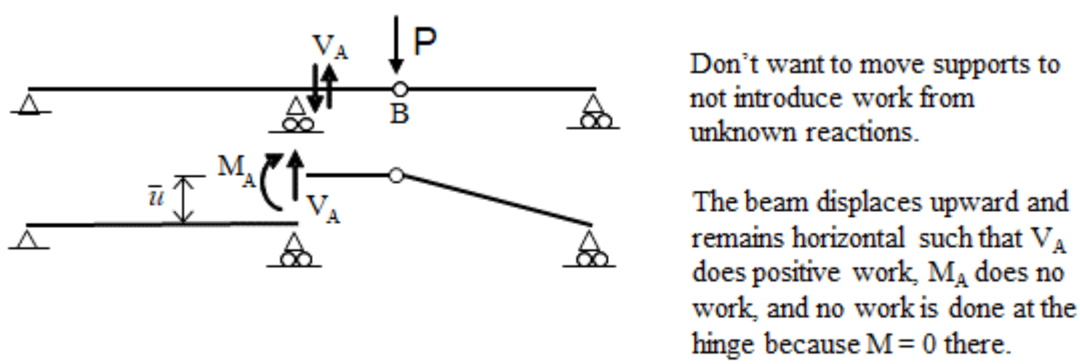
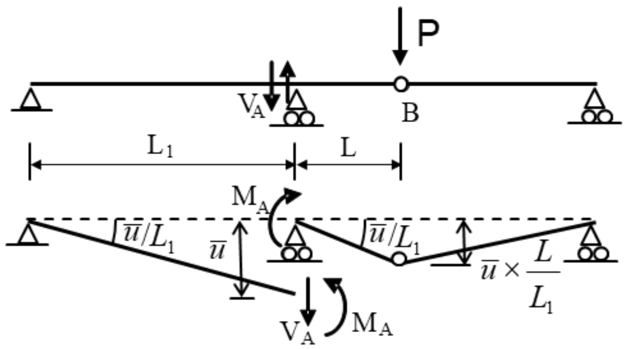



**Figure 5.10 Example 5.**

$$V_A \times \bar{u} - P \times \bar{u} = 0 \quad So \quad V_A = P$$

**Example 6**: Find the shear to the left of the interior roller support.




Don't want to move supports to not introduce work from unknown reactions.

The beams must slope at the same angles so that the work done by $M_A$ cancels out.

**Figure 5.11 Example 6.**

$$-V_A\times\overline{u}+M_A\times\frac{\overline{u}}{L_1}-M_A\times\frac{\overline{u}}{L_1}+P\times\overline{u}\times\frac{L}{L_1}=0 \quad So \quad V_A=P\times\frac{L}{L_1}$$

**Example 7**: Solve for shear and moment at point A.

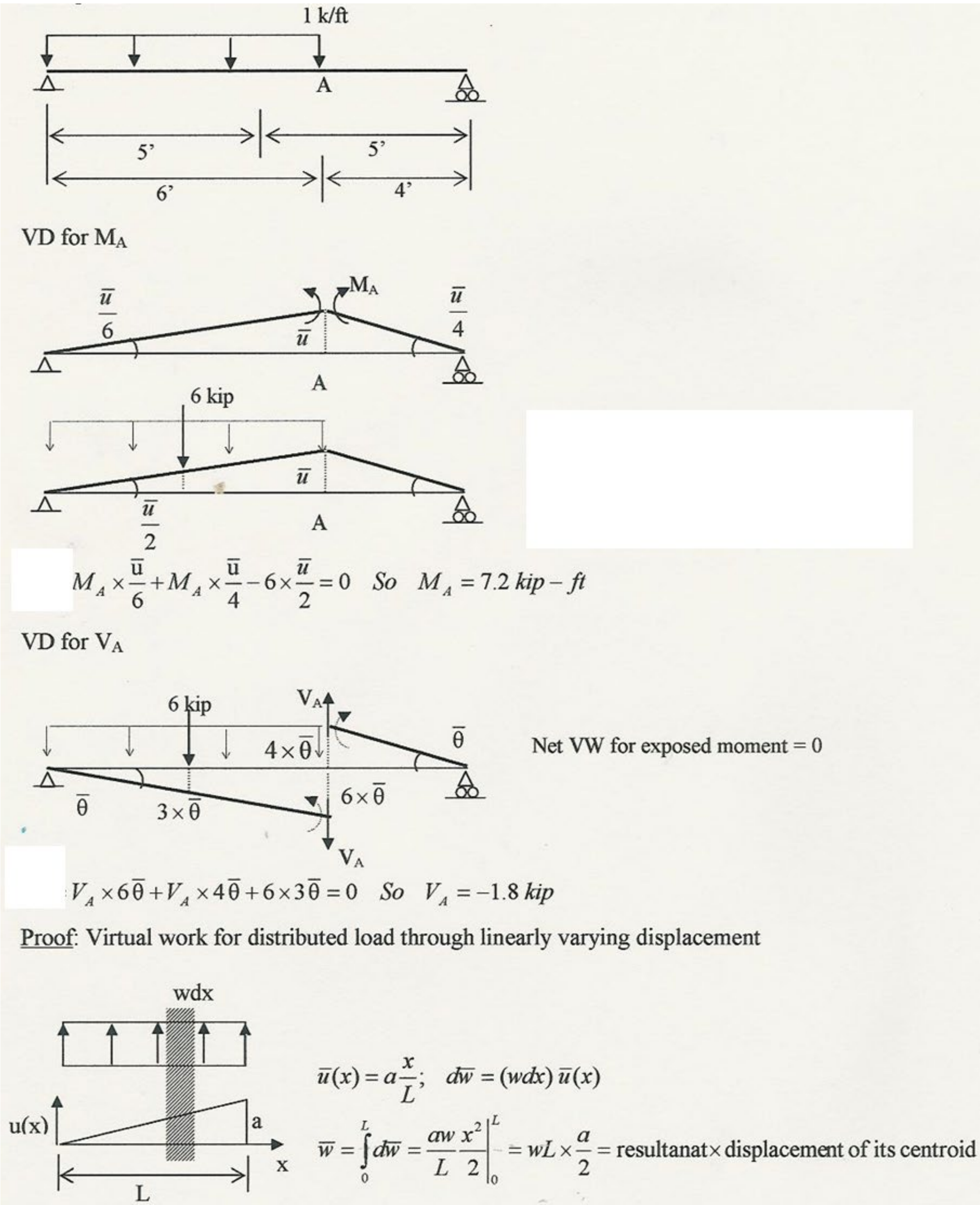



**Figure 5.12 Example 7.**

**Example 8**: What virtual displacements are required to determine (a) the moment at B and (b) the horizontal reaction at A?

See solution below.

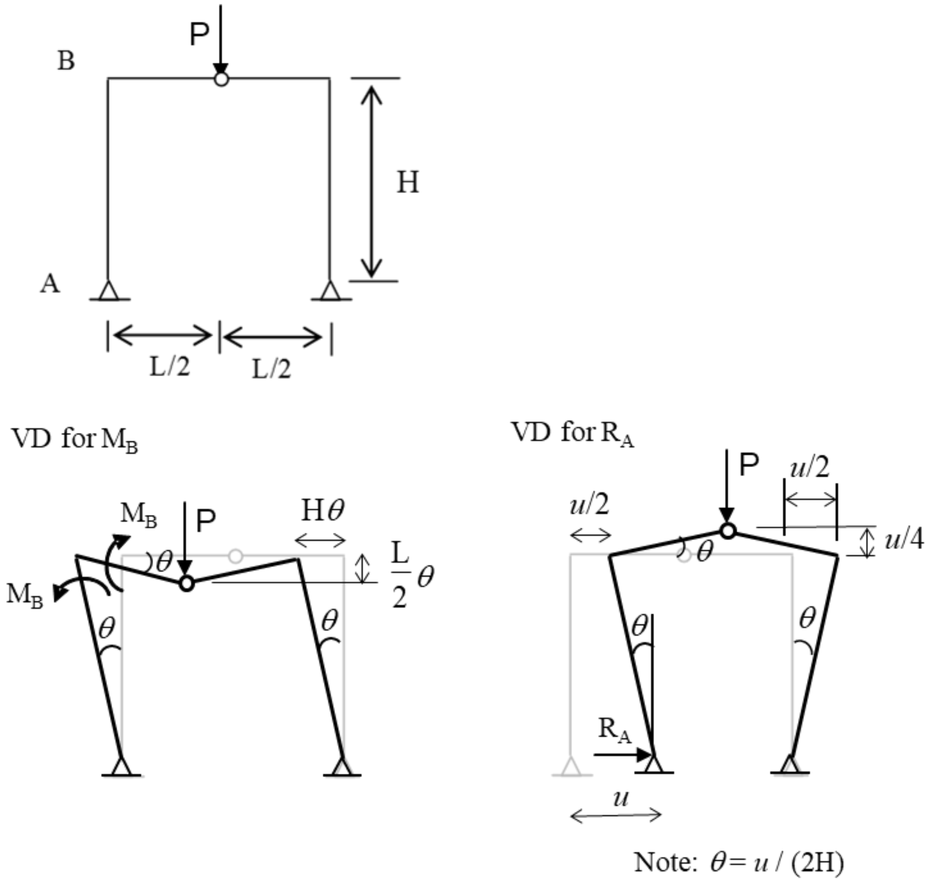



**Figure 5.13 Example 8.**

## 5.5. INFLUENCE LINES

So far we have performed the following steps in analyzing structures:

- Given the geometry and support conditions of a structure, and
- given the magnitude and position of the loads,
- calculate the reactions and internal forces at locations of interest in the structure.

The preceding steps apply if the magnitude and position of loads is known. This is generally the case for dead loads (as they are calculated based on known quantities that are fixed in both magnitude and position). The same may not be true for live loads, whose magnitude and position may vary. The structural engineer needs to determine the maximum internal forces that a live load can produce in a structure. In very simple structures it may be possible by inspection to identify how to place live loads so as to produce maximum effects, but for more complicated structures a rigorous method will be required. In this section, we will introduce influence lines as a method for determining how to place live loads.

An *influence line* is defined as the plot of the forces or moments at one point in a structure as a function of the position of a unit load placed anywhere in the structure.

To illustrate the concept of the influence line, we first consider a simply supported beam (Figure 5.14a). We wish to calculate the influence line for the vertical reaction $R_1$. For this purpose, a unit load is positioned a distance x from the left-hand support. Summing moments about the roller at the right-hand end, we can write $R_1(x)L - 1(L - x) = 0 \rightarrow R_1(x) = \left(1 - \frac{x}{L}\right)$. We can plot this function beneath the beam, as shown in Figure 5.14b. This plot is the influence line for the vertical reaction $R_1(x)$. We can repeat the exercise for the vertical reaction $R_2(x)$. The influence line for $R_2(x)$ is plotted in Figure 5.14c.

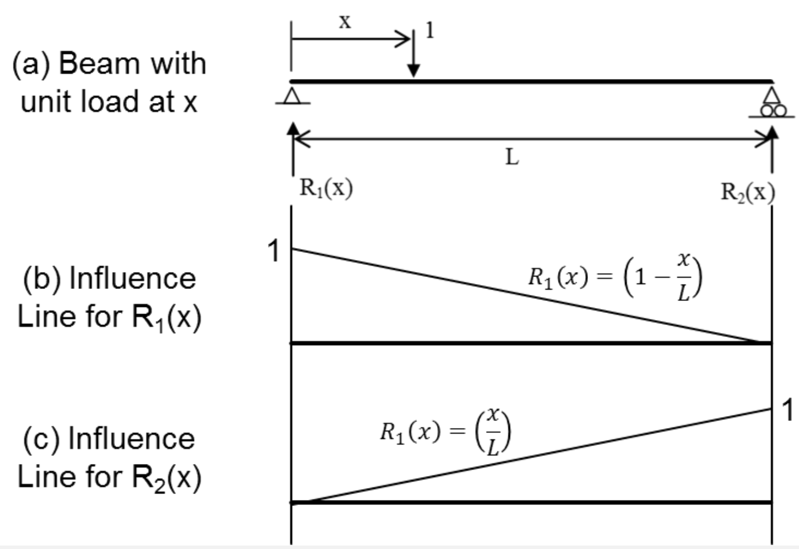



**Figure 5.14 Example 9: Influence lines for reactions of simply supported beam.**

The shape of the influence in Figure 5.14 looks like the virtual displaced shape for a beam in which the virtual displacement is unity at the location and in the direction of the internal force that we are seeking. This is explored further in Figure 5.15, where a unit virtual displacement has been imposed at $R_1(x)$. We can write the virtual work equation using methods presented earlier in this chapter as

$$R_1(x) \times 1 - 1 \times \delta_x = 0 \rightarrow R_1(x) = \delta_x = \left(1 - \frac{x}{L}\right)$$

This demonstrates the following principle: *The influence line for a force or moment is equal to the virtual displaced shape having a unit displacement in that force or moment, provided no other unknown reactions or internal forces do any virtual work.* (This was first expressed by the Muller-Breslau principle in1880.)

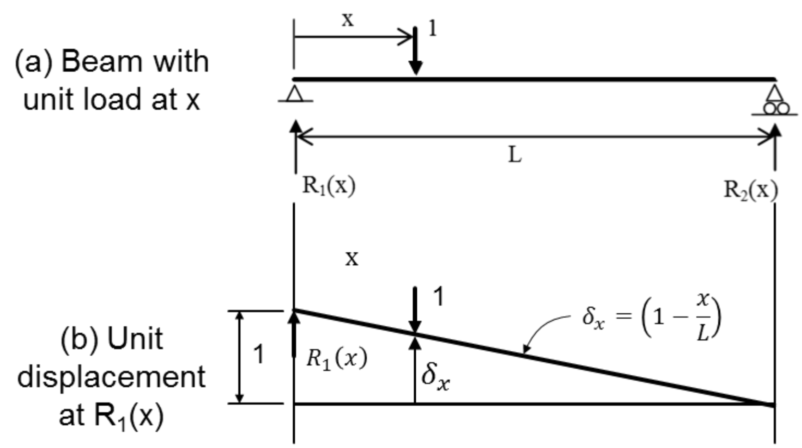



**Figure 5.15 Unit virtual displacement of simply supported beam at R1(x).**

**Example 10**: Calculate the influence line for moment at center of simply supported beam. The solution is shown in Figure 5.16. To determine the relations between the angles and the height of the virtual displaced shape at the center of the beam, consider the following:

1. The vertical displacement of AB and BC must be equal such that the shear does not contribute to the virtual work.
2. Therefore, we can write: $\theta_A \times L_{AB} = \theta_C L_{BC}$.
3. We also know that we want $\theta_A + \theta_C = 1$.
4. Solving these two equations in two unknowns, we find $\theta_A = \theta_C = 0.5$.
5. Therefore, the height of the virtual displacement diagram at B is $\delta_B = \theta_A \frac{L}{2} = \frac{L}{4}$.

Example 11 will present an alternative way of calculating the angles.

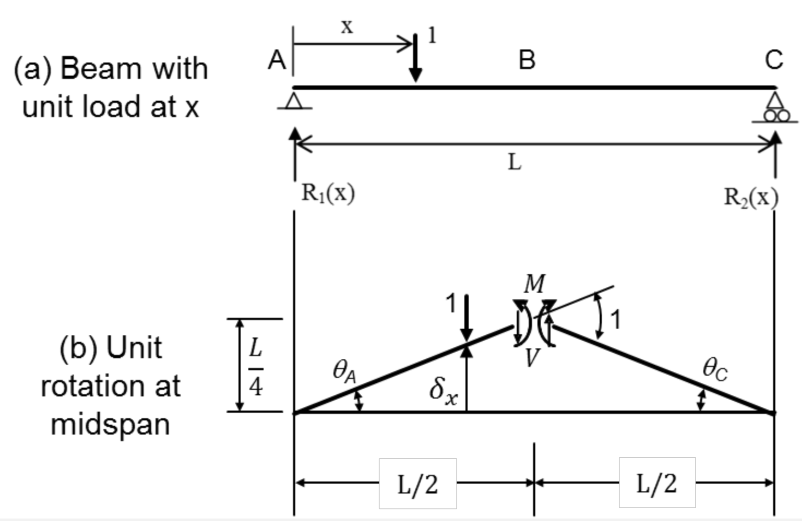



**Figure 5.16 Example 10.**

## 5.6. USE OF INFLUENCE LINES TO CALCULATE MAXIMUM LOAD EFFECTS

The previous section introduced influence lines. In this section we demonstrate the use of influence lines to calculate maximum load effects. We do this through an example.

**Example 11**: Consider the beam of Figure 5.17a. The beam is loaded by 200 plf dead load, 300 plf live load, plus a concentrated live load if 4 kips. Calculate the maximum positive (bottom in tension) moment at point D.

We begin by sketching a virtual displaced shape for which positive moment at D will do positive virtual work while no other unknown quantity does work. We give the shape a displacement amplitude of $\bar{u}$ at point D (see Figure 5.17b). To get the influence line for positive moment at D, we need the total rotation at D to be equal to 1. Therefore, divide the virtual displacement amplitude of Figure 5.17b by the quantity $\frac{3}{10}\bar{u}$, which is the rotation at D as shown in Figure 5.17b. This gives us the influence line of Figure 5.17c.

The influence line tells us that positive moment at D will be produced wherever the influence line for $+M_D$ is positive. Thus, we should place distributed load along CE, we should place the concentrated live load at point D (because the ordinate of the influence line is most positive there), and dead load should be placed across the entire span AE (by definition, dead load cannot be moved). Figure 5.17d shows the loading that will produce the greatest value of $+M_D$.

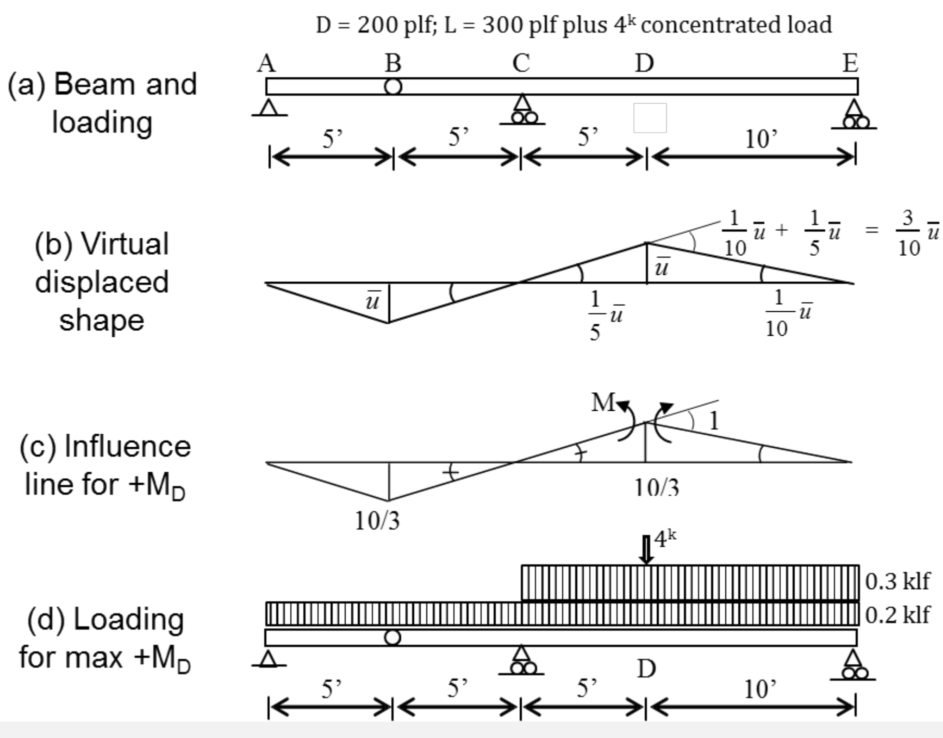



**Figure 5.17 Example 11.**

<u>We next use the principle of virtual work to calculate the moment at D, as follows:</u>

Dead load:

$$M_D = (0.2)\left[-\frac{1}{2}(\frac{10}{3})(10) + \frac{1}{2}(\frac{10}{3})(15)\right] = 1.67\ kip - ft$$

Uniform live load:

$$M_D = (0.3)\left[\frac{1}{2}(\frac{10}{3})(15)\right] = 7.50\ kip - ft$$

Concentrated live load:

$$M_D = (4)(\frac{10}{3}) = 13.33\ kip - ft$$

Use superposition to obtain the total moment:

$$M_D = 1.67\ k - ft + 7.50\ k - ft + 13.33\ k - ft = 22.5\ k - ft$$

One can also find minimum bending moment at D, in this case by placing the dead load uniformly along AE, distributed live load along AC, and concentrated live load at B. The resulting moment is 1.67 – 5 - 13.33 = -16.7 k-ft.

Thus, Beam AE must be designed such that it can resist bending moments at point D in the range of +22.50 kip-ft to –16.66 kip-ft.

# 6. Principle of Virtual Forces for Determination of Displacements

## 6.1. INTRODUCTION

This chapter introduces the principle of virtual forces as a method for the determination of displacements in structures.

## 6.2. WORK AND VIRTUAL WORK[1]

To begin, we define *work* as the vector product of force and the displacement through which it moves (Figure 6.1). We denote work by the variable $W$. For a force $\bar{P}$ moving through a differential displacement $d\bar{\Delta}$, the differential work is defined as $dW = \bar{P} \cdot d\bar{\Delta}$. Integrating, we obtain the work as $W = \int_0^{\Delta} \bar{P} \cdot d\bar{\Delta}$. If $\bar{P}$ is a constant, then the work is simply $W = \bar{P} \cdot \bar{\Delta}$.

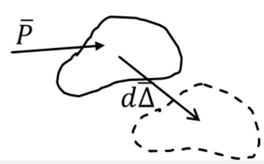



**Figure 6.1 Force moving through a differential displacement.**

Work can be either *real work* or *virtual work*. *Real work* is the work done by a real force (or moment) and the conjugate[2] displacement (or rotation) produced by that force. For example, consider a cantilever with horizontal force $Q_0$ at the free end (Figure 6.2). The reactions required for equilibrium are horizontal force H and moment $M_0$ at the base. Load $Q_0$ induces the displacements shown. Because the top displacement $\delta$ is conjugate to $Q_0$, that is, it is at the same location and in the same direction as $Q_0$, the real work done by $Q_0$ is the product $Q_0\delta$. We note that the reactions H and $M_0$ at the base perform no work, because there is no displacement of the structure that is conjugate to either H or $M_0$. Put more simply, horizontal force H at the base of the structure undergoes zero horizontal displacement and therefore produces no real work; likewise for $M_0$.

[1] Note: This section repeats the text of Section 5.2.
[2] As used here, conjugate means at the same location and in the same direction.

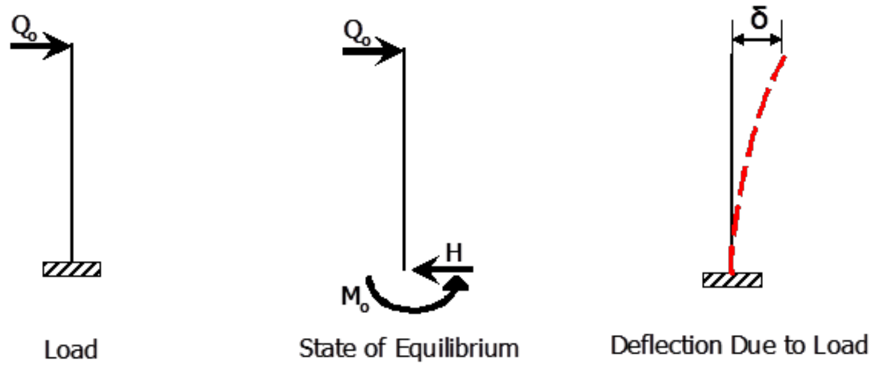



**Figure 6.2 Equilibrium and deflections of a cantilever loaded laterally at its free end.**

In structural engineering we are mainly interested in real forces and real displacements. However, we will use virtual forces and virtual displacements as constructs to calculate real displacements and real forces (we'll see how later). A *virtual force* is a force that is imagined to act on a structure, but it does not correspond to any real states of equilibrium arising from the real loads on the structure. Using the example of Figure 6.2, subjected to a real force $Q_0$ and experiencing real displacement δ, we could imagine a virtual moment m applied at the free end (Figure 6.3). Obviously, this virtual moment has nothing to do with the real equilibrium or the real displacements. (You may be wondering why we would do this. As we will see later in this chapter, we would apply virtual moment m as shown in order to calculate the rotation of the free end in the direction of m for a beam loaded by the real load $Q_0$.)

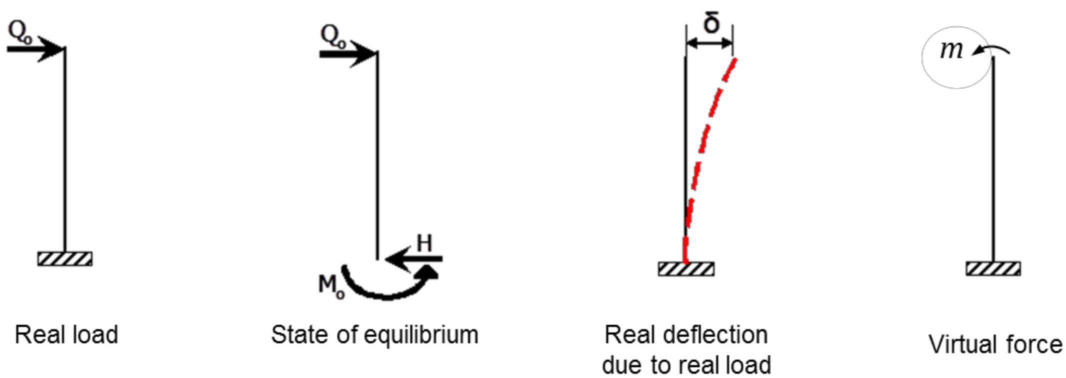



**Figure 6.3 Introduction of a virtual force to a cantilever.**

Similarly, we could introduce a virtual displacement to the cantilever of Figure 6.2, as shown in Figure 6.4. Obviously this virtual displacement field has nothing to do with the real displacements. (As discussed in Chapter 5, we can use the virtual rotation at point a to determine the moment at point a.)

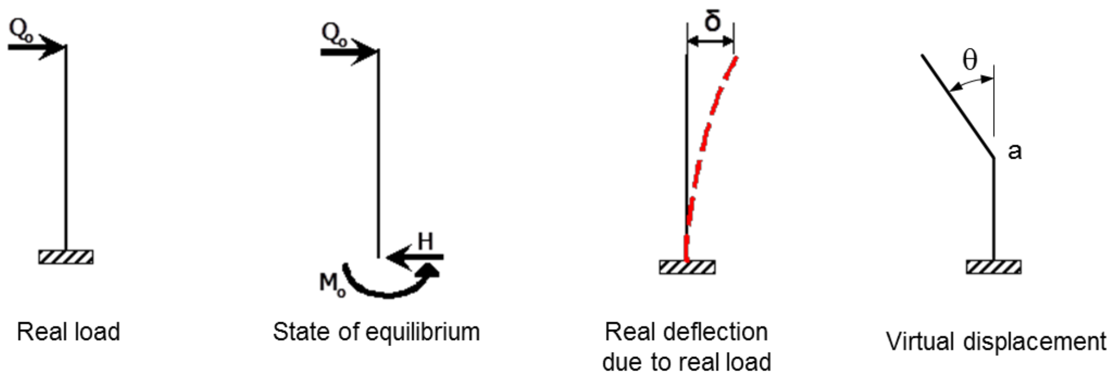



**Figure 6.4 Introduction of a virtual displacement to a cantilever.**

*Virtual work* is work that is produced by either virtual forces or virtual displacements. It is work that exists "in effect" or is "imagined," but is not real work. We denote virtual work by the variable $\delta W$. Virtual work can be one of either (a) or (b):

a) Virtual work produced by real forces moving through virtual (or imagined) displacements. We will use this for the purpose of finding the real forces (reactions, or internal forces and moments);
b) Virtual work produced by virtual (or imagined) forces moving through real displacements. We will use this for the purpose of finding the real displacements.

In this chapter, we emphasize virtual work produced by virtual forces moving through real displacements. Chapter 5 considers virtual work due to real forces moving through virtual displacements.

## 6.3. PRINCIPLE OF CONSERVATION OF ENERGY

When a deformable body is acted on by external forces, those external forces do external work $W_{ext}$ equal to the product of the external forces and the conjugate external displacements (that is, the displacements at the locations and in the direction of the applied external forces). Internal stresses and deformations are also developed, the internal stresses being in equilibrium with the external forces, and internal deformations being compatible with the external deformations. As the internal stresses move through the internal deformations, they do internal work $W_{int}$. The *principle of conservation of energy* states that the work done by the external forces acting on a deformable body is transformed into internal work or strain energy within the body as it deforms. This is expressed by Eq. (6.1).

$$W_{ext} = W_{int} \tag{6.1}$$

The principle of conservation of energy is applicable to any type of structure. To derive the general principle, consider the planar deformable body of Figure 6.5a in equilibrium under a set of external forces $P$. External work $W_{ext}$ is done by these forces moving through the conjugate deformations around the exterior of the body. Next consider a small particle internal to the deformable body, such as the one shaded. This particle is acted in by a set of internal forces $P_i$ that are in equilibrium with the external forces. Although these forces $P_i$ are internal to the deformable body, they can also be considered as external forces acting on the particle. These forces do external work on the particle defined by $dW_{ext} = P_i \cdot \delta_i$, in which $\delta_i$ are conjugate displacements around the particle due to the deformations of the deformable body. Part of this work, $dW_R$, will be due to rigid body movements of the particle, and the remainder, $dW_{int}$, will be due to deformations of the particle. According to the principle of virtual displacements, the work done by a set of forces in equilibrium moving through a rigid body displacement is zero, hence, $dW_R = 0$. Thus, at the particle level, we can write $dW_{ext} = dW_R + dW_{int} = dW_{int}$. We can add up the work done on all the particles of the deformable body, resulting in the expression $W_{ext} = W_{int}$. The term on the left-hand side is the sum of all of the external work done on all of the particles. Note, however, that adjacent particles are subjected to equal and opposite forces, such that the external work terms for forces acting between individual particles must all cancel, leaving only the external work acting around the free surface of the deformable body. The term on the right-hand side is the sum of all of the internal work done on all of the particles. Thus, we have demonstrated that the external work acting on the outside of the deformable body and the internal work within the deformable body are equal, as expressed by Eq. (6.1).

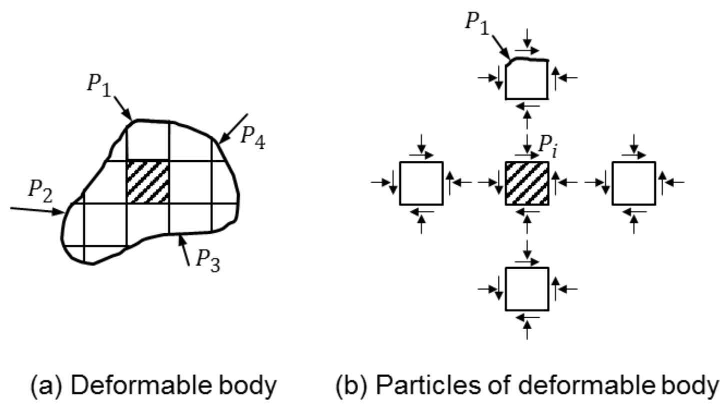



**Figure 6.5 Deformable body in equilibrium under external forces P.**

We can demonstrate the principle numerically for a linear-elastic rod of constant cross-sectional area $A$ and length $L$ subjected to concentrated forces at the free end (Figure 6.6). Under load $P$, the rod elongates by $\delta_P$. Note that the force $P$ increases from zero to $P$ in proportion with the elongation of the rod, that is, $P_x = P\frac{x}{\delta_P}$. The external work is

$$W_{ext,P} = \int_0^{\delta_P} P_x dx = \frac{P\delta_P}{2} \tag{6.2}$$

Note that $W_{ext,P}$ is equal to the diagonally cross-hatched triangular area in Figure 6.6c.

To derive the internal work due to the application of load $P$, consider the internal deformations and forces at a point along the rod (Figure 6.6b). The axial strain is $\epsilon_P = \frac{P}{A}$. The differential internal work along $dx$ is $dW_{int,P} = \int_0^{\epsilon_P} P\frac{x}{\epsilon_P} dx$. Integrating this along the length we obtain the total internal work due to application of the load $P$ as

$$W_{int,P} = \int_0^L \int_0^{\epsilon_P} P\frac{x}{\epsilon_P} dx = \frac{PL\epsilon_P}{2} = \frac{P\delta_P}{2} \tag{6.3}$$

Note that this is equal to the external work $W_{ext,P}$, demonstrating again that $W_{ext} = W_{int}$.

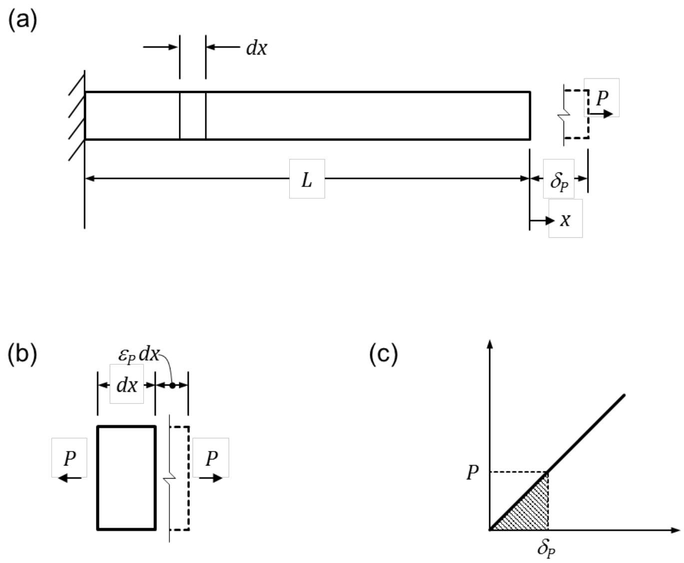



**Figure 6.6 Internal work on a axially loaded linear-elastic rod subjected to force *P*.**

Suppose now that the load $P$ is already in place and an unrelated force $F$ is added to the rod causing an additional displacement $\delta_F$ (Figure 6.7). In this case, the external work done by the pre-existing force $P$ is simply

$$W_{ext,P} = P\delta_F \tag{6.4}$$

which is represented by the vertically cross-hatched rectangular area of Figure 6.7c. The internal work done by the pre-existing force $P$ is

$$W_{int,P} = \int_0^L P\epsilon_F dx = P\epsilon_F L = P\delta_F \tag{6.5}$$

Equations (6.4) and (6.5) again demonstrate that $W_{ext} = W_{int}$.

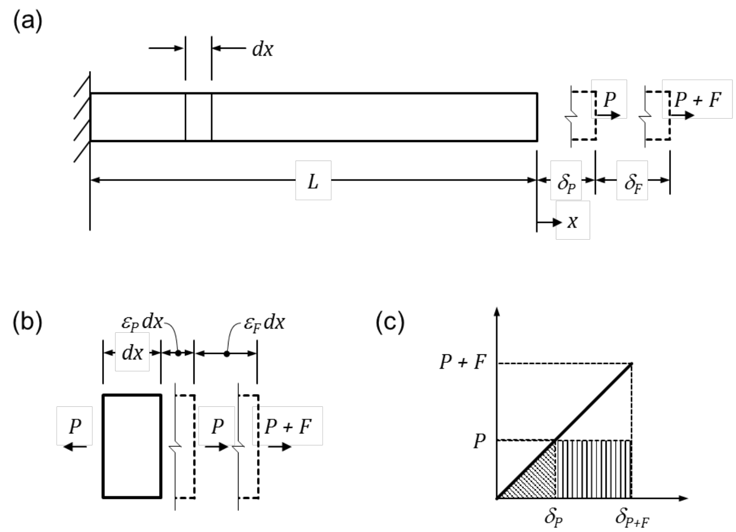



**Figure 6.7 Internal work on a axially loaded linear-elastic rod subjected to force *P* + *F*.**

## 6.4. PRINCIPLE OF VIRTUAL FORCES

In Figure 6.7, the force *P* was assumed to be a real force. However, the results apply equally to the case where *P* is a virtual force, that is a force that is imagined to act on the structure. This leads to the *principle of virtual forces*, which can be stated as follows: *For a deformable body that is in equilibrium under a virtual force system and remains in equilibrium while it is subjected to real deformations due to a real force system, the external virtual work done by the external virtual forces is equal to the internal virtual work done by the internal virtual stresses*.

We will use the principle of virtual forces to calculate the real displacements. Before we can do this, we need to develop expressions for internal work. This is done in Section 6.5.

It will also be useful to establish a convention for designating real versus virtual forces, shears, and moments, as follows:

- Real forces, shears, and moments are designated with upper case letters *P*, *V*, and *M*.
- Virtual forces, shears, and moments are designated with lower case letters *p*, *v*, and *m*.

## 6.5. INTERNAL VIRTUAL WORK

As a structure deforms, *internal virtual work* is done by internal virtual stresses/forces acting through real internal deformations. We need to be able to calculate this internal work. Figure 6.8 shows conjugate real internal displacements associated with virtual axial force, shear, and bending moment. (We could also show real forces, but our interest here is to apply the principle of virtual forces to the solution of displacement problems.) Internal work associated with each is described in the following text.

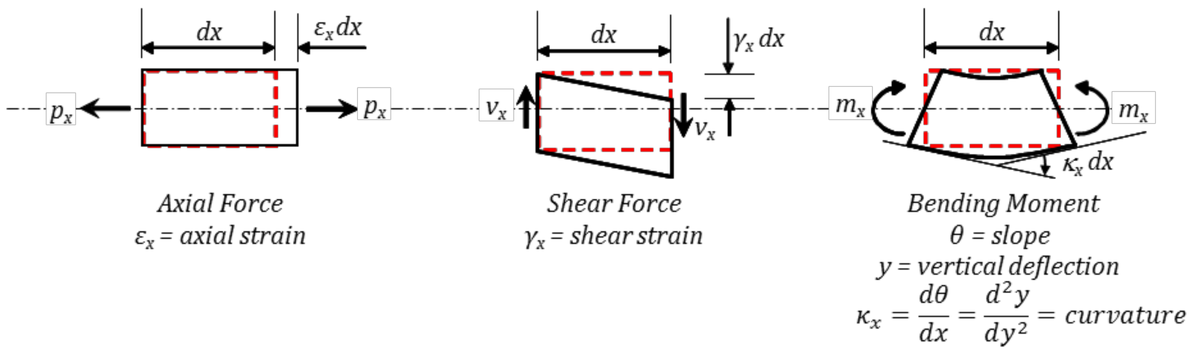



**Figure 6.8 Conjugate internal displacements and forces for axial, shear, and bending actions.**

*Axial Force*

Virtual axial force $p_x$ is applied to a differential element of length $dx$. The internal real displacement that is conjugate to this force is the elongation of the element, which is equal to $\varepsilon_x dx$, where $\varepsilon_x$ is axial strain. The differential internal work performed by axial force $p_x$ in differential element $dx$ is given by the following expression:

$$dW_i = p_x \cdot \epsilon_x dx \tag{6.6}$$

For a member of length $L$, with constant axial stiffness $EA$, subjected to constant axial force $p$,

$$W_{int} = \int_0^L p_x \epsilon_x dx = p\epsilon L \tag{6.7}$$

If the real axial strain is due to a real axial force $P$ in the member, then the internal work can be expressed as

$$W_{int} = \int_0^L p_x \epsilon_x dx = p\epsilon L = p\frac{PL}{EA} \tag{6.8}$$

Note that in Eq. (6.8), the term $p$ represents the virtual internal force and the term $\frac{PL}{EA}$ represents the real elongation under real axial force $P$. The real elongation could alternatively be due to temperature change or fabrication error that results in a change of length in the absence of externally applied force.

*Shear Force*

Virtual shear force $v_x$ is applied to a differential element of length $dx$. The internal conjugate real displacement is the vertical displacement of the force on the right side of the differential element, which is equal to $\gamma_x dx$, where $\gamma_x$ is shear strain. The differential internal work performed by shear force $v_x$ in differential element $dx$ is thus given by the following expression:

$$dW_i = v_x \cdot \gamma_x dx \tag{6.9}$$

*Bending Moment*

Virtual bending moment $m_x$ is applied to a differential element of length $dx$. The internal conjugate displacement is the angle of rotation of one side of the element relative to the other.

This is equal to the change in slope of the element, which is given by the expression $(d\theta/dx)dx$, where $\theta$ is the slope. But change of slope $d\theta/dx$ is approximately equivalent to curvature $\kappa_x$, so we can express the real internal conjugate displacement as $\kappa_x dx$. The internal work performed by moment $m_x$ in differential element $dx$ is thus given by the following expression:

$$dW_i = m_x \cdot \kappa_x dx \qquad (6.10)$$

## 6.6. APPLICATIONS OF THE PRINCIPLE OF VIRTUAL FORCES

The primary application of the principle of virtual forces is in the calculation of displacements. The steps in the method are as follows:

1. Consider a real state of deformation that is produced by some real effect. For example, the real state of deformation could be the external and internal displacements of the structure due to dead load, or due to a uniform drop in temperature.
2. Identify a displacement of interest, including both its location and its direction. The displacement can be a translation, a rotation, or a relative movement between two points. Apply a unit virtual force at the location and in the direction of the displacement of interest, and calculate the internal virtual stresses/forces and reactions resulting from the application of the unit virtual force.
3. Write the expressions for $W_{ext}$ and $W_{int}$, where
   - $W_{ext}$ = work of the unit virtual force and its reactions moving through the real displacements of the structure; and
   - $W_{int}$ = work of the internal forces produced by the unit virtual force moving through the real internal deformations of the structure.
4. Set $W_{ext} = W_{int}$ and solve for the displacement of interest.

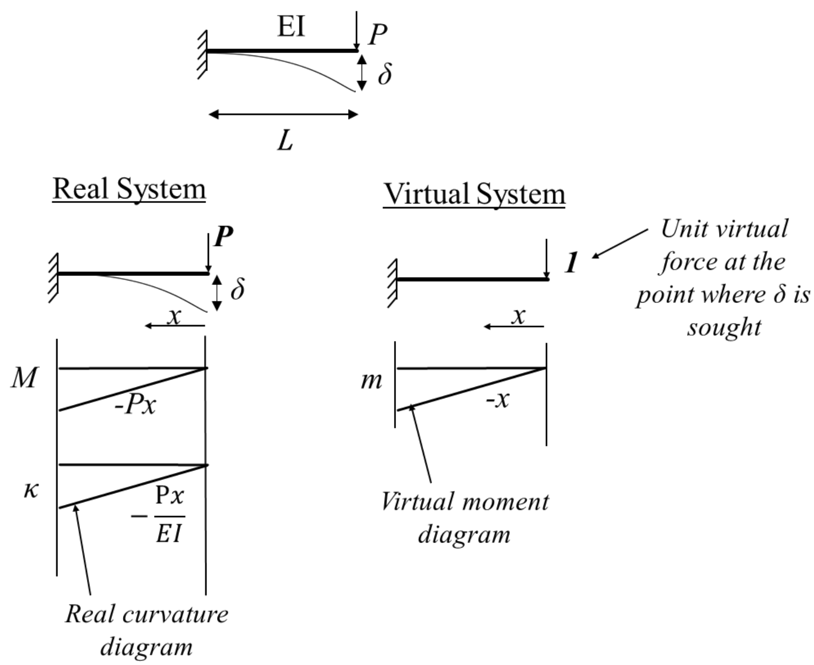
### 6.6.1. Deflections of beams and frames by integration of the work equations

For beams and frames, deflections are primarily due to flexural curvature. Therefore, in the examples that follow, only flexural curvature is considered.

**Example 1**: For the beam shown in Figure 6.9, calculate the downward deflection at the point of load application.

On the left-hand side, we show the real system, including the real load $P$ and tip deflection $\delta$, the real moment diagram $M$, and the real curvature $\kappa$ resulting from $M$. On the right-hand side, we show the virtual force system selected to determine the real displacement $\delta$. Because we want the downward displacement at the free end of the cantilever, we place a unit load at that point and in the downward direction. The virtual moment diagram $m$ is shown.

To solve the problem, we set $W_{ext} = W_{int}$. The external work is the product of the virtual unit force and the real deflection. The internal virtual work is obtained by integrating the product of the virtual moment and the real curvature, in accordance with Eq. (6.10). We then solve for the deflection, which is given in the boxed equation at the bottom of the figure.



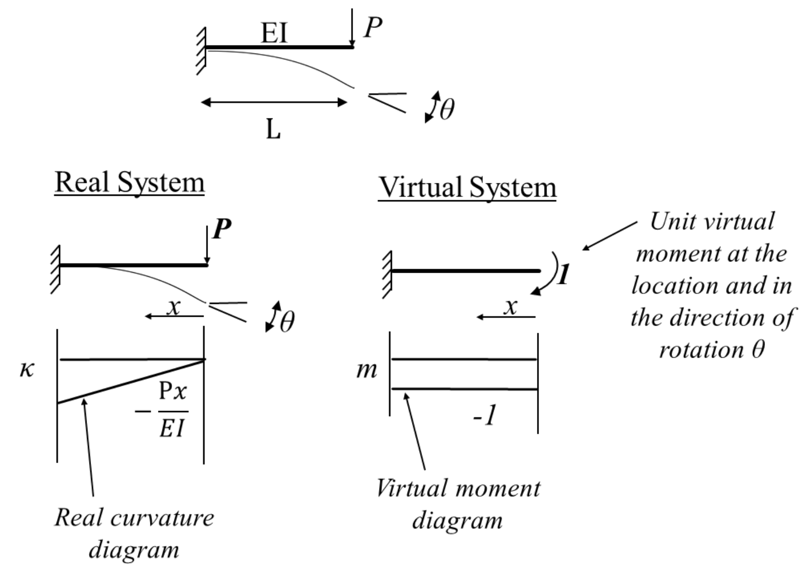

$$W_{ext} = W_{int}$$

$$1 * \delta = \int_0^L (m_x)(\kappa_x)dx = \int_0^L (-x)\left(-\frac{Px}{EI}\right)dx = \left(\frac{Pl^3}{3EI}\right) \rightarrow \delta = \frac{PL^3}{3EI}$$

**Figure 6.9 Deflection at free end of a cantilever with concentrated load.**

**Example 2**: For the same beam and loading as Example 1, determine the end rotation.

The real loading, tip rotation, and curvature are shown on the left-hand side of Figure 6.10. The virtual external force is a unit moment at the location and in the direction of the rotation that is being sought. Setting $W_{ext} = W_{int}$, we solve to find the end rotation. We can check the answer by noting that the change in rotation from the fixed end to the free end is the area under the curvature diagram. Therefore, $\theta = \frac{1}{2}\left(\frac{Pl}{EI}\right) l = \frac{Pl^2}{2EI}$, which matches the result shown in Figure 6.10.




$$W_{ext} = W_{int}$$

$$1 * \theta = \int_0^L (m_x)(\kappa_x)dx = \int_0^L (-1)\left(-\frac{Px}{EI}\right)dx = \left(\frac{Pl^2}{2EI}\right) \rightarrow \theta = \frac{PL^2}{2EI}$$

**Figure 6.10 Rotation at free end of a cantilever with concentrated load.**

### 6.6.2. Deflections of beams and frames by simplified relations

For beams and frames in linearly elastic structures, the internal virtual work generally is of the form

$$W_{int} = \int_0^l (m_x)\left(\frac{M_x}{EI}\right)dx \tag{6.11}$$

If we assume the common case of $EI$ = constant and $m_x$ = linear function = $a + bx$, then

$$\begin{aligned} W_{int} &= \int_0^l (a + bx)\left(\frac{M_x}{EI}\right)dx \\ &= \frac{1}{EI}\left[a\int_0^l M_x dx + b\int_0^l xM_x dx\right] \end{aligned} \tag{6.12}$$

Let $A_m$ = area under the real moment diagram. Then,

$$W_{int} = \frac{A_m}{EI}\left[a + b\frac{\int_0^l xM_x dx}{A_m}\right] \tag{6.13}$$

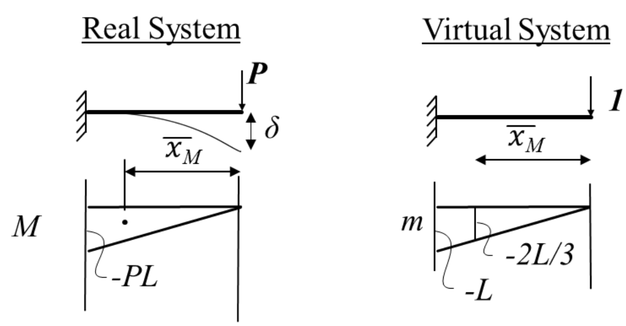
Noting that the fraction to the right of coefficient $b$ defines the centroid of the real moment diagram, we can now write

$$W_{int} = \frac{A_m}{EI}[a + b\overline{x_m}] = \frac{1}{EI}A_m m_{\overline{x_m}} \tag{6.14}$$

in which $A_m$ = area under the real moment diagram, $\overline{x_m}$ = centroidal location of the real moment diagram, and $m_{\overline{x_m}}$ = value of the virtual moment diagram at the centroid of the real moment diagram. Using Eq. (6.14) requires some practice, but once the analyst becomes skilled at its use, the calculations of displacements by virtual work become greatly simplified.

**Example 3**: Rework Example 1 using Eq. (6.14).

The real loading, displacement, and moment diagrams are shown on the left-hand side of Figure 6.11. The centroid of the real moment diagram is located at $\overline{x_m} = \frac{2}{3}l$ from the free end of the beam. Turning now to the right-hand side of Figure 6.11, we show the virtual loading and virtual moment diagram. We seek $m_{\overline{x_m}}$, which is the value of the virtual moment diagram at the centroid of the real moment diagram. As shown, this value is $-\frac{2}{3}l$. We then set $W_{ext} = W_{int}$, with $W_{int}$ defined according to Eq. (6.14). The answer matches the answer obtained in Example 2.



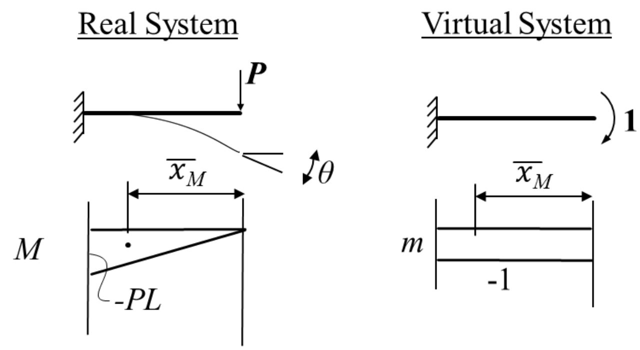

$$\mathrm{W_{ext}} = \mathrm{W_{int}}$$

$$1 * \delta = \frac{1}{EI} * A_M * m_{\overline{x_M}} = \frac{1}{EI} * \frac{(-\mathrm{PL})^2}{2} * \frac{(-2\mathrm{L})}{3} = \frac{PL^3}{3EI}$$

**Figure 6.11 Example 3 using Eq. (6.14).**

**Example 4**: Rework Example 2 using Eq. (6.14).

The real loading, rotation, and moment diagrams are shown on the left-hand side of Figure 6.12. The centroid of the real moment diagram is located at $\overline{x_m} = \frac{2}{3}l$ from the free end of the beam. Turning now to the right-hand side of Figure 6.12, we show the virtual loading and virtual moment diagram. We seek $m_{\overline{x_m}}$, which is the value of the virtual moment diagram at the centroid of the real moment diagram. As shown, this value is $-1$. We then set $W_{ext} = W_{int}$, with $W_{int}$ defined according to Eq. (6.14). The answer matches the answer obtained in Example 2.



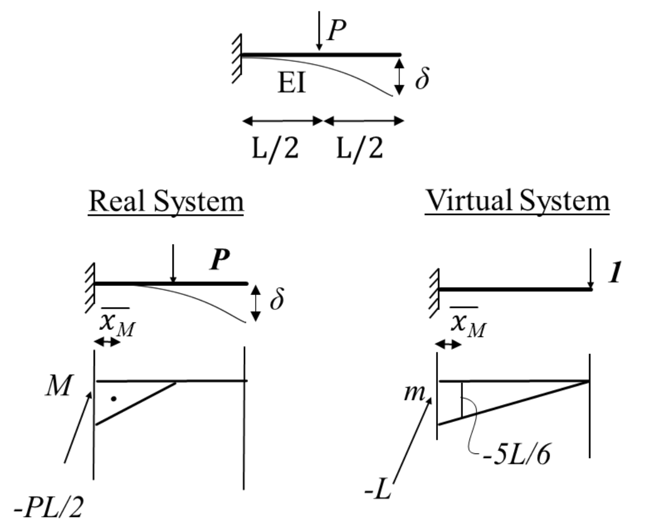

$$W_{ext} = W_{int}$$

$$1 * \theta = \frac{1}{EI} * A_M * m_{\overline{x_M}} = \frac{1}{EI} * \frac{(-PL)^2}{2} * (-1) = \frac{PL^2}{2EI}$$

**Figure 6.12 Example 4 using Eq. (6.14).**

**Example 5**: Find the tip displacement for the beam shown in Figure 6.13.

As in the previous examples, we draw the real and virtual parts of the solution. Note that the real loading involves a force at mid-span, but the displacement is sought at the free end of the beam. Thus, it is required to place the unit virtual load at the free end of the beam. We next locate the centroid $\overline{x_m}$ of the real moment diagram, find the value of the virtual moment diagram at that position along the beam, and solve the problem using Eq. (6.14).



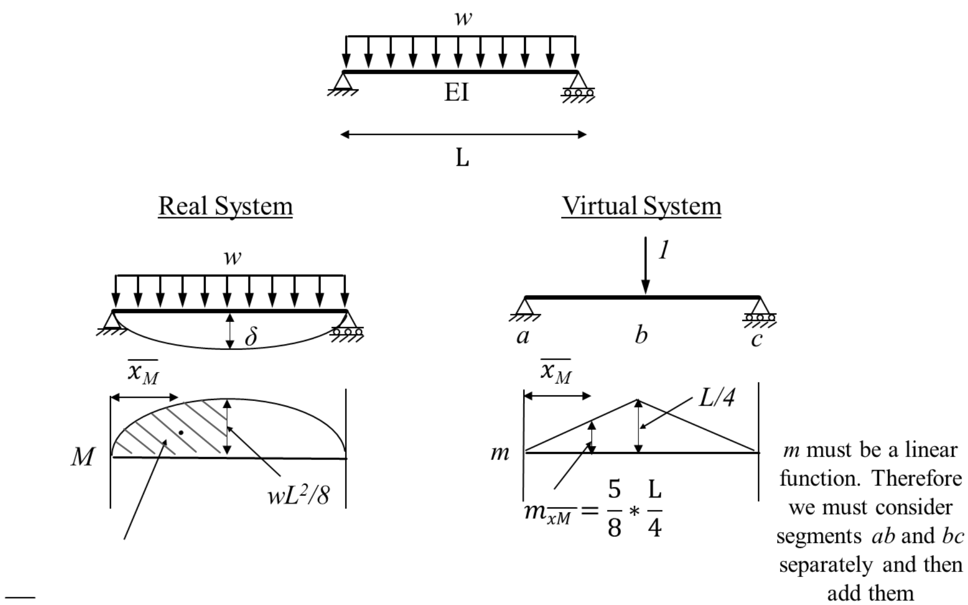

$$W_{ext} = W_{int}$$

$$1 * \delta = \frac{1}{EI} * A_M * m_{\overline{x_M}} = \frac{1}{EI} * \frac{(-\mathrm{PL})}{2} * \frac{L}{2} * \frac{1}{2} * \frac{(-5*L)}{6} = \frac{5\mathrm{P}L^3}{48EI}$$

**Figure 6.13 Example 5 using Eq. (6.14).**

**Example 6**: Find the midspan deflection for the beam shown in Figure 6.14.

We first draw the real and virtual parts of the solution, with the virtual unit load at the midspan where the deflection is sought. Note that the virtual moment diagram does not consist of a single line segment as was assumed in the derivation of Eq. (6.14), so we cannot directly apply Eq. (6.14) without modification. To solve the problem, we split the beam into two separate parts, one involving segment ab and the other involving segment bc, such that the virtual moment diagram is a straight line segment along each part. Having done this, we proceed as usual to find the centroid of the real moment diagram and the value of the virtual moment diagram at that location. Given symmetry of the beam, the loading, and the virtual moment diagram, we simply solve segment ab and multiply by 2 to obtain the answer.




$$W_{ext} = W_{int}$$

$$1 * \delta = 2\left(\frac{1}{EI} * A_M * m_{\overline{x_M}}\right) = 2\frac{1}{EI} * \frac{2}{3} * \frac{L}{2} * \frac{wL^2}{8} * \frac{5}{8} * \frac{L}{4} = \frac{5wL^4}{384EI}$$

Adding two equal parts

**Figure 6.14 Example 6 using Eq. (6.14).**

**Example 7**: Find the horizontal displacement at point D due to a point load Q at B.

Real and virtual systems are shown. Centroids of the real moment diagrams are at $\frac{2}{3}l$ for members AB and BC. Values of the virtual moment diagram at these locations are $\frac{2}{3}h$ and $h$, respectively. Areas of the real moment diagrams are $\frac{Qh^2}{2}$ and $\frac{QhL}{2}$ for members AB and BC respectively. Thus, the answer is $\delta_D = \frac{1}{EI_c}\frac{Qh^2}{2}\frac{2}{3}h + \frac{1}{EI_G}\frac{QhL}{2}h = \frac{Qh^3}{3EI_c} + \frac{Qh^2L}{2EI_G}$.

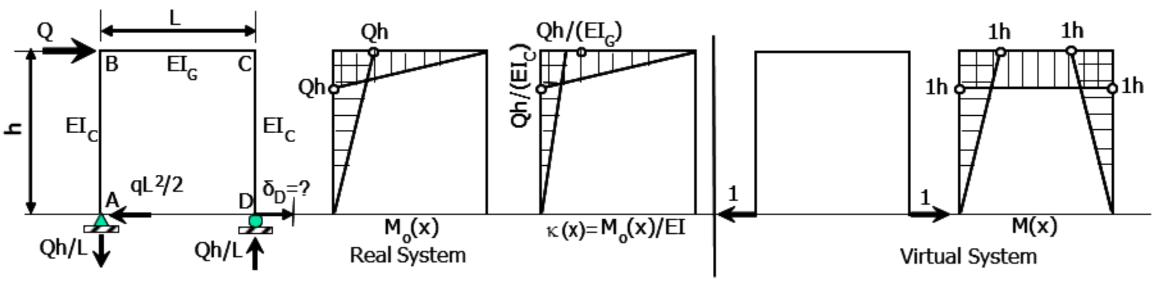



**Figure 6.15 Example 7.**

**Example 8**: Find the vertical displacement at the free end of the L-shaped frame due to the temperature differential shown.

The real curvatures in this example are due to temperature differential, rather than externally applied force. The real curvatures are calculated based on the temperature differential. Virtual moments are due to a unit load at free end of the frame. To solve the problem, we could use Eq. (6.10) to define $dW_{int}$, and then integrate $dW_{int}$ along the column and beam. Alternatively, we could observe that Eq. (6.14) uses $A_m/EI$, which is the area of the moment diagram divided by EI, but this is identically equal to the area under the real curvature diagram. Values of the virtual moment diagrams at the centroids of the real curvature diagrams are 8m for the column and 4m for the beam. Thus, the deflection is $\delta = (0.00016/m)(6m)(8m) + (0.00024/m)(8m)(4m) = 0.01536m = 15.4\ mm$.

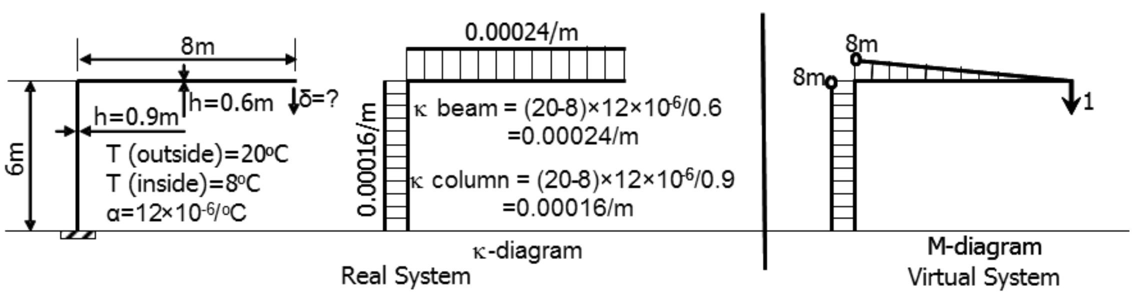



**Figure 6.16 Example 8 - Deflection due to temperature differential.**

### 6.6.3. Deflection of Truss Structures

Truss deflections are due primarily to axial deformation of the truss members (plus deflections of the supports, if any). In general, we do not consider deformations due to bending or shear. Thus, the internal work $W_{int}$ of interest is the work due to axial forces acting through axial deformations of the truss members, as defined by Eq. (6.8). When considering internal virtual work, we are interested in the virtual work of virtual axial forces acting through real axial deformations.

**Example 9**: For the truss in Figure 6.17, calculate the downward deflection of point c under the action of a 5 kip load.

The solution is separated into real and virtual parts. The real forces in the truss members and the real deflection $\delta$ are shown in the sketch on the left-hand side. Because we seek the downward deflection at point c, we place a unit virtual work at point c and calculate the virtual internal forces, all shown in the sketch on the right-hand side. To facilitate the calculations, we create a table in which we identify the members, member lengths, the real forces $P_i$, the resulting member elongations $\delta_i$, the virtual forces $p_i$, and finally the virtual work in each member $p_i\delta_i$. We sum up the $p_i\delta_i$ terms to obtain the total internal virtual work $W_{int}$, equate this to the external virtual work $W_{ext} = 1 \times \delta$, and solve for the deflection $\delta$.

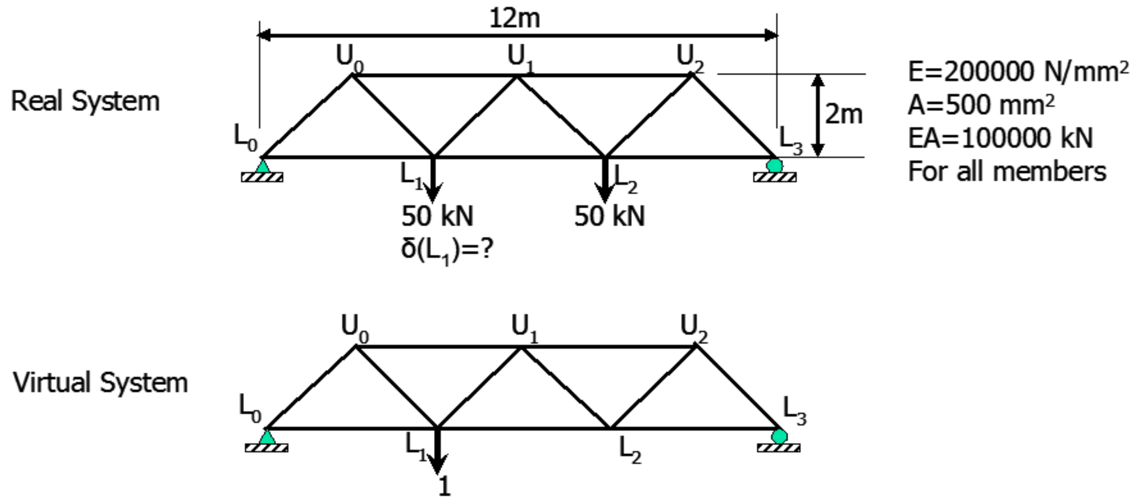

Real system Virtual system Virtual work

| Member | L | $P_i$ | $\delta_i = \frac{PL}{EA}$ | $p_i$ | $p_i * \delta_i$ |
|---|---|---|---|---|---|
| *ac* | 100” | -5k | -0.0167” | -1 | 0.0167 |
| *bc* | 141” | 7.07k | -0.0332” | $\sqrt{2}$ | $0.0332\sqrt{2}$ |
| | | | | Σ= | 0.0637 |

$$W_{ext} = W_{int}$$
$$1 * \delta = 0.0637\ in$$

**Figure 6.17 Example 9.**

**Example 10**: Calculate the downward deflection of joint $L_1$, $\delta(L_1)$, due to the shown loads. The solution proceeds in similar fashion as for Example 9.




| Member | Length $L_i$ (m) | Real Force, $P_i$ (kN) | Real elongation, $\delta_i = P_iL_i/EA_i$ | Virtual Force, $p_i$ | Internal Virtual Work, $p_i\delta_i$ |
|---|---|---|---|---|---|
| $L_0L_1$ | 4.0 | 50 | 0.002 | 0.667 | 0.00133 |
| $L_1L_2$ | 4.0 | 100 | 0.0040 | 1.000 | 0.00400 |
| $L_2L_3$ | 4.0 | 50 | 0.0025 | 0.333 | 0.00067 |
| $L_0U_0$ | 2.83 | -70.7 | -0.002 | -0.943 | 0.00189 |
| $U_0L_1$ | 2.83 | 70.7 | 0.002 | 0.943 | 0.00189 |
| $L_1U_1$ | 2.83 | 0 | 0 | 0.471 | 0 |
| $U_1L_2$ | 2.83 | 0 | 0 | -0.471 | 0 |
| $L_2U_2$ | 2.83 | 70.7 | 0.000707 | 0.471 | 0.00094 |
| $U_2L_3$ | 2.83 | -70.7 | -0.000707 | -0.471 | 0.00094 |
| $U_0U_1$ | 4.0 | -100 | -0.0010 | -1.333 | 0.00533 |
| $U_1U_2$ | 4.0 | -100 | -0.0010 | -0.667 | 0.00267 |

$\sum$ 0.0197 m

$$W_{ext} = W_{int} \Rightarrow 1 \times \delta(L_1) = 0.0197 \text{ m, or } \delta(L_1) \sim 20 \text{ mm.}$$

**Figure 6.18 Example 10.**

**Example 11**: Reconsider the truss of Example 10. The external loads are removed. Instead, various members are subjected to temperature change $T$, as indicated in the third column of the table within Figure 6.19. The coefficient of thermal expansion is $\alpha=12\times10^{-6}/^{o}C$. Calculate the downward deflection $\delta(L_1)$ of joint $L_1$.

The solution requires determination of the real change in length of each of the members due to the real temperature change. Column 4 in the table below lists the strain due to temperature change. Column 5 repeats the internal forces due to the unit load applied at joint $L_1$. Column 6 determines the internal virtual work associated with the product of the internal virtual force and the internal real deformation. The internal virtual work is summed, equated with external virtual work, and the deflection is solved.

| **Member** | **Length** $L_i$ (m) | **Real Temp. Change,** $T_i\ (^oC)$ | **Real strain,** $\varepsilon_i$ | **Virtual Force,** $p_i$ | **Internal Virtual Work,** $p_i\times\varepsilon_i\times L_i$ |
|---|---|---|---|---|---|
| $L_0L_1$ | 4.0 | -10 | -0.00012 | 0.667 | -0.00032 |
| $L_1L_2$ | 4.0 | -10 | -0.00012 | 1.000 | -0.00032 |
| $L_2L_3$ | 4.0 | 0 | 0 | 0.333 | 0 |
| $L_0U_0$ | 2.83 | 0 | 0 | -0.943 | 0 |
| $U_0L_1$ | 2.83 | 0 | 0 | 0.943 | 0 |
| $L_1U_1$ | 2.83 | 0 | 0 | 0.471 | 0 |
| $U_1L_2$ | 2.83 | 0 | 0 | -0.471 | 0 |
| $L_2U_2$ | 2.83 | 0 | 0 | 0.471 | 0 |
| $U_2L_3$ | 2.83 | 0 | 0 | -0.471 | 0 |
| $U_0U_1$ | 4.0 | +10 | +0.00012 | -1.333 | -0.00064 |
| $U_1U_2$ | 4.0 | +10 | +0.00012 | -0.667 | -0.00032 |
| | | | | $\sum$ | -0.00176 m |

$$W_{ext} = W_{int} \Rightarrow 1 \times \delta(L_1) = 0.00176\text{ m, or } \delta(L_1) \sim 2\text{ mm.}$$

**Figure 6.19 Example 11.**

**Example 12**: Reconsider the truss of Example 10. All loads are removed, but the support at $L_3$ settles by 30 mm, as shown in Figure 6.20. What is the downward movement at point $L_1$?

To solve this problem, we treat the displacement at $L_3$ as a real external displacement. We then apply a downward unit virtual force at joint $L_1$ (the location where we want to find the vertical movement) and calculate the external virtual reaction at joint $L_3$, which is 0.33 upward. The settlement at joint $L_3$ does not create any internal stresses or strains, so there is no internal work. Consequently, the work equation becomes $W_{ext} = (1)\big(\delta(L_1)\big) + (-0.333)(30\ mm) = 0$. Therefore, $\delta(L_1) = 10\ mm$, which we can confirm by inspection of the geometry.

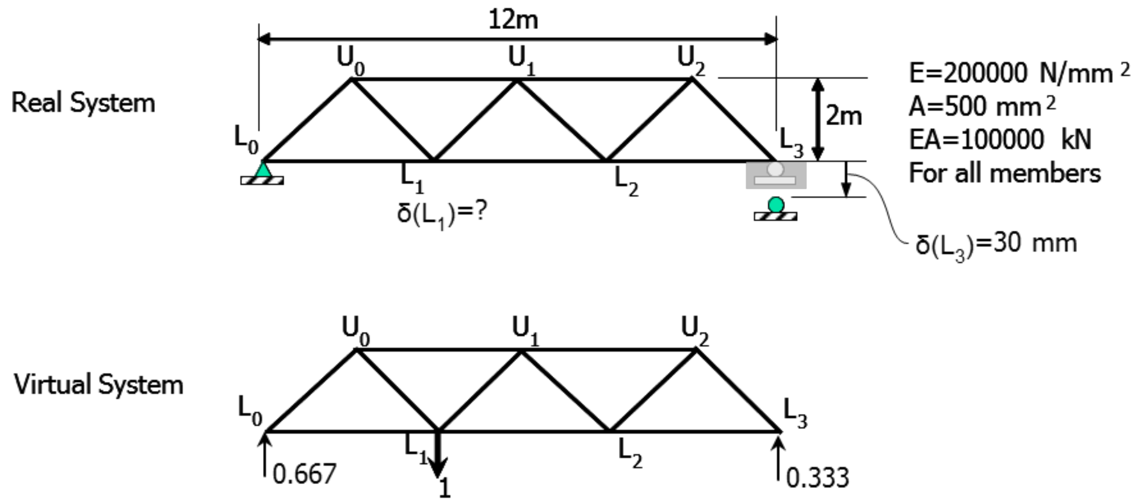



**Figure 6.20 Example 12.**

# 7. Use of Computer Software for Structural Analysis

## 7.1. INTRODUCTION

Computer methods are an integral part of modern structural engineering. In addition to software for routine numerical work (such as EXCEL and MATLAB), specialized structural engineering software is used to calculate response of structures to external loads, both static and dynamic, and structural engineers interact with geotechnical engineers using specialized software for soil-structure interaction and with construction engineers, architects, and owners using Building Information Management (BIM) software.

This chapter introduces the use of specialized structural engineering software that is used to calculate response of structures to external loads. The discussion is limited to static loading. Emphasis is on how to set up the structural analysis problem and how to interpret and check the results. For the theory and its implementation in computer software, the student is referred to CE 121.

## 7.2. STRUCTURAL IDEALIZATIONS

As with hand calculations, structural analysis using computer software generally requires some idealization of the real-world problem. As an example, consider the three-dimensional structure of Figure 7.1. Although some computer software will enable the structure to be modeled using numerical representations of the members that very closely replicate the entire structure, most analyses are done on a much more simplified representation of the structure. For example, the structure of Figure 7.1 might be idealized as a three-dimensional frame comprising beams, columns, beam-column joints, and supports as shown in Figure 7.2.

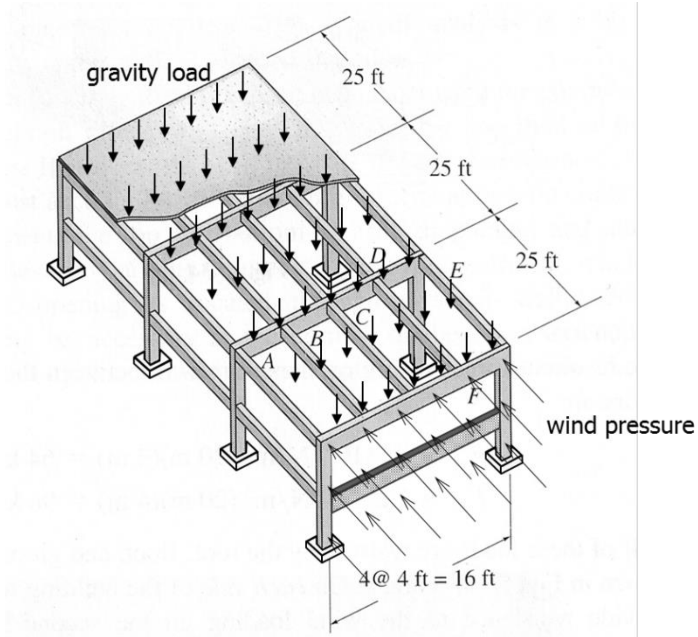



**Figure 7.1 Three-dimensional structure and loading.**

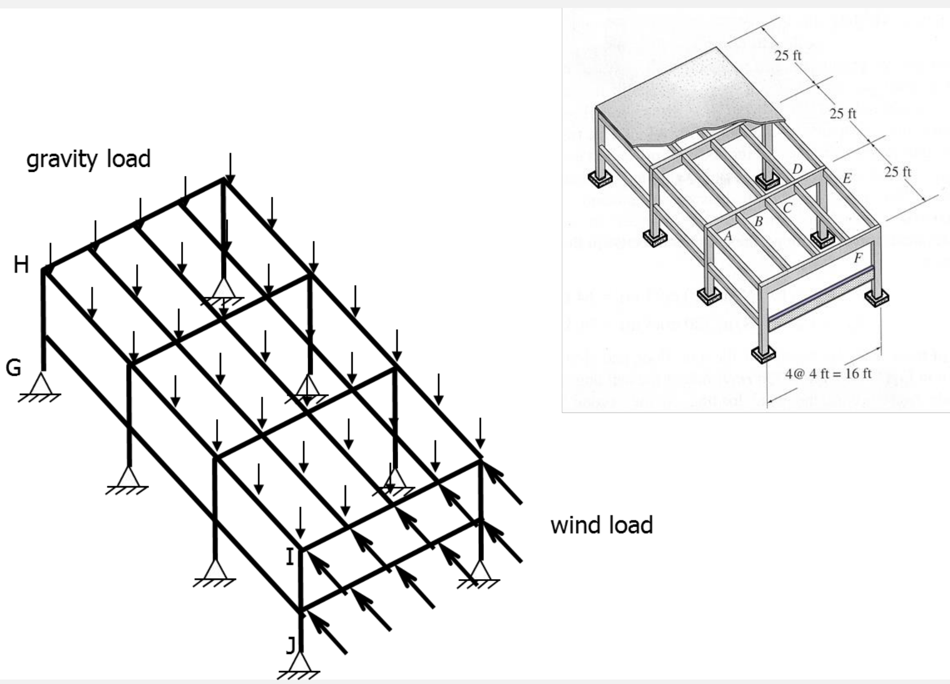



**Figure 7.2 Three-dimensional frame model of the structure of Figure 7.1.**

While three-dimensional models are widely used in engineering practice today, some problems are further idealized as being two dimensional. Figure 7.3 depicts a two-dimensional model that might be used to analyze for the internal forces and deformations of the primary vertical and lateral-load-resisting system of the subject building in the longitudinal direction. Two-dimensional models are used in CE 120, in part because they provide an good introduction to the issues of using structural analysis software, and in part because some student versions of software have limited functionality that is more suitable for analysis of two-dimensional structures.

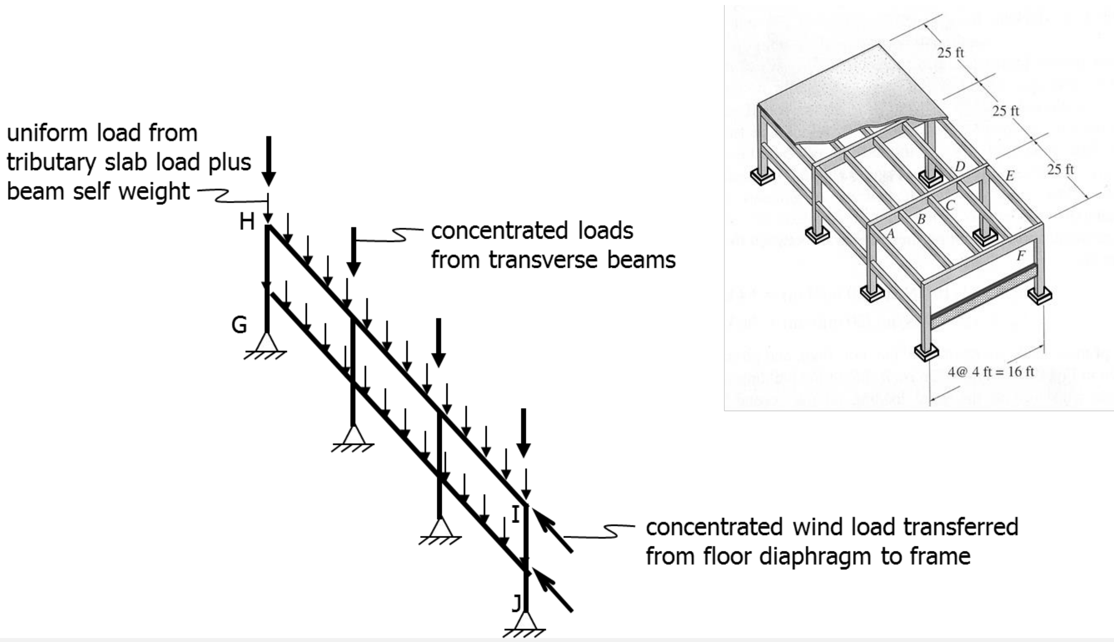



**Figure 7.3 Two-dimensional idealization of the longitudinal framing of the structure of Figure 7.1.**

## 7.3. GEOMETRY AND CONNECTIVITY

To set up a problem for structural analysis, most software packages require the analyst to establish a system for numbering the joints and the members. Figure 7.4 illustrates a common implementation. First, joints are numbered in some logical sequence from 1 through $n$, where $n$ = the number of joints in the structure. In the example shown, $n = 12$. Next, the members are numbered in some logical sequence from 1 through $m$, where $m$ = the number of members in the structure. In the example shown, $m = 14$. Note that the numbering sequence for joints and members is somewhat arbitrary. A logical sequence is one that is relatively easy for the analyst to keep track of. In the early days of structural analysis software, when computer speeds were much slower than they are today, the numbering scheme was important to the solution time. While such aspects can still affect software speed, the consequences are not so important for most problems that we will be solving.

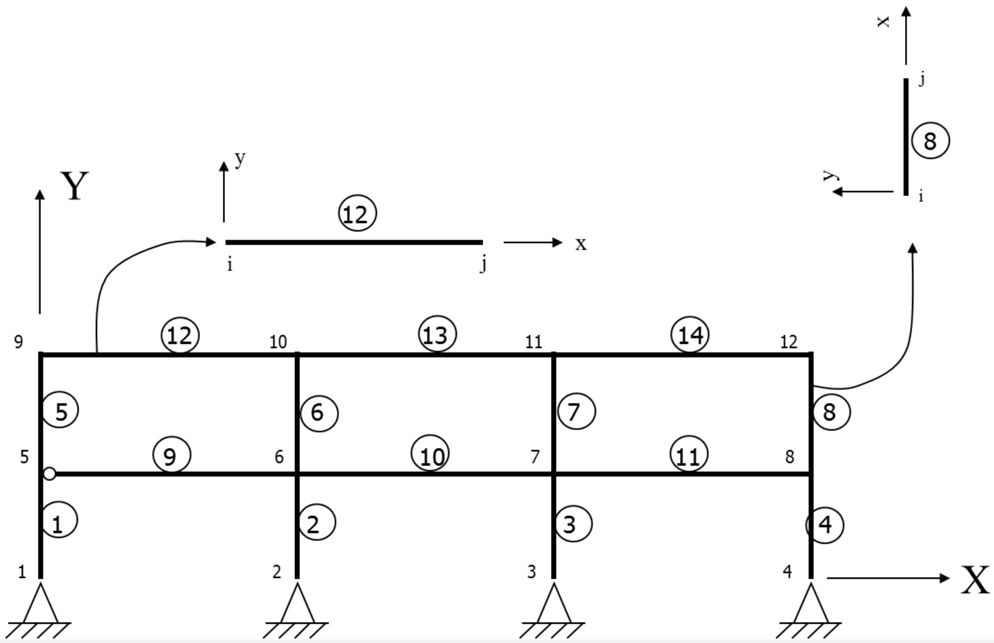



**Figure 7.4 Geometry, connectivity, and global and local coordinate systems.**

Our structural models will be limited to those that can be reasonably represented by line elements (beam-column elements). Note that each line element must be assigned a member number and an *i* end and a *j* end. In Figure 7.4, beam 12 extends between joints 9 and 10, with the *i* end attached at joint 9 and the *j* end attached at joint 10. As another example, column 8 extends from joint 8 to joint 12, with the *i* end attached at joint 8 and the *j* end attached at joint 12. There is no restriction that the *i* end has to be attached to the joint with the lesser number while the *j* end is attached to the joint with the greater number. Rather, the analyst just needs to know which end (*i* or *j*) is attached to which joint. To facilitate keeping track of input and output quantities, it generally is best to number members in a consistent way, that is, all beams from left to right and columns from bottom to top.

A *global coordinate* system X-Y (or X-Y-Z in 3-dimensional problems) needs to be established such that the locations of all the joints can be defined. It is up to the analyst to establish this coordinate system. In the example of Figure 7.4, the origin of the X-Y coordinate system coincides with joint 1.

Additionally, a *local coordinate* system x-y-z has to be established for every line member. In most software packages, the x-axis has its origin at end *i* and extends along the member axis to joint *j*. As shown in Figure 7.4, for some members the local coordinate system may be parallel to the global coordinate system (for example, member 12) while for some other members the local coordinate system is rotated relative to the global coordinate system (for example, member 8). For the members, the structural analysis software will need to define a rule to relate x, y, and z axes. Most software will use the right-hand rule.

Once all the joint positions are established, and the members are connected to the joints, the next step is to define the boundary conditions for the global system and for the individual members. At the global level, software will need to know which of the joints correspond to support points, and it will need to know the type of support. In the example of Figure 7.4, joints 1 through 4 would be identified as pinned supports. Depending on the software, the analyst may simply be able to specify that these are pinned supports, or she might need to specify which *degrees of freedom* are fixed and which are restrained. For fixed supports, the support is fixed against translation in X and Y directions and against rotation in the Z direction. Stated differently, the three degrees of freedom within the plane are fixed. Pinned supports would be fixed against translation in X and Y directions but would be free to rotate in the Z direction. Roller supports would be fixed against translation in one direction but would be free to translate in the orthogonal direction and to rotate in the Z direction.

Member connectivity also needs to be specified. In the example of Figure 7.4, all members except member 9 are rigidly connected to the joints such that they can transfer axial force, shear, and moment. For member 9, the *i* end has a pin that enables rotation without moment about the local z axis. Thus, for member 9, end *i* would be specified to have a release about the z axis. Note that some software may have restrictions about the number of members that can have releases at a joint. The restriction relates to avoiding singularities in the stiffness matrices used in the calculations. The software will issue an error if this problem arises.

## 7.4. LOADS

Most software for structural analysis permits two types of loads, as follows:

- Nodal loads – these are loads (forces or moments) applied directly to the joints/nodes.
- Member loads – these are loads applied directly to the members, and can be either concentrated loads or distributed loads. The position and direction (positive or negative) of loads usually needs to be specified. In some software, the member loads are defined relative to the local coordinate system of the member; thus, a concentrated load of -5 kips at x = 3 ft on member 9 in Figure 7.4 would be a load in the negative y direction (that is, downward in the global coordinate system) at 3 feet to the right of end *i*. Alternatively, the software may define the loads in the global coordinate system, or may provide the option of using either the local or the global coordinate system.

## 7.5. UNITS

Software for structural analysis uses consistent units, solving the structural analysis problem solely as a numerical solution. Thus, the analyst is required to specify all input data in consistent units, with the output being in those same units. For example, the input could be in US customary units of kips and inches, as in:

- lengths and coordinates are in inches;
- member properties – Areas are in.$^2$, moments of inertia are in.$^4$;
- forces are kips, kips per in., or ksi, and moments are kip-in.;
- member stiffnesses are kip/in, and material properties are ksi;

The output would all be in the same units, as in:

- deflections in inches;
- forces and reactions in kips; moments in kip-in.
- etc.

Some software will have built-in member properties, in which case may be necessary to specify the system of units being used.

## 7.6. MEMBER PROPERTIES

The analyst generally will be required to specify the member material properties. These properties include quantities such as the Young's Modulus, *E*; shear modulus, *G*; cross-sectional area, *A*; and moment of inertia, *I*. The units should be consistent with the units selected for the problem, as noted in Section 7.5.

Most software permits the analyst to specify a member material type, which will include all the relevant properties (*E*, *G*, *A*, *I*, etc.) for that member type. Then the member type can be assigned to any of the members. For the structure shown in Figure 7.4, if all the beams have the same cross sections and materials, then the analyst can define material type *Beam 1* and assign that material type to members 9 through 14.

## 7.7. COMPUTER ANALYSIS OUTPUT

Output from computer analysis will include support reactions; joint rotations and deflections; and member internal forces. The results should always be checked to be certain they are consistent with expectations. Usually, it is a good idea to start with a simple problem for which the results are known and verify that the computer results are consistent with that known answer.

Different software packages treat member internal forces differently. Some will define member moments as being positive for moment that puts the local y side of the member in tension, others will define positive moments if they put the local y side of the member in compression, and yet others will define moments as positive if they act clockwise or counter-clockwise on the ends of the member *i* and *j*. The analyst needs to examine the results to be certain that the sign convention is understood. Free-body cuts can be used to check results.

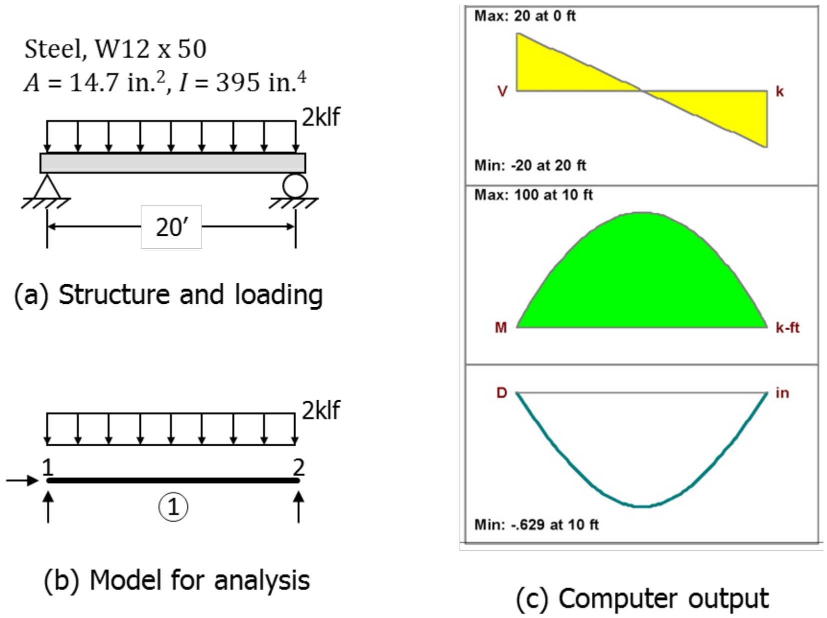
## 7.8. EXAMPLES

Some examples are outlined below.

**Example 1**: Use computer software to analyze a simply supported beam under uniformly distributed load (Figure 7.5a). Check that the output results are consistent with known solutions.

**Solution**: The software RISA 2D is used. There problem involves one member and two nodes (Figure 7.5b). The output results are shown in Figure 7.5c. The results can be checked by hand calculations.

- The maximum shears should be $V = \pm\frac{wl}{2} = \pm\frac{2\times 20}{2} = \pm 20\ kips$. <u>OK</u>.
- The maximum moment should be $M = \frac{wl^2}{8} = \frac{2\times 20^2}{8} = 100\ k - ft$. <u>OK.</u>
- First, we get all the input quantities in consistent units of kips and inches. The load is $w$ = 2 klf = 0.167 kip/in. The span is $l$ = 20 ft = 240 in. Using results from Chapter 6, the maximum deflection should be $\delta = \frac{5}{384}\frac{wl^4}{EI} = \frac{5}{384}\frac{0.167k/in.\times 240in.^4}{29000ksi\times 395in.^4} = 0.629\ in.$ <u>OK.</u>




(a) Structure and loading

(b) Model for analysis

(c) Computer output

**Figure 7.5 Example 1.**

**Example 2**: Use computer software to analyze how the answer to Example 1 is affected if the ends are fixed against rotation rather than being simply supported (Figure 7.6a).

**Solution**: The software RISA 2D is used. The output results are in Figure 7.6c, with the following observations:

- The shears are unaffected by fixing the beam ends. $V = \pm\frac{wl}{2} = \pm\frac{2\times20}{2} = \pm20\ kips$.
- The moments are strongly affected by end fixity, as should be expected. The fixed end moments are $M = \frac{wl^2}{12}$. The midspan moments are $M = \frac{wl^2}{8}$.
- The deflections are decreased from 0.629 in. to 0.126 in., or to one-fifth of the deflection of the simply supported beam.

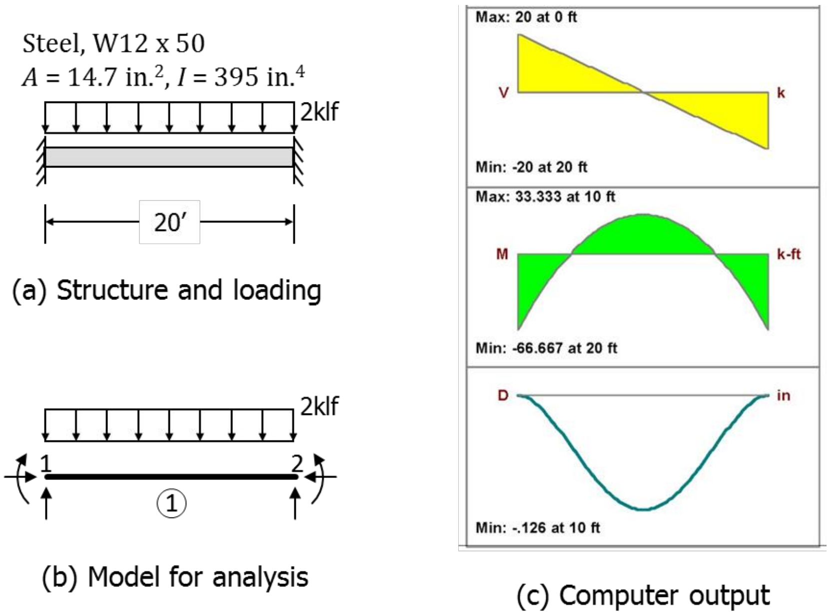



**Figure 7.6 Example 2.**

**Example 3**: Use computer software to determine the vertical reactions in a continuous beam subjected to uniformly distributed load *w*.

**Solution**: The software RISA 2D is used. The output results are in Figure 7.7. We can check that the sum of vertical reactions balances the vertical loads. OK. The shears are similar to $wl/2$, obtained for a simply supported beam or a fixed-fixed beam. OK. Interestingly, the exterior reactions are close to $wl/2$ and the interior reactions are close to $wl$. We will use this result in the tributary area method introduced in Chapter 9.

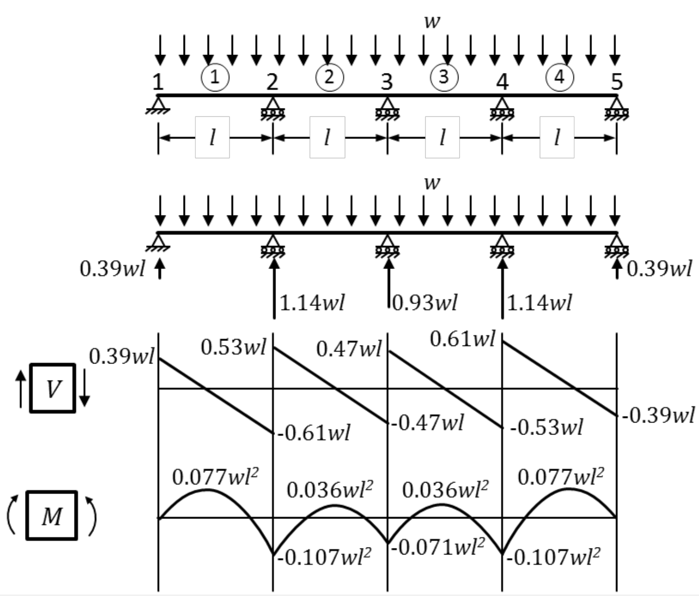



**Figure 7.7 Example 3: Continuous beam.**

# 8. Building Codes

## 8.1. INTRODUCTION

A *building code* is a set of rules that specify minimum acceptable requirements for the design and construction of a building. The main purpose of a building code is to protect the welfare, safety, and health of the public. Building codes cover a range of subjects, from administration and enforcement, general building requirements, loads, and material-specific design requirements. This chapter introduces the building codes that are widely used in the United States, with emphasis on those codes that are used in California. Subsequent chapters will present requirements from these codes related to loads, design methods, and proportioning and detailing of structural members in wood, steel, and reinforced concrete.

## 8.2. BUILDINGS CODES IN THE UNITED STATES

Different countries have different laws governing the construction of buildings. The organization of building codes in the different countries follows from those laws. For example, in some countries the construction of buildings is under the jurisdiction of the national government. In others, however, enforcement of building requirements is delegated to entities other than the national government. In the United States, the tenth amendment of the U.S. Constitution reads:

> *The powers not delegated to the United States by the Constitution, nor prohibited by it to the States, are reserved to the States respectively, or to the people*.

Because the U.S. Constitution does not address building regulation, the power to establish and enforce laws for buildings and other construction is passed, through the 10th amendment, to the individual states. Some states have adopted state-wide building codes, while others have delegated code adoption to the local level. Regardless, there is no national building code in the United States.

The wide range of code adoption possibilities in the various states has the potential to lead to an equally wide range of different codes in different states and local regions. This would make it very difficult for an engineer in one city to perform designs in another city or state. To solve this problem, the building regulation community in the United States has developed a series of model codes. A *model building code* is a document that is written in a building code format such that it can be adopted by a responsible jurisdiction and thereby can become the legally binding building code.

## 8.3. BUILDINGS CODES IN CALIFORNIA

In the State of California, the main building codes are those illustrated in Figure 8.1. They include:

- The *California Building Code (CBC)* – This code is developed by the California Building Standards Commission (http://www.bsc.ca.gov). Established by the California Building Standards Law, the California Building Standards Commission is organized within the Department of General Services of the State of California. It is charged with codifying and publishing approved building standards in one state building standards code (California Code of Regulations, Title 24). At the time of this writing, the current version of the CBC was published in 2013. The CBC mainly adopts the provisions of the International Building Code.
- The *International Building Code (IBC)* – This model code is developed by the International Code Council, which is non-profit association that develops model codes and standards for buildings. The IBC addresses the establishment of a *Department of Building Safety* and a *building official*, who is an officer or other designated authority charged with the administration and enforcement of the building code for buildings constructed within the *jurisdiction*[1]. It also covers building use categories and specific requirements for different use categories; fire protection systems; accessibility and egress; and height limits. At the time of this writing, the current version of the IBC was published in 2012. It references ASCE 7 for determination of loads and load effects, and it references materials codes for design of structures using wood, steel, and reinforced concrete.
- *ASCE 7* – This document has the title *Minimum Design Loads for Buildings and Other Structures (ASCE/SEI 7-10)*. ASCE 7 provides requirements for general structural design and includes means for determining dead, live, soil, flood, snow, rain, atmospheric ice, earthquake, and wind loads, as well as their combinations, which are suitable for inclusion in building codes and other documents. At the time of this writing, the current version of ASCE 7 was published in 2010.
- *Materials Codes* – There are different materials codes, one or more for each of the main structural materials.
    - *National Design Specification (NDS) for Wood Construction* contains requirements for wood construction.
    - *Steel Construction Manual (AISC)* contains requirements for structural steel construction.
    - *Building Code Requirements for Structural Concrete (ACI 318)* contains requirements for reinforced and prestressed concrete construction.

The aforementioned building codes cover most of the requirements in California, and other states adopt almost the exact same set of requirements from the IBC, ASCE 7, and the materials codes. However, each local jurisdiction can adopt exceptions to the provisions of these codes in order to accommodate special conditions that affect the jurisdiction and that are not adequately covered in the more generally adopted codes. Therefore, it is important to check with the local building official to determine any local requirements.

[1] A jurisdiction is defined as a governmental unit that has adopted a code under due legislative authority, and that has authority over the construction of buildings within a geographic region.

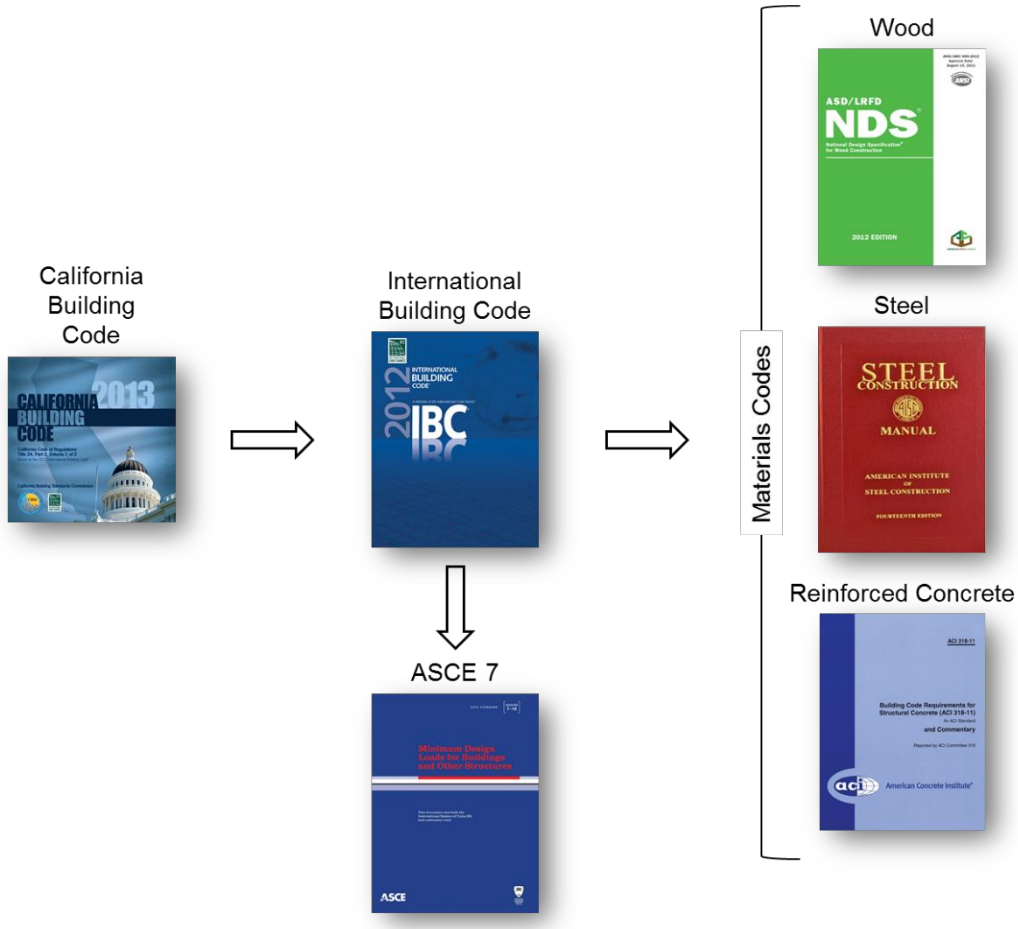



Figure 8.1 Building Codes adopted in California.

# 9. Gravity Loads and Load Paths

## 9.1. INTRODUCTION

This chapter introduces how a structural engineer determines the gravity loads and the loading combinations for design of a structure. The concept of a load path is introduced and used to identify how a structure resists vertical and lateral loads. The loads and methods generally follow the procedures of ASCE 7-10.[1]

## 9.2. LOAD TYPES

*Loads* on structures can be either *externally applied forces* (e.g., self-weight, live loads, wind loads) or *imposed deformations* (e.g., expansion due to temperature change or foundation settlement). In some documents, loads are referred to by the term *actions*. We will use the two terms *loads* and *actions* interchangeably in this text.

Building codes classify loads based on their origin. This is convenient because some loads are determined by the structure itself, some by its occupancy, and some by the environment in which the structure is located. The different load types have different variability, duration, and directionality effects that may need to be considered in design. The main load types that are considered in ASCE 7 are:

D = dead load
E = earthquake load
F = load due to fluids with well-defined pressures and maximum heights
$F_a$ = flood load
H = load due to lateral earth pressure, ground water pressure, or pressure of bulk materials
L = live load
$L_r$ = roof live load
R = rain load
S = snow load
T = self-straining load
W = wind load

Among these, the following merit additional discussion:

**Dead load (D)** – These are loads due to self-weight and items that are permanently attached to a structure, such as floor finishes, HVAC (heating, ventilation, and air

[1] ASCE 7-10, Minimum Design Loads for Buildings and Other Structures (ASCE/SEI 7-10), American Society of Civil Engineers, 2010,

conditioning). Dead loads are constant in magnitude, direction, and position in the structure.

**Live load (L)** – These are loads due to occupancy and use, such as occupants, furnishings, and traffic. Some live loads may be relatively long-term, such as books in a library stack. However, live loads are usually considered to be short term loadings that are not constant in magnitude or location.

**Snow load (S), Rain load (R), Wind load (W), and Earthquake Load (E)** – These are loads attributed to the environment, and are generally of short duration.

This chapter is concerned mainly with gravity loads, with emphasis on dead, live, rain, and snow loads. Subsequent chapters consider wind and earthquake loading.

## 9.3. DEAD LOADS

Dead loads include self-weight of the structure and items that are permanently attached to the structure. For some structural materials, self-weight may be tabulated on a unit-area basis. For others, the unit density is used with the volume of material to calculate the self-weight. Some pieces of fixed equipment such as roof chillers introduce large concentrated loads that should be considered directly. Others dead loads such as HVAC may be applied as concentrated or line loads, but their exact position in the structure is not known at the time the structure is designed and their magnitude is not large relative to other loads, and, therefore, as a matter of convenience, they are treated as average uniformly distributed loads. Distributed floor, roof, and wall (or cladding) loads refer to loads per square foot (or square meter) of the floor, roof, or wall surface, respectively.

Table 9.1 lists major dead loads tabulated in ASCE 7-10. Table 9.2 lists unit densities from ASCE 7-10.

**Table 9.1 Minimum Design Dead Loads**

| Component | Load (psf) |
|---|---|
| **CEILINGS** | |
| Acoustical fiber board | 1 |
| Gypsum board (per 1/8-in. thickness) | 0.55 |
| Mechanical duct allowance | 4 |
| Plaster on tile or concrete | 5 |
| Plaster on wood lath | 8 |
| Suspended steel channel system | 2 |
| Suspended metal lath and cement plaster | 15 |
| Suspended metal lath and gypsum plaster | 10 |
| Wood furring suspension system | 2.5 |
| **COVERINGS, ROOF, AND WALL** | |
| Asbestos-cement shingles | 4 |
| Asphalt shingles | 2 |
| Cement tile | 16 |
| Clay tile (for mortar add 10 psf) | |
| Book tile, 2-in. | 12 |
| Book tile, 3-in. | 20 |
| Ludowici | 10 |
| Roman | 12 |
| Spanish | 19 |
| Composition: | |
| Three-ply ready roofing | 1 |
| Four-ply felt and gravel | 5.5 |
| Five-ply felt and gravel | 6 |
| Copper or tin | 1 |
| Corrugated asbestos-cement roofing | 4 |
| Deck, metal, 20 gage | 2.5 |
| Deck, metal, 18 gage | 3 |
| Decking, 2-in. wood (Douglas fir) | 5 |
| Decking, 3-in. wood (Douglas fir) | 8 |
| Fiberboard, 1/2-in. | 0.75 |
| Gypsum sheathing, 1/2-in. | 2 |
| Insulation, roof boards (per inch thickness) | |
| Cellular glass | 0.7 |
| Fibrous glass | 1.1 |
| Fiberboard | 1.5 |
| Perlite | 0.8 |
| Polystyrene foam | 0.2 |
| Urethane foam with skin | 0.5 |
| Plywood (per 1/8-in. thickness) | 0.4 |
| Rigid insulation, 1/2-in. | 0.75 |
| Skylight, metal frame, 3/8-in. wire glass | 8 |
| Slate, 3/16-in. | 7 |
| Slate, 1/4-in. | 10 |
| Waterproofing membranes: | |
| Bituminous, gravel-covered | 5.5 |
| Bituminous, smooth surface | 1.5 |
| Liquid applied | 1 |
| Single-ply, sheet | 0.7 |
| Wood sheathing (per inch thickness) | 3 |
| Wood shingles | 3 |
| **FLOOR FILL** | |
| Cinder concrete, per inch | 9 |

## Table 9.1 Minimum Design Dead Loads (continued)

| Component | Load (psf) |
|---|---|
| Lightweight concrete, per inch | 8 |
| Sand, per inch | 8 |
| Stone concrete, per inch | 12 |
| **FLOORS AND FLOOR FINISHES** | |
| Asphalt block (2-in.), 1/2-in. mortar | 30 |
| Cement finish (1-in.) on stone–concrete fill | 32 |
| Ceramic or quarry tile (3/4-in.) on 1/2-in. mortar bed | 16 |
| Ceramic or quarry tile (3/4-in.) on 1-in. mortar bed | 23 |
| Concrete fill finish (per inch thickness) | 12 |
| Hardwood flooring, 7/7-in. | 4 |
| Linoleum or asphalt tile, 1/4-in. | 1 |
| Marble and mortar on stone–concrete fill | 33 |
| Slate (per mm thickness) | 15 |
| Solid flat tile on 1-in. mortar base | 23 |
| Subflooring, 3/4-in. | 3 |
| Terrazzo (1-1/2-in.) directly on slab | 19 |
| Terrazzo (1-in.) on stone–concrete fill | 32 |
| Terrazzo (1-in.), 2-in. stone concrete | 32 |
| Wood block (3-in.) on mastic, no fill | 10 |
| Wood block (3-in.) on 1/2-in. mortar base | 16 |
| **FLOORS, WOOD-JOIST (NO PLASTER)** | |
| DOUBLE WOOD FLOOR | |

| Joint sizes (in.) | 12-in. spacing (1b/ft$^2$) | 16-in. spacing (1b/ft$^2$) | 24-in. spacing (1b/ft$^2$) |
|---|---|---|---|
| 2 × 6 | 6 | 5 | 5 |
| 2 × 8 | 6 | 6 | 5 |
| 2 × 10 | 7 | 6 | 6 |
| 2 × 12 | 8 | 7 | 6 |

| Component | Load (psf) |
|---|---|
| **FRAME PARTITIONS** | |
| Movable steel partitions | 4 |
| Wood or steel studs, 1/2-in. gypsum board each side | 8 |
| Wood studs, 2 × 4, unplastered | 4 |
| Wood studs, 2 × 4, plastered one side | 12 |
| Wood studs, 2 × 4, plastered two sides | 20 |
| **FRAME WALLS** | |
| Exterior stud walls: | |
| 2 × 4 @ 16-in., 5/8-in. gypsum, insulated, 3/8-in. siding | 11 |
| 2 × 6 @ 16-in., 5/8-in. gypsum, insulated, 3/8-in. siding | 12 |
| Exterior stud walls with brick veneer | 48 |
| Windows, glass, frame, and sash | 8 |
| Clay brick wythes: | |
| 4 in. | 39 |
| 8 in. | 79 |
| 12 in. | 115 |
| 16 in. | 155 |

Hollow concrete masonry unit wythes:

| Wythe thickness (in inches)<br>Density of unit (105 pcf) | | 4 | 6 | 8 | 10 | 12 |
|---|---|---|---|---|---|---|
| No grout | | 22 | 24 | 31 | 37 | 43 |
| 48 in. o.c. | | | 29 | 38 | 47 | 55 |
| 40 in. o.c. | grout | | 30 | 40 | 49 | 57 |
| 32 in. o.c. | spacing | | 32 | 42 | 52 | 61 |
| 24 in. o.c. | | | 34 | 46 | 57 | 67 |
| 16 in. o.c. | | | 40 | 53 | 66 | 79 |
| Full grout | | | 55 | 75 | 95 | 115 |

## Table 9.1 Minimum Design Dead Loads (continued)

| **Component** | | | | | **Load (psf)** |
|---|---|---|---|---|---|
| Density of unit (125 pcf) | | | | | |
| No grout | 26 | 28 | 36 | 44 | 50 |
| 48 in. o.c. | | 33 | 44 | 54 | 62 |
| 40 in. o.c. grout | | 34 | 45 | 56 | 65 |
| 32 in. o.c. spacing | | 36 | 47 | 58 | 68 |
| 24 in. o.c. | | 39 | 51 | 63 | 75 |
| 16 in. o.c. | | 44 | 59 | 73 | 87 |
| Full grout | | 59 | 81 | 102 | 123 |
| Density of unit (135 pcf) | | | | | |
| No grout | 29 | 30 | 39 | 47 | 54 |
| 48 in. o.c. | | 36 | 47 | 57 | 66 |
| 40 in. o.c. grout | | 37 | 48 | 59 | 69 |
| 32 in. o.c. spacing | | 38 | 50 | 62 | 72 |
| 24 in. o.c. | | 41 | 54 | 67 | 78 |
| 16 in. o.c. | | 46 | 61 | 76 | 90 |
| Full grout | | 62 | 83 | 105 | 127 |
| Solid concrete masonry unit wythes (incl. concrete brick): | | | | | |
| Wythe thickness (in mm) | 4 | 6 | 8 | 10 | 12 |
| Density of unit (105 pcf) | 32 | 51 | 69 | 87 | 105 |
| Density of unit (125 pcf) | 38 | 60 | 81 | 102 | 124 |
| Density of unit (135 pcf) | 41 | 64 | 87 | 110 | 133 |

## Table 9.2 Minimum Densities for Design Loads from Materials (after ASCE 7-10)

| Material | Density ($lb/ft^3$) | Density ($kN/m^3$) | Material | Density ($lb/ft^3$) | Density ($kN/m^3$) |
|---|---|---|---|---|---|
| Aluminum | 170 | 27 | Soil, submerged | 70 | 11.0 |
| Pitch | 69 | 10.8 | River mud, submerged | 90 | 14.1 |
| Tar | 75 | 11.8 | Sand or gravel, submerged | 60 | 9.4 |
| Cast-stone masonry (cement, stone, sand) | 144 | 22.6 | Sand or gravel and clay, submerged | 65 | 10.2 |
| | | | Glass | 160 | 25.1 |
| Cement, portland, loose | 90 | 14.1 | Gypsum, wallboard | 50 | 7.9 |
| Ceramic tile | 150 | 23.6 | Ice | 57 | 9.0 |
| Concrete, plain, normalweight | 144 | 22.6 | Masonry, brick | | |
| Concrete, reinforced, normalweight | 150 | 23.6 | Hard (low absorption) | 130 | 20.4 |
| Concrete, lightweight, structural | 70–105 | 11.0– | Medium (medium absorption) | 115 | 18.1 |
| Copper | 556 | 87.3 | Soft (high absorption) | 100 | 15.7 |
| Clay, dry | 63 | 9.9 | Masonry, concrete | | |
| Clay, damp | 110 | 17.3 | Lightweight units | 105 | 16.5 |
| Clay, submerged | 80 | 12.6 | Medium weight units | 125 | 19.6 |
| Clay and gravel, dry | 100 | 15.7 | Normal weight units | 135 | 21.2 |
| Gravel, dry | 104 | 16.3 | Particleboard | 45 | 7.1 |
| Silt, moist, loose | 78 | 12.3 | Plywood | 36 | 5.7 |
| Silt, moist, packed | 96 | 15.1 | Steel, cold-drawn | 492 | 77.3 |
| Silt, flowing | 108 | 17.0 | Terra cotta, architectural | | |
| Sand and gravel, dry, loose | 100 | 15.7 | Voids filled | 120 | 18.9 |
| Sand and gravel, dry, packed | 110 | 17.3 | Voids unfilled | 72 | 11.3 |
| Sand and gravel, wet | 120 | 18.9 | Fir, Douglas, coast region | 34 | 5.3 |
| | | | Redwood | 28 | 4.4 |

**Example 1**: Consider the two normalweight reinforced concrete T beams shown in Figure 9.1. Calculate the weight that must be supported by one T beam per foot of length.

The solution is provided in tabular form.

| Element | Unit Weight | Weight/ft |
|---|---|---|
| Waterproofing membrane | 5.5 psf | $5.5\,psf \times \frac{36}{12} ft \times 1 ft$ $= 16.5\; lb\ per\ foot\ length, or\ 16.5\ plf$ |
| Insulation board | 1.1 psf per inch | $1.1 \frac{psf}{in} \times 2" \times \frac{36}{12} ft = 6.6\, plf$ |
| Reinforced concrete | 150 pcf | $150\,pcf \times \left(\left(\frac{4}{12} ft \times \frac{36}{12} ft\right) + \left(\frac{14}{12} ft \times \frac{10}{12} ft\right)\right) \times 1\,ft$ $= 295.8\,plf$ |
| Total per T beam | | $16.5 + 6.6 + 295.8 = 319$ plf |

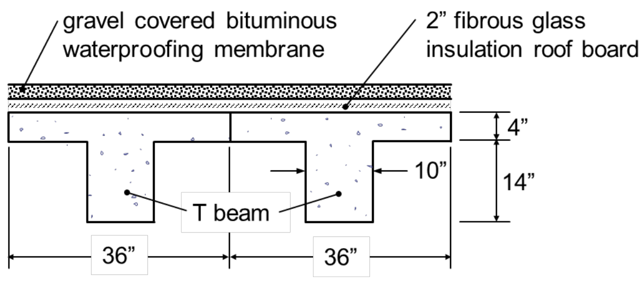



Figure 9.1 Example 1, beam cross section.

## 9.4. LIVE LOADS

### 9.4.1. Basic requirements

Live load is a load produced by the use and occupancy of the building, not including construction or environmental loads (wind load, snow load, rain load, earthquake load, or flood load). Roof live load is a load on a roof produced (1) during maintenance by workers, equipment, and materials, or (2) during the life of the structure by movable objects, such as planters or other similar small decorative appurtenances that are not occupancy related. Table 9.3 tabulates minimum live loads as specified by ASCE 7.

Note that the live loads specified in Table 9.3 include both a uniformly distributed live load $L_0$ and a concentrated live load. The structure is to be designed to support the uniformly distributed live load or the concentrated live load, whichever produces the greater load effect. The concentrated load is to be uniformly distributed over an area 2.5 ft (762 mm) by 2.5 ft (762 mm) and shall be located so as to produce the maximum load effects in the members.

In office buildings or other buildings where partitions will be erected or rearranged, provision for partition weight should be made, whether or not partitions are shown on the plans. According to ASCE 7, partition live load shall not be less than 15 psf (0.72 kN/m2), except partition live load is not required where the minimum specified live load exceeds 80 psf (3.83 kN/$m^2$).

**Table 9.3 Minimum Uniformly Distributed Live Loads, $L_o$, and Minimum Concentrated Live Loads**

| Occupancy or Use | Uniform psf (kN/m²) | Conc. lb (kN) |
|---|---|---|
| **Assembly areas** | | |
| Fixed seats (fastened to floor) | 60 (2.87)[a] | |
| Lobbies | 100 (4.79)[a] | |
| Movable seats | 100 (4.79)[a] | |
| Platforms (assembly) | 100 (4.79)[a] | |
| Stage floors | 150 (7.18)[a] | |
| Other assembly areas | 100 (4.79)[a] | |
| **Corridors** | | |
| First floor | 100 (4.79) | |
| Other floors | Same as occupancy served except as indicated | |
| **Dining rooms and restaurants** | 100 (4.79)[a] | |
| **Garages** | | |
| Passenger vehicles only | 40 (1.92)[a,b,c] | |
| **Hotels** (see Residential) | | |
| **Libraries** | | |
| Reading rooms | 60 (2.87) | 1,000 (4.45) |
| Stack rooms | 150 (7.18)[a,h] | 1,000 (4.45) |
| Corridors above first floor | 80 (3.83) | 1,000 (4.45) |
| **Manufacturing** | | |
| Light | 125 (6.00)[a] | 2,000 (8.90) |
| Heavy | 250 (11.97)[a] | 3,000 (13.40) |
| **Office buildings** | | |
| File and computer rooms shall be designed for heavier loads based on anticipated occupancy | | |
| Lobbies and first-floor corridors | 100 (4.79) | 2,000 (8.90) |
| Offices | 50 (2.40) | 2,000 (8.90) |
| Corridors above first floor | 80 (3.83) | 2,000 (8.90) |
| **Residential** | | |
| One- and two-family dwellings | | |
| Uninhabitable attics without storage | 10 (0.48)[l] | |
| Uninhabitable attics with storage | 20 (0.96)[m] | |
| Habitable attics and sleeping areas | 30 (1.44) | |
| All other areas except stairs | 40 (1.92) | |
| All other residential occupancies | | |
| Private rooms and corridors serving them | 40 (1.92) | |
| Public rooms[a] and corridors serving them | 100 4.79) | |
| **Roofs** | | |
| Ordinary flat, pitched, and curved roofs | 20 (0.96)[n] | |
| Roofs used for roof gardens | 100 (4.79) | |
| Roofs used for other occupancies | Same as occupancy served | |
| Roofs used for other special purposes | o | o |
| All other construction | 20 (0.96) | |
| Primary roof members, exposed to a work floor | | |
| Single panel point of lower chord of roof trusses or any point along primary structural members supporting roofs over manufacturing, storage warehouses, and repair garages | | 2,000 (8.9) |
| All other primary roof members | | 300 (1.33) |
| All roof surfaces subject to maintenance workers | | 300 (1.33) |

**Table 9.3 Minimum Uniformly Distributed Live Loads, Lo, and Minimum Concentrated Live Loads (continued)**

| Occupancy or Use | Uniform psf (kN/m$^2$) | Conc. lb (kN) |
|---|---|---|
| **Schools** | | |
| Classrooms | 40 (1.92) | 1,000 (4.45) |
| Corridors above first floor | 80 (3.83) | 1,000 (4.45) |
| First-floor corridors | 100 (4.79) | 1,000 (4.45) |
| **Stairs and exit ways** | 100 (4.79) | 300[r] |
| One- and two-family dwellings only | 40 (1.92) | 300[r] |
| **Storage areas above ceilings** | 20 (0.96) | |
| **Storage warehouses** (shall be designed for heavier loads if required for anticipated storage) | | |
| Light | 125 (6.00)[a] | |
| Heavy | 250 (11.97)[a] | |
| **Stores** | | |
| Retail | | |
| First floor | 100 (4.79) | 1,000 (4.45) |
| Upper floors | 75 (3.59) | 1,000 (4.45) |
| Wholesale, all floors | 125 (6.00)[a] | 1,000 (4.45) |

[a]Live load reduction for this use is not permitted by Section 4.7 unless specific exceptions apply.

[b]Floors in garages or portions of a building used for the storage of motor vehicles shall be designed for the uniformly distributed live loads of Table 4-1 or the following concentrated load: (1) for garages restricted to passenger vehicles accommodating not more than nine passengers, 3,000 lb (13.35 kN) acting on an area of 4.5 in. by 4.5 in. (114 mm by 114 mm); and (2) for mechanical parking structures without slab or deck that are used for storing passenger vehicles only, 2,250 lb (10 kN) per wheel.

[c]Design for trucks and buses shall be in accordance with AASHTO LRFD Bridge Design Specifications; however, provisions for fatigue and dynamic load allowance therein are not required to be applied.

[h]The loading applies to stack room floors that support nonmobile, double-faced library book stacks subject to the following limitations: (1) The nominal book stack unit height shall not exceed 90 in. (2,290 mm); (2) the nominal shelf depth shall not exceed 12 in. (305 mm) for each face; and (3) parallel rows of double-faced book stacks shall be separated by aisles not less than 36 in. (914 mm) wide.

[l]Uninhabitable attic areas without storage are those where the maximum clear height between the joist and rafter is less than 42 in. (1,067 mm), or where there are not two or more adjacent trusses with web configurations capable of accommodating an assumed rectangle 42 in. (1,067 mm) in height by 24 in. (610 mm) in width, or greater, within the plane of the trusses. This live load need not be assumed to act concurrently with any other live load requirement.

[m]Uninhabitable attic areas with storage are those where the maximum clear height between the joist and rafter is 42 in. (1,067 mm) or greater, or where there are two or more adjacent trusses with web configurations capable of accommodating an assumed rectangle 42 in. (1,067 mm) in height by 24 in. (610 mm) in width, or greater, within the plane of the trusses. For attics constructed of trusses, the live load need only be applied to those portions of the bottom chords where both of the following conditions are met:

3. The attic area is accessible from an opening not less than 20 in. (508 mm) in width by 30 in. (762 mm) in length that is located where the clear height in the attic is a minimum of 30 in. (762 mm); and
4. The slope of the truss bottom chord is no greater than 2 units vertical to 12 units horizontal (9.5% slope).

The remaining portions of the bottom chords shall be designed for a uniformly distributed nonconcurrent live load of not less than 10 lb/ft$^2$ (0.48 kN/m$^2$).

[n]Where uniform roof live loads are reduced to less than 20 lb/ft$^2$ (0.96 kN/m$^2$) in accordance with Section 4.8.2 and are applied to the design of structural members arranged so as to create continuity, the reduced roof live load shall be applied to adjacent spans or to alternate spans, whichever produces the greatest unfavorable load effect.

[o]Roofs used for other special purposes shall be designed for appropriate loads as approved by the authority having jurisdiction.

[r]Minimum concentrated load on stair treads (on area of 2 in. by 2 in. [50 mm by 50 mm]) is to be applied nonconcurrent with the uniform load.

### 9.4.2. Live load reductions

Live load in a building is variable both in magnitude and position. While it is reasonable to assume that an office may contain a safe weighing 2000 lb over a 2.5 ft by 2.5 ft area, resulting in a local pressure of 500 psf, it is not reasonable to assume that an office will be filled (wall-to-wall) with safes. Instead, a 12 ft by 12 ft office might have one safe, file cabinets, and desk weighing around 4000 lb plus a small gathering of 20 people weighing around 3000 lb. Thus, the average floor live load is around (4000 + 3000 lb)/144 ft$^2$ = 50 psf. Note that this is 0.1 times the average pressure under the safe. Considering an entire building, it is increasingly unlikely that every room has a safe and a small gathering, so a live load smaller than 50 psf would be reasonable. *We therefore recognize that the value of live load should vary with the size of the loaded area.*

**Floors**

According to ASCE 7, it is permitted to reduce the uniformly distributed floor live loads, $L_0$, in Table 9.3 where the following conditions are satisfied:

1. The live load, $L_0$, shall be restricted to floor loads. (The live load reduction specified here is not applicable to roofs. See roof reduction later.)
2. The live load shall not exceed 100 psf (4.79 kN/m$^2$). (These are storage loads, and it is possible that the storage facility will be completely filled.)
3. The reduction shall not apply to assembly uses, or to garages.
4. The value of the influence area $K_{LL}A_T$ shall be at least 400 ft$^2$ (37.2 m$^2$).

Where all of the aforementioned conditions are satisfied, it is permitted to reduce the design live load in accordance with the following formula:

$$L = L_0\left(0.25 + \frac{15}{\sqrt{K_{LL}A_T}}\right), \text{US customary units}$$

$$L = L_0\left(0.25 + \frac{4.57}{\sqrt{K_{LL}A_T}}\right), \text{SI units} \qquad (9.1)$$

where

$L$ = reduced design live load per ft$^2$ (m$^2$) of area supported by the member;
$L_0$ = unreduced design live load per ft$^2$ (m$^2$) of area supported by the member (see Table 9.3);
$K_{LL}$ = live load element factor (see Table 4-2);
$A_T$ = tributary area in ft$^2$ (m$^2$);

Additionally, the live load for members supporting multiple floors is limited as follows:

5. $L$ shall not be less than $0.5L_0$ for members supporting one floor; and
6. $L$ shall not be less than $0.4L_0$ for members supporting two or more floors.

| Table 9.4 Live Load Element Factor, $K_{LL}$. | | |
|---|---|---|
| **Element** | $K_{LL}^*$ | |
| | ***w/o cantilever slabs*** | ***w/ cantilever slabs*** |
| Interior columns | 4 | 4 |
| Edge columns | 4 | 3 |
| Corner columns | * | 2 |
| Interior beams | 2 | 2 |
| Edge beams | 2 | * |
| All other members not identified, including:<br>Edge beams with cantilever slabs<br>Cantilever beams<br>One-way slabs<br>Two-way slabs<br>Members without provisions for continuous shear transfer normal to their span | 1 | 1 |

*It is permitted to calculate $K_{LL}$ rather than using the tabulated values.

Tabulated values of $K_{LL}$ in Table 9.4 cover most conditions, but there are some conditions for which it is useful to be able to calculate $K_{LL}$. For this purpose, we define

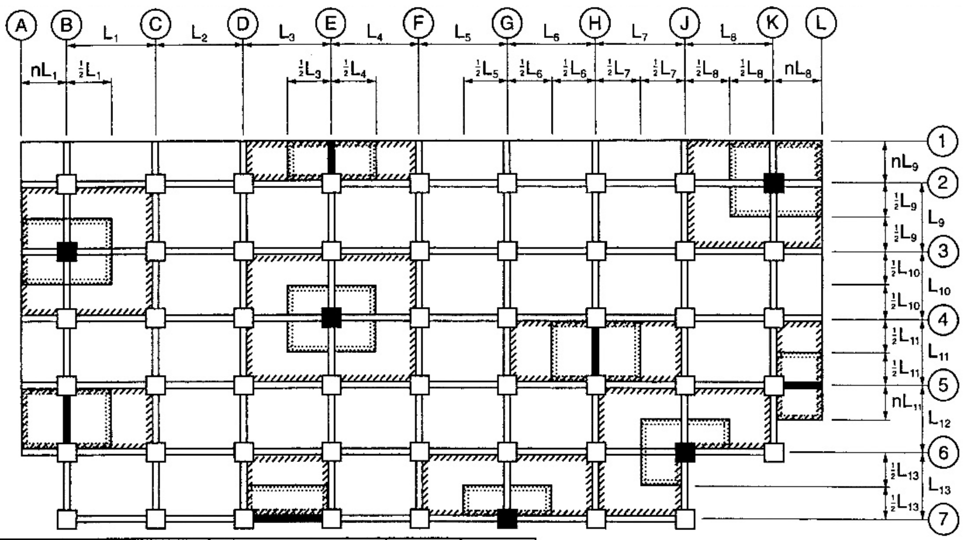
$$K_{LL} = \frac{A_I}{A_T} \tag{9.2}$$

in which

$A_I$ = the influence area for the member. The influence area is that area of the structure for which an applied load will appreciably affect the member under consideration. In general, $A_I$ is greater than the tributary area $A_T$.

$A_T$ = tributary area for the member. The tributary area is an area of the structure whose loads can be considered to produce the member load. For a floor under uniformly distributed loading, the product of the loading and the tributary area is equal to the load transmitted to the member. (The tributary area concept will be described in greater detail later in this chapter.)

Figure 9.2 illustrates typical influence areas and tributary areas for a structure with regular bay spacings. For example, consider the corner column at the intersection of axes K and 2. The tributary area $A_T$ extends from the column half-way to adjacent columns, and also includes the area of the cantilever slabs. The live load times that area would produce a good estimate of the total live load supported by the column. The influence area $A_I$ extends all the way to the adjacent columns, as any load placed within the panel bounded by axes J to L and 1 to 3 will contribute to and thereby influence the column. For this example, assuming the cantilever span $nL_a = 0.5L_a$, we obtain $K_{LL} = A_I/A_T = 9/4 = 2.25$. Table 9.4 lists $K_{LL} = 2$ for this case, which is close to the calculated value. Values for other members are listed in the inset table of Figure 9.2.




| Element | Actual $K_{LL}$ Members w/o Cantilevers | Actual $K_{LL}$ Members w/ Cantilevers | | | $K_{LL}$ (Table 4-2) | Example Member (Fig. C4-1) |
|---|---|---|---|---|---|---|
| | | n = 0 | n = 0.5 | n = 1.0 | | |
| Interior Columns | 4 | - | - | - | 4 | E4 |
| Exterior Columns w/o cantilevers | 4 | - | - | - | 4 | G7, J6 |
| Edge Columns w/ cantilevers | - | 4 | 3 | 2.67 | 3 | B3 |
| Corner Columns w/ cantilevers | - | 4 | 2.25 | 1.78 | 2 | K2 |
| Edge Beams w/o cantilever slabs | 2 | - | - | - | 2 | D7 - E7 |
| Interior Beams | 2 | - | - | - | 2 | H4 - H5 |
| Edge Beams w/ cantilever slabs | - | 2 | 1.5 | 1.33 | 1 | B5 - B6 |
| Cantilever Beams w/o cantilever slabs | 2 | - | - | - | 1 | E1 - E2 |
| Cantilever Beams w/ cantilever slabs | - | 2* | 1.5* | 1.33* | 1 | K5 - L5 |

* The value of n for member K5-L5 is used to calculate the distance $nL_{11}$

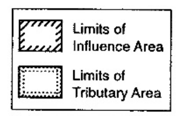



Figure 9.2 Typical tributary areas and influence areas.

**Example** – Consider a typical interior column from the example in Figure 9.2. The floor supports typical office loading. Assume $L_3 = L_4 = 30$ ft and $L_{10} = L_{11} = 20$ ft. Find the design live load for (a) a column supporting a single floor and (b) a column supporting five floors.

Solution: A tabular solution is provided

| Case | $L_0$ | $A_T$ | $K_{LL}$ | $K_{LL}A_T > 400\text{ft}^2$ | $L$ [Eq. (9.1)] | Limit | $P$ ($L$ x $A_T$) |
|---|---|---|---|---|---|---|---|
| 1 floor | 50 psf | 1x30x20=600ft$^2$ | 4 | Yes | $0.56L_0$ | $0.50L_0$ | 16.8 kips |
| 5 floors | 50 psf | 5x600=3000ft$^2$ | 4 | Yes | $0.39\ L_0$ | $0.40L_0$ | 60 kips |

Discussion: To solve this problem, we find the office load from Table 9.3. For live load reduction to be applicable, the influence area $A_I = K_{LL}A_T$ must be at least 400 ft$^2$, which is satisfied in both cases. The reduced live load is calculated from Eq. (9.1), but must not exceed the limit $0.5L_0$ for floors supporting one floor and $0.4L_0$ for floors supporting more than one floor. The final axial force is $0.56L_0A_T$ for 1 floor and $0.40L_0A_T$ for 5 floors.

## Roofs

ASCE 7 treats live load reductions for roofs differently from how floors are treated. The main points are considered below.

For ordinary flat, pitched, and curved roofs, and awning and canopies other than those of fabric construction supported by a skeleton structure, live load reductions are permitted. On such structures, the minimum roof live load shall be 12 psf (0.58 kN/m$^2$).

$$L_r = L_0 R_1 R_2, where\ 12 \le L_r \le 20, \text{US customary units}$$
$$L_r = L_0 R_1 R_2, where\ 0.58 \le L_r \le 0.96, \text{SI units} \tag{9.3}$$

where

$L_r$ = reduced roof live load per ft$^2$ (m$^2$) of horizontal projection supported by the member

$L_0$ = unreduced design roof live load per ft$^2$ (m$^2$) of horizontal projection supported by the member

The reduction factors $R_1$ and $R_2$ shall be determined as follows:

$$\begin{aligned} R_1 &= 1\ for\ A_T \le 200\ ft^2 \\ &= 1.2 - 0.001A_T\ for\ 200\ ft^2 < A_T < 600ft^2 \\ &= 0.6\ for\ A_T \ge 600\ ft^2, US\ customary\ units \\ R_1 &= 1\ for\ A_T \le 18.6\ m^2 \\ &= 1.2 - 0.011A_T\ for\ 18.6\ m^2 < A_T < 55.7\ m^2 \\ &= 0.6\ for\ A_T \ge 55.7\ m^2 \end{aligned} \tag{9.4}$$

$$\begin{aligned} R_2 &= 1\ for\ F \le 4 \\ &= 1.2 - 0.05F\ for\ 4 < F < 12 \\ &= 0.6\ for\ F \ge 12 \end{aligned} \tag{9.5}$$

where, for a pitched roof, $F$ = number of inches of rise per foot (in SI: $F = 0.12 \times$ slope, with slope expressed in percentage points) and, for an arch or dome, $F$ = rise-to-span ratio multiplied by 32.

### 9.4.3. Impact Loads

The live loads specified in ASCE 7 can be assumed to include adequate allowance for ordinary impact conditions. However, provision shall be made in the structural design for uses and loads that involve unusual vibration and impact forces. Such effects include:

**Elevators** - All elements subject to dynamic loads from elevators should be designed for impact loads and deflection limits prescribed by ASME A17.1.

**Machinery** - For the purpose of design, the weight of machinery and moving loads should be increased as follows to allow for impact: (1) light machinery, shaft- or motor-driven, 20 percent; and (2) reciprocating machinery or power-driven units, 50 percent. All percentages should be increased by alternative factors where specified by the manufacturer.

### 9.4.4. Placement for Maximum Load Effect

The design must consider the possibility that live loads will be placed in patterns that produce the maximum load effects. Influence lines, introduced in Chapter 5, are an efficient tool for identifying where to place loads for maximum effects. The reader is referred to Chapter 5 for additional discussion.

The term *pattern load* describes a load being positioned in a pattern that may produce maximum load effects. For example, storage loads can be placed in alternate bays, with the bays between those storage loads being unloaded so as to form corridors (Figure 9.3 b and c). This loading will produce maximum positive moments in the loaded bays. Alternatively, two adjacent bays can be fully loaded with the next bays unloaded. This loading, along with alternate bays also being loaded, will produce the maximum negative moments at supports (Figure 9.3 d, e, f, and g).

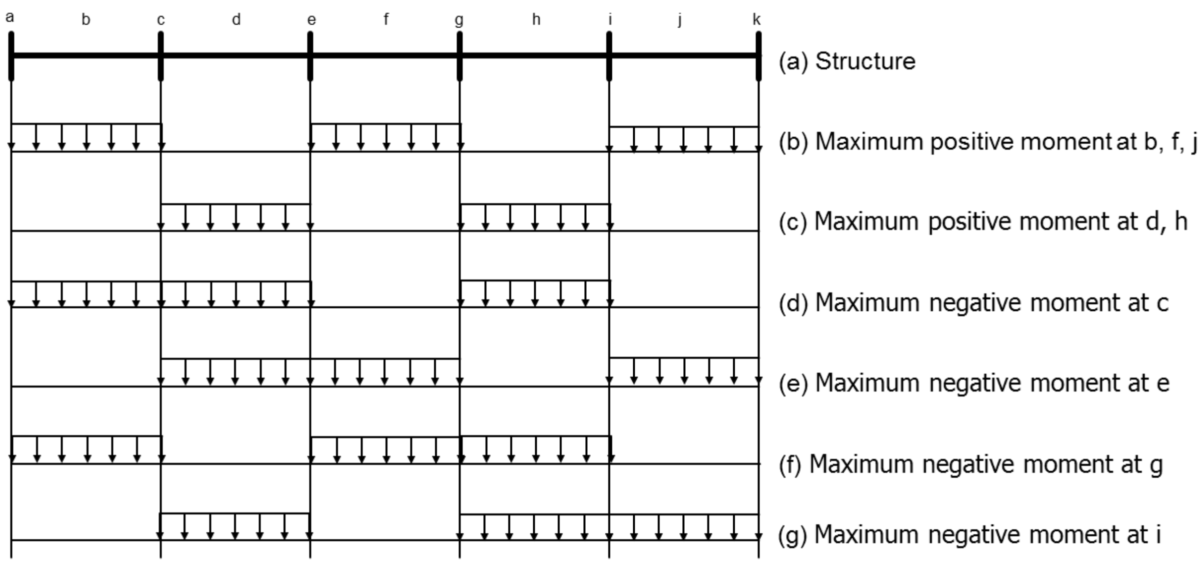



Figure 9.3 Pattern loads for maximum moment effects.

## 9.5. SNOW LOADS

Snow loads vary with geographic location. Snow loads on structures also vary with configuration of the structure, and must consider exposure and local projections where drifts can accumulate (Figure 9.4). The interested reader should refer to ASCE 7 and to local codes.

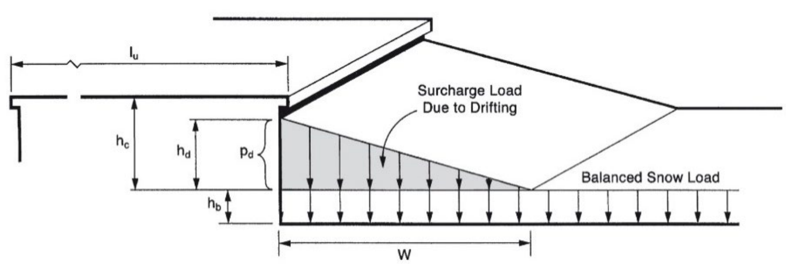



Figure 9.4 Configuration of snow drifts on lower roofs. (From ASCE 7)

## 9.6. LOAD PATHS AND TRIBUTARY AREA CONCEPT

### 9.6.1. Load paths

Roof and floor systems commonly are constructed using a series of surface structural elements supported by larger elements capable of spanning greater distances to the supporting columns or walls. For example, consider the framing system shown in Figure 9.5. Floor load is applied to surface elements (which could be wood planks, plywood, or concrete slab). Although these elements are continuous in EW and NS directions, the shortest and, hence, stiffest load path is in the NS direction, where they are supported by joists. The joists support the reactions from the surface elements plus their own weight, and span EW to supporting beams. The beams support the joist loads plus self-weight, and span those loads NS to girders. The girders in turn span EW to supporting columns, which transmit loads through axial forces to the foundations or other supporting elements.

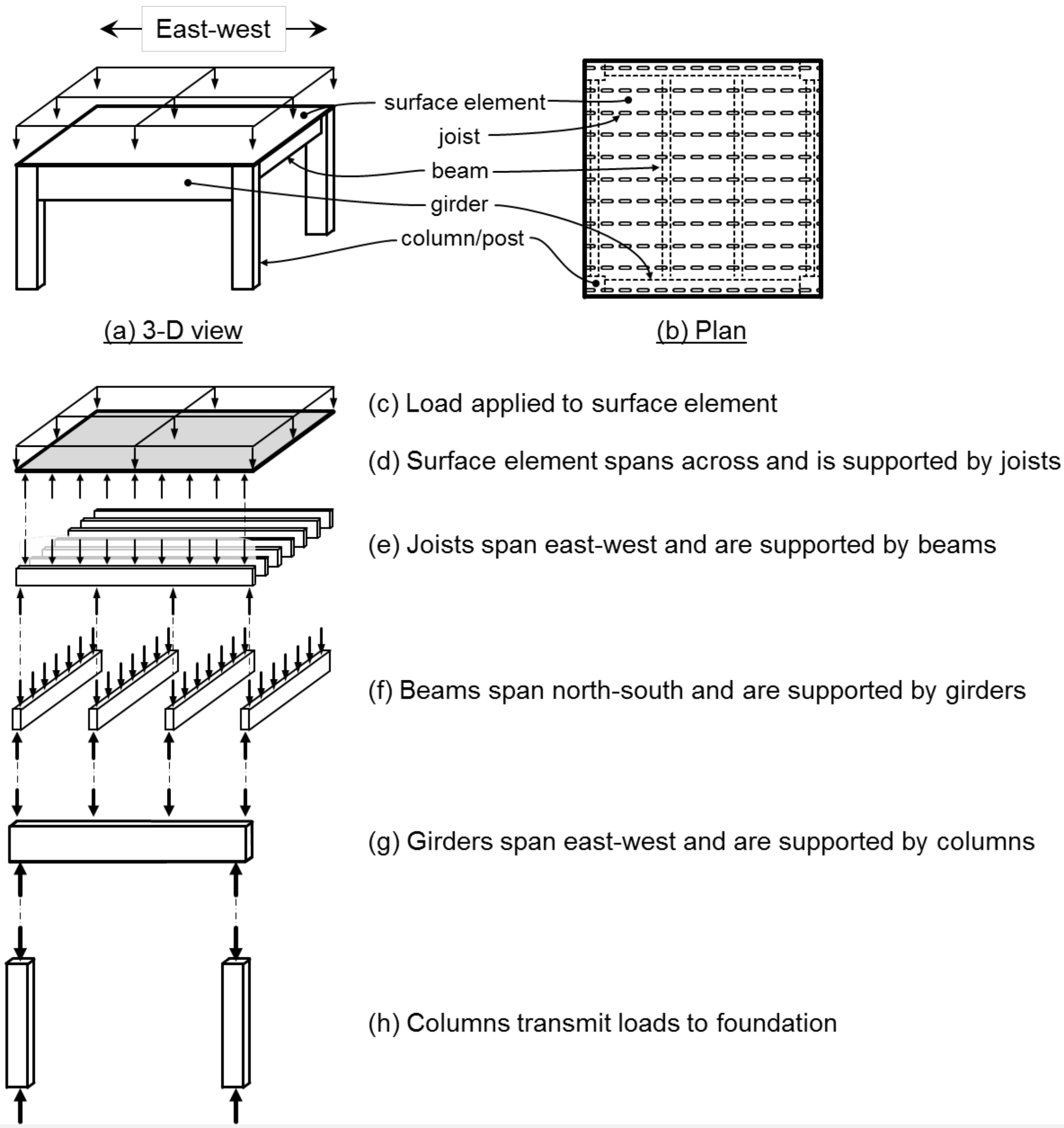



Figure 9.5 Load path for gravity loads in a floor framing system.

The structural elements need not be stacked atop one another as implied by the exploded diagram of Figure 9.5. Greater economy in construction and operations can sometimes be achieved by framing structural members into one another such that they have the same top elevation (Figure 9.6). Regardless, the conceptualization of the load path is the same as depicted in Figure 9.5.

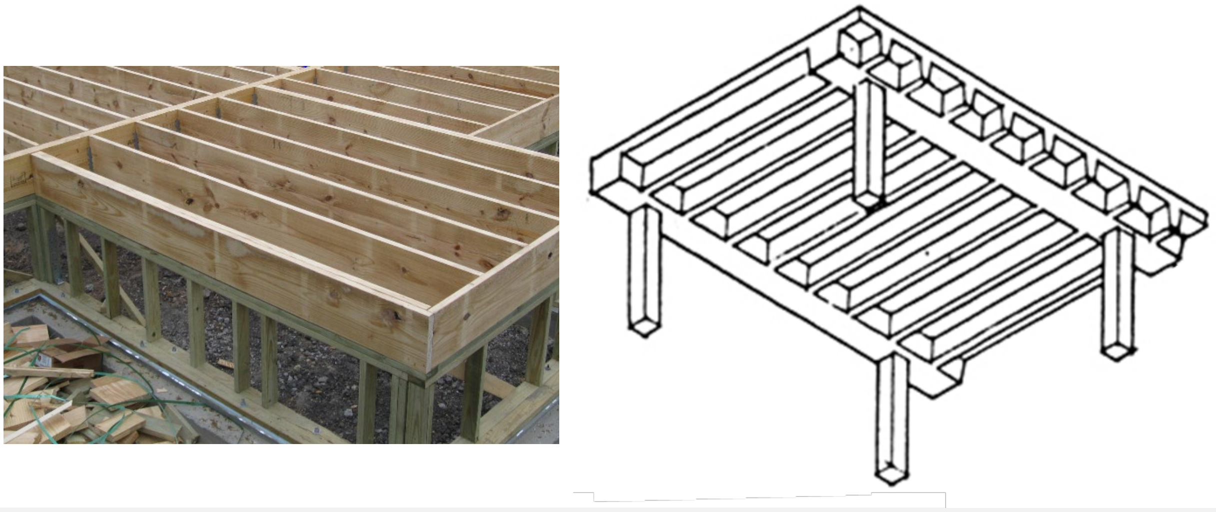


Figure 9.6 Joists framed into beams so as to have the same top elevation. (a) Wood framing (Southern Forest Products Association); (b) Reinforced concrete framing (Idees Deco Maison).

The surface elements of Figure 9.5 are supported only by joists, such that it is certain that the surface elements frame in one direction between joists. Such surface elements or slabs are referred to as *one-way elements*. Where the joists, beams, and girders all frame together at the same top elevation, the framing action of the surface element occurs in two directions. Such surface elements or slabs are referred to as *two-way elements* and, in reinforced concrete construction, *two-way slabs*.

To better understand the behavior of two-way elements, consider a rectangular slab supported on un-yielding walls around the perimeter (Figure 9.7). Two strips of the slab that intersect at the mid-span must have the same deflection, $\delta$. Idealizing the system as two strips supporting uniformly distributed load $w$, we can write

$$\delta_a = \delta_b = \frac{5w_a l_a^4}{384EI_a} = \frac{5w_b l_b^4}{384EI_b} \qquad (9.6)$$

Recognizing that this representation of the two-way slab is a major simplification, it is not unreasonable to further simplify it by assuming $EI_a = EI_b$. Thus, we arrive at

$$\frac{w_b}{w_a} = \frac{l_a^4}{l_b^4} \qquad (9.7)$$

According to Eq. (9.7), the amount of load carried in the b direction, $w_b$, is 16 times that carried in the a direction, $w_a$, when $l_a/l_b = 2$. We could change the assumptions of the idealization but the conclusion will remain effectively the same, specifically:

*For a slab having length-to-width ratio of 2 or greater, it is reasonable to assume that the slab is a one-way element framing in the short-span direction.*

*For a slab having length-to-width ratio less than approximately 2, the slab should be designed as a two-way element with some load framing in each direction.*

This text only addresses one-way elements.

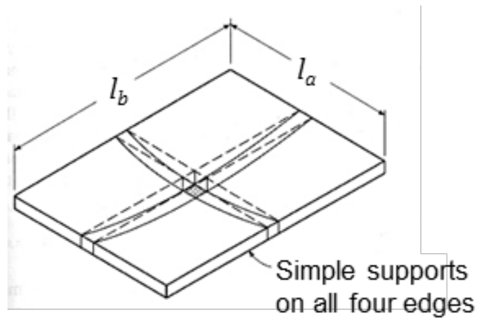



Figure 9.7 Two-way slab supported on unyielding walls.

### 9.6.2. Tributary width and tributary area

The tributary width or tributary area concept is an approximate analysis method used for estimating the load path in structural systems. To develop the basis for the method, consider the continuous and discontinuous beams supporting uniformly distributed loads shown in Figure 9.8. The continuous beam was analyzed using the computer software RISA 2D, while the discontinuous beam was analyzed by hand. From the results we can observe the following:

- The reactions for the continuous beam are similar to the reactions from the discontinuous beam. The exterior reactions in the continuous beam are conservatively estimated by the results from the discontinuous beam, while the first interior reaction is underestimated by 14%.
- The shear diagrams for the continuous and discontinuous beams are also similar.
- The moment diagrams for the two beams are markedly different.

From the preceding observations, we conclude that reactions can be reasonably approximated by modeling the beam as a discontinuous beam. Moments, however, are strongly affected by continuity and cannot be accurately estimated by considering the beam to be discontinuous.

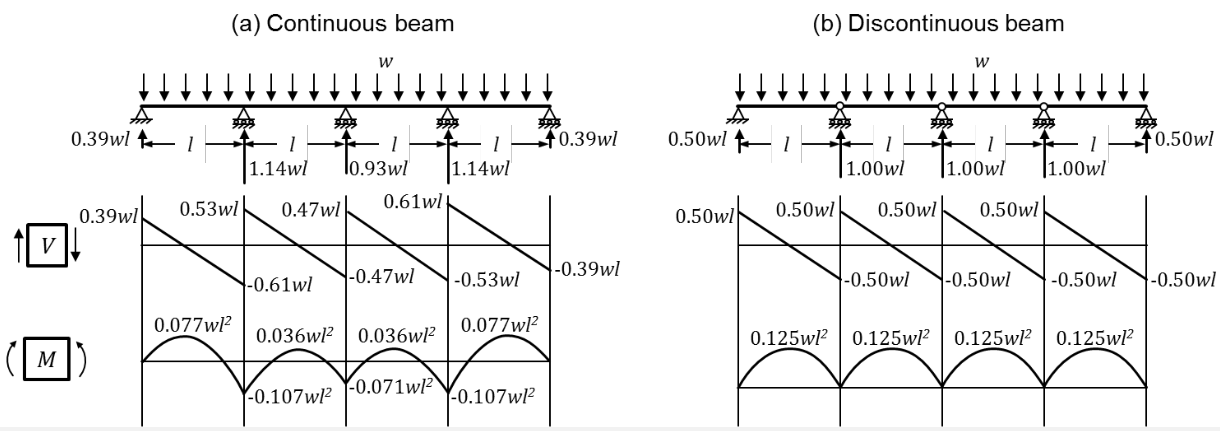



Figure 9.8 Comparison of reactions, shears, and moments for continuous and discontinuous beams.

We can obtain the same results as obtained in Figure 9.8 by using the tributary width concept. According to the *tributary width* concept, the load transferred to a beam support is equal to the load acting within the tributary width, where the tributary width is a width extending halfway to each of the adjacent supports (Figure 9.9). This method works very well where loads are uniformly distributed. Where loads are not uniformly distributed, it is preferably to treat the beam as a discontinuous beam, as in Figure 9.8b, and calculate the reactions using equilibrium, or, alternatively, to analyze it as a continuous beam, as in Figure 9.8a. Treating the beam as a discontinuous beam, for a concentrated load halfway between two supports, half the concentrated load would be transferred to one support and half to the other. If the concentrated load was positioned three-quarters of the way along the support, three quarters of the load would go to the closer support with the remainder going to the more distant support.

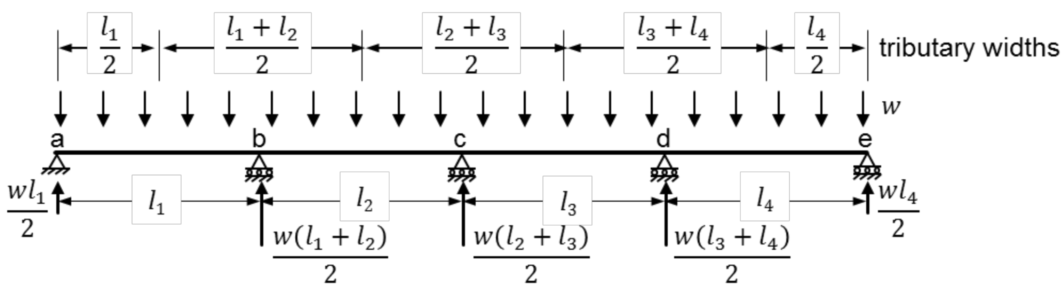



Figure 9.9 Tributary widths for a continuous beam.

The concept can be expanded to tributary areas, as depicted in Figure 9.10. For a beam along axis 2 between axes a and b, the tributary area is $A_{T1}$. For the girder along axis c between axes 1 and 2, the tributary area $A_{T2}$ is the area from two beams supported by the girder. We could also add the small area immediately above the girder, but this is too detailed for the approximate nature of the calculation. For the interior column at the intersection of axes 3 and b, the area is $A_{T3}$. A similar approach is used for the corner column at 4d.

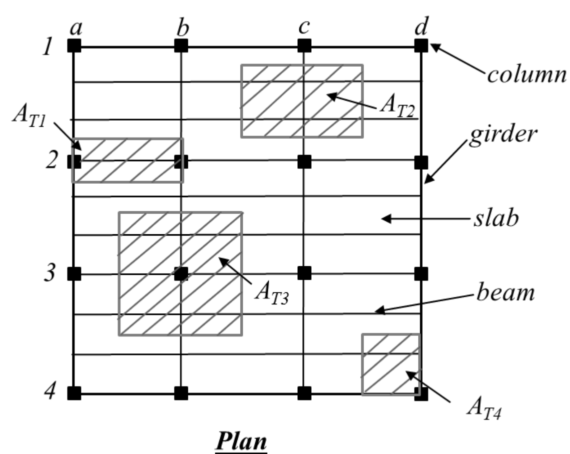
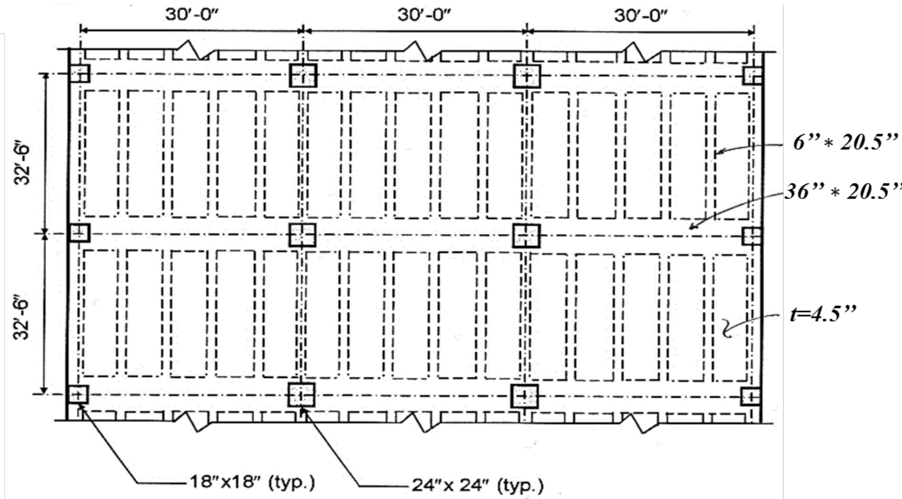



Figure 9.10 Tributary areas for a floor system.

**Example**: For the floor system shown in Figure 9.11, determine the design gravity loads for a typical (a) slab, (b) interior joist, (c) interior beam, and (d) interior column supporting a single floor. The solution is shown in the figure. Note that the labelled depth of the interior joist and beam include the depth of the slab.




<u>**Design Data:**</u>

<u>Materials</u>: Normal-weight concrete, 150 pcf

<u>Loads:</u>

Superimposed: $w_D = 30$ psf , $w_L = 100$ psf

Slab: $w = \left(\frac{4.5}{12} ft\right)(150\ pcf) = 56$ psf

Joist: $w = \left(\frac{6}{12} ft\right)\left(\frac{20.5-4.5}{12} ft\right)(150\ pcf) = 100$ plf

Beam: $w = \left(\frac{36}{12} ft\right)\left(\frac{20.5-4.5}{12} ft\right)(150\ pcf) = 600$ plf

Figure 9.11 Tributary area and live load reduction example.

**Example (continued)**

Slab This is a one-way slab. Special rules apply to live load reduction of one-way slabs. Per ASCE 7-10, the tributary area is limited to the product of the span and a width of 1.5 times the span:

$A_T = 6' * (1.5 * 6') = 54\ ft^2$

$K_{LL} * A_T = 1 * 54 = 54 ft^2 < 400\ ft^2$, no live load reduction is permitted

$w_D = 30\ psf + 56\ psf = \underline{86\ psf}$

$w_L = \underline{100\ psf}$

Joist $A_T = 6' * (32.5' - 3') = 177\ ft^2$

$K_{LL} * A_T = 2 * 177\ ft^2 = 354\ ft^2 < 400\ ft^2$, no live load reduction is permitted

$w_D = (86\ psf)(6') + 100\ plf = \underline{616\ plf}$

$w_L = (100\ psf)(6) = \underline{600\ plf}$

Beam $A_T = 32.5' * 30' = 975\ ft^2$

$K_{LL} * A_T = 2 * 975\ ft^2 = 1950\ ft^2 > 400\ ft^2$, reduction is permitted

$L = L_0\ (0.25 + \frac{15}{\sqrt{1950}}) = 0.59 \quad L_0$, not permitted to be less than $0.50\ L_0$

$w_L = (0.59)(100psf) = 59\ psf$

Because there are several joists, we treat the load as being uniformly distributed:

$w_D = \frac{(616\ plf)(32.5' - 3')}{6'} + \left(\frac{36}{12}'\right)\left(\frac{20.5}{12}'\right)(150\ pcf) = 3030\ plf + 769\ plf = \underline{3.8\ klf}$

$w_L = (59\ psf)(32.5') = \underline{1.9\ klf}$

Column (supporting 'single floor)

$A_T = 975\ ft^2$

$K_{LL} * A_T = 4 * 975\ ft^2 = 3900\ ft^2 > 400\ ft^2$, reduction is permitted

$L = L_0\ (0.25 + \frac{15}{\sqrt{3900}}) = 0.49 \quad L_0 < 0.50L_0 \rightarrow L = 0.5L_0$

$w_L = (0.50)(100psf) = 50\ psf$

$P_D = (3.8\ klf)(4x6') + (0.616\ klf)(32.5'-2') = 110\ kips$

The calculation of $P_D$ missed a small portion of the girder weighting approximately:

$\left(\frac{36}{12}\right)\left(\frac{20.5}{12}\right)(30' - 4 * 6' - 2')\ (150\ pcf) = 3.1$ kips, so the correct $P_D$ is:

$P_D = 110 + 3.1k = \underline{113\ kips}$

$P_L = (50\ psf)(32.5')(30') = \underline{48.8\ kips}$

# 10. Wind Loads and Load Paths

## 10.1. INTRODUCTION

This chapter introduces how a structural engineer determines the wind loads for design of a structure. The concept of a load path for wind loading is introduced and used to identify how a structure resists vertical and lateral loads. Examples of lateral-force-resisting systems are shown. The loads and methods generally follow the procedures of ASCE 7-10.[1]

## 10.2. GENERAL NATURE OF WIND LOADS

Globally, wind arises from differential heating between the equator and the poles, leading to buoyancy forces and movement of the air. As heated air rises near the equator and moves north, it is further affected by the Coriolis effect as the planet rotates. Regionally, wind is affected by differences in atmospheric pressure, with air moving from regions of high pressure to low pressure.

Wind speed for building design is based on meteorological records for different regions. Design wind speed is based on recordings of the wind speed sustained for 3 seconds at elevation of 33 ft above the ground. Traditionally, design has been based on a speed having a 50-year return period (equivalent to a 2% probability of exceedance annually), with design loads factored by 1.6 to achieve an appropriate level of safety. In regions not prone to hurricanes, this factor increases the return period to approximately 700 years. In 2010, ASCE 7-10 revised the design basis to use the basic wind speed associated with a return period of 700 years (equivalent to an annual exceedance probability of 0.00140), but with a load factor of 1.0. As shown in Figure 10.1, the basic wind speed for ordinary buildings in California is 110 mph.

[1] ASCE 7-10, Minimum Design Loads for Buildings and Other Structures (ASCE/SEI 7-10), American Society of Civil Engineers, 2010,

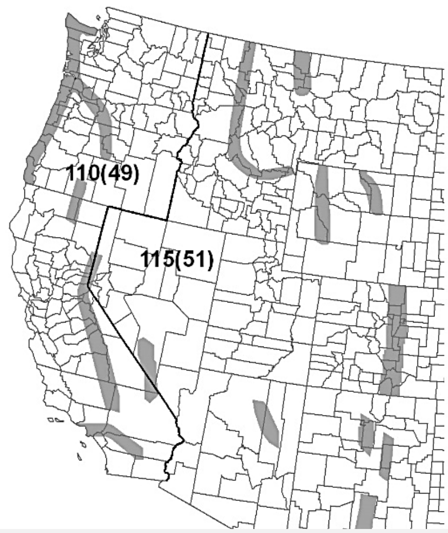



Figure 10.1 Basic wind speed in miles per hour (meters per second) for Risk Category II Buildings (After ASCE 7-10).

Locally, wind is affected by *topographic effects*, which can slow or divert the moving air mass. Friction of the surface of the earth leads to variation of wind speed with height (Figure 10.2). This effect is incorporated in the ASCE 7 wind design provisions through a *wind exposure coefficient*. ASCE 7 also includes terrain effects, such as wind *speed-up* as a mass of air moving across a flat plane is constricted by sudden rise in the land elevation. (Details of terrain effects are not covered in this text.)

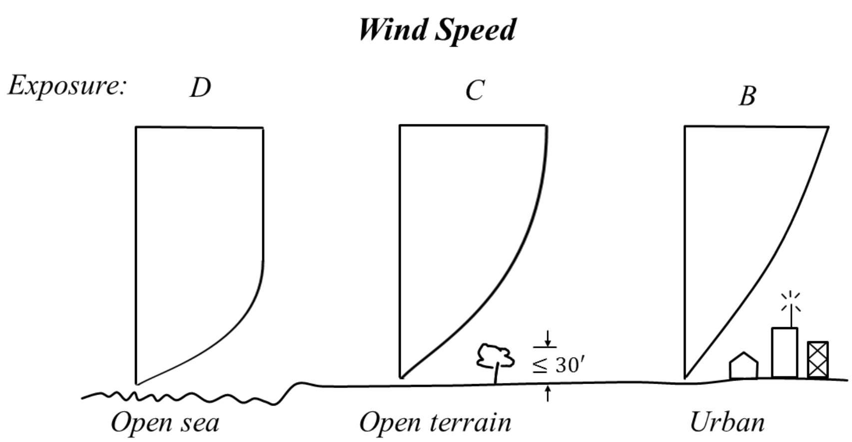



Figure 10.2 Velocity profile of wind as affected by surface roughness.

When wind passes by a structure, the paths of some of the air particles are diverted around the structure (Figure 10.3). This causes an increase in the velocity necessary to maintain continuity of the flowing air. Where the velocity increases, the pressure reduces. This can create an uplift on a roof (Figure 10.3a) or outward pressure (suction) on the side walls (Figure 10.3b). Pressure is also created on windward walls and suction on leeward walls. ASCE 7 contains pressure coefficients to account for these effects. (Uplift on roofs can carry away a poorly anchored roof.)

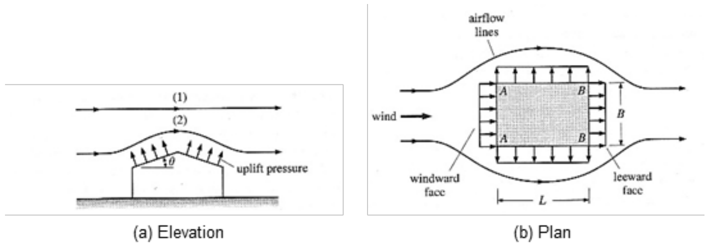



Figure 10.3 Flow of air particles around structures and the resulting air pressures. (Drawing from Uang et al., 2011)[2].

In wind engineering, where the static pressure can be ignored, the stagnation pressure is the pressure exerted on an object when all kinetic energy has been converted to pressure energy. The stagnation pressure is:

$$q_s = \frac{\rho V^2}{2} \tag{10.1}$$

in which $\rho$ = mass density of air and $V$ = wind velocity. Importantly, the stagnation pressure is proportional to the square of the wind velocity.

When air particles are diverted around an object, as in the figure above, a pressure coefficient is used to express the a portion of the stagnation pressure gets transferred to the building. This is discussed in further detail later in this chapter.

In the next three pages from Uang et al. (2011), wind loading is introduced in more detail. Note that while the discussion of vortex shedding is correct, the example of the Tacoma Narrows bridge is not. The Tacoma Narrows bridge actually failed due to aeroelastic flutter, a slightly different type of wind excitation.

[2] Fundamentals of Structural Analysis, Leet, Uang, and Gilbert, McGraw-Hill, 2011.

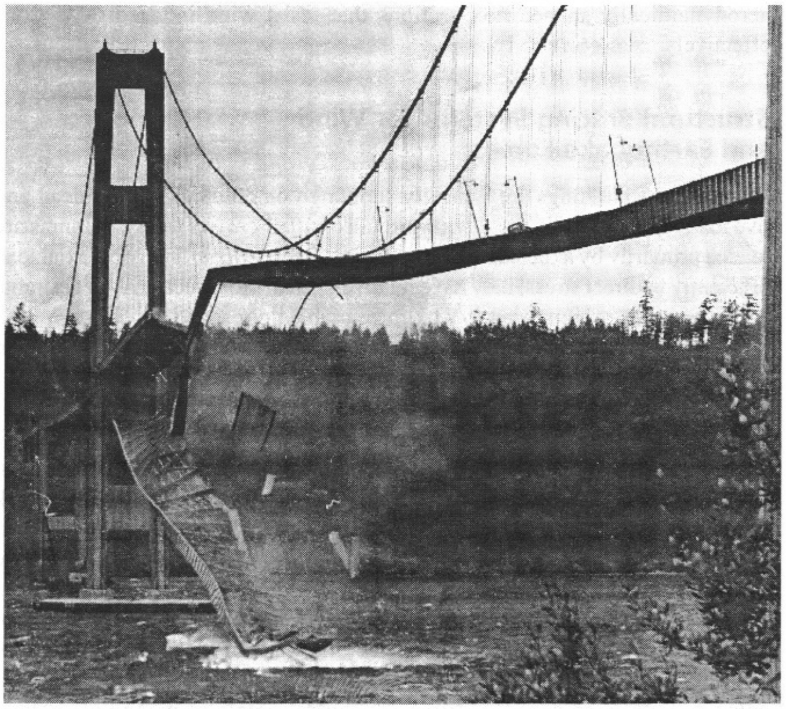


**Photo 2.1:** Failure of the Tacoma Narrows Bridge showing the first section of the roadway as it crashes into Puget Sound. The breakup of the narrow, flexible bridge was produced by large oscillation induced by the wind.

**Vortex Shedding.** As wind moving at constant velocity passes over objects in its path, the air particles are retarded by surface friction. Under certain conditions (critical velocity of wind and shape of surface) small masses of restrained air periodically break off and flow away (see Figure 2.12). This process is called *vortex shedding*. As the air mass moves away, its velocity causes a change in pressure on the discharge surface. If the period (time interval) of the vortices leaving the surface is close to that of the natural period of the structure, oscillations in the structure will be induced by the pressure variations. With time these oscillations will increase and shake a structure vigorously. The Tacoma Narrows Bridge failure shown in Photo 2.1 is a dramatic example of the damage that vortex shedding can wreak. Tall chimneys and suspended pipelines are other structures that are susceptible to wind-induced vibrations. To prevent damage to vibration-sensitive structures by vortex shedding, *spoilers* (see Figure 2.13), which cause the vortices to leave in a random pattern, or *dampers*, which absorb energy, may be attached to the discharge surface. As an alternative solution, the natural period of the structure may be modified so that it is out of the range that is sensitive to vortex shedding. The natural period is usually modified by increasing the stiffness of the structural system.

For several decades after the Tacoma Narrows Bridge failure, designers added stiffening trusses to the sides of suspension bridge roadways to minimize bending of the decks (Photo 2.2). Currently designers use stiff

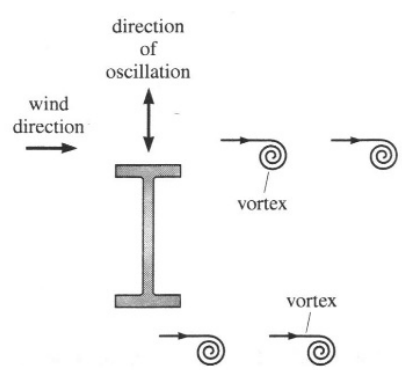
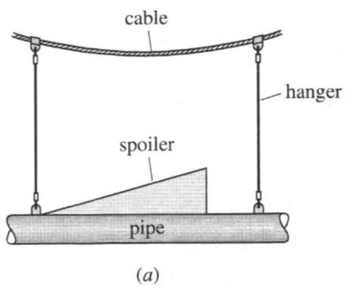



**Figure 2.12:** Vortices discharging from a steel girder. As vortex speeds off, a reduction in pressure occurs, causing girder to move vertically.



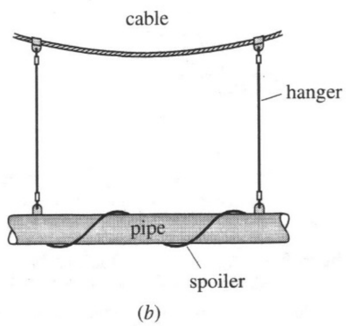




**Figure 2.13:** Spoilers welded to a suspender pipe to change the period of vortices: (*a*) triangular plate used as a spoiler; (*b*) spiral rod welded to pipe used as spoiler.

aerodynamically shaped box sections that resist wind-induced deflections effectively.

## Structural Bracing Systems for Wind and Earthquake Forces

The floors of buildings are typically supported on columns. Under dead and live loads that act vertically downward (also called *gravity load*), columns are loaded primarily by axial compression forces. Because columns carry axial load efficiently in direct stress, they have relatively small cross sections—a desirable condition since owners want to maximize usable floor space.

When lateral loads, such as wind or the inertia forces generated by an earthquake, act on a building, lateral displacements occur. These displacements are zero at the base of the building and increase with height. Since slender columns have relatively small cross sections, their bending stiffness is small. As a result, in a building with columns as the only supporting elements, large lateral displacements can occur. These lateral displacements can crack partition walls, damage utility lines, and produce motion sickness in occupants (particularly in the upper floors of multistory buildings where they have the greatest effect).

To limit lateral displacements, structural designers often insert, at appropriate locations within the building, structural walls of reinforced masonry or reinforced concrete. These *shear walls* act in-plane as deep cantilever beam-columns with large bending stiffnesses several orders of magnitude greater than those of all the columns combined. Because of their large stiffness, shear walls often are assumed to carry all transverse loads from wind or earthquake into the foundation. Since the lateral loads act normal to the longitudinal axis of the wall, just as the shear acts in a beam, they are called shear walls (Figure 2.14*a*). In fact, these walls must also be reinforced for bending along

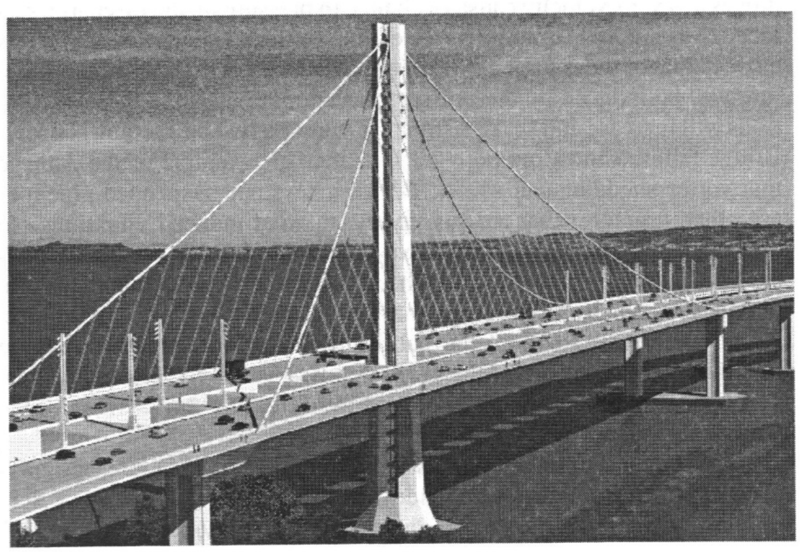


**Photo 2.2:** The main span of the new San Francisco-Oakland Bay Bridge is a single-tower and self-anchored suspension bridge. The suspension cable loops around the near bent and anchors to the bridge deck at the far bent.

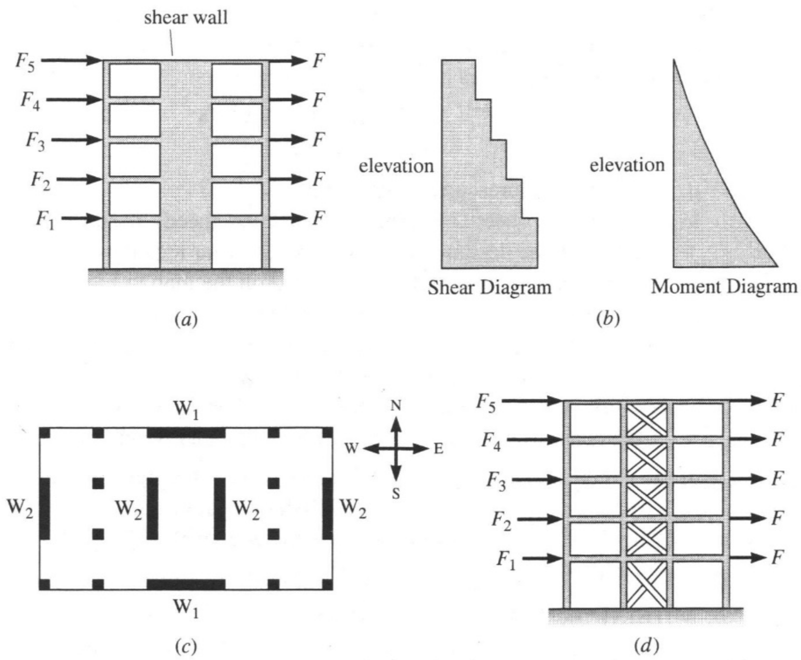



**Figure 2.14:** Structural systems to resist lateral loads from wind or earthquake. (*a*) Reinforced concrete shear wall carries all lateral wind loads; (*b*) shear and moment diagrams for shear wall produced by the sum of wind loads on the windward and leeward sides of the building in (*a*); (*c*) plan of building showing position of shear walls and columns; (*d*) cross-bracing between steel columns; forms a truss to carry lateral wind loads into the foundations.

both vertical edges since they can bend in either direction. Figure 2.14*b* shows the shear and moment diagrams for a typical shear wall.

Loads are transmitted to the walls by continuous floor slabs that act as rigid plates, termed *diaphragm action* (Figure 2.14*a*). In the case of wind, the floor slabs receive the load from air pressure acting on the exterior walls. In the case of earthquake, the combined mass of the floors and attached construction determines the magnitude of the inertia forces transmitted to the shear walls as the building flexes from the ground motion.

Shear walls may be located in the interior of buildings or in the exterior walls (Figure 2.14*c*). Since only the in-plane flexural stiffness of the wall is significant, walls are required in both directions. In Figure 2.14*c* two shear walls, labeled $W_1$, are used to resist wind loads acting in the east-west direction on the shorter side of the building; four shear walls, labeled $W_2$, are used to resist wind load, in the north-south direction, acting on the longer side of the building.

In buildings constructed of structural steel, as an alternative to constructing shear walls, the designer can add X-shaped or V-shaped cross-bracing between columns to form deep wind trusses, which are very stiff in the plane of the truss (Figure 2.14*d* and Photo 2.3).

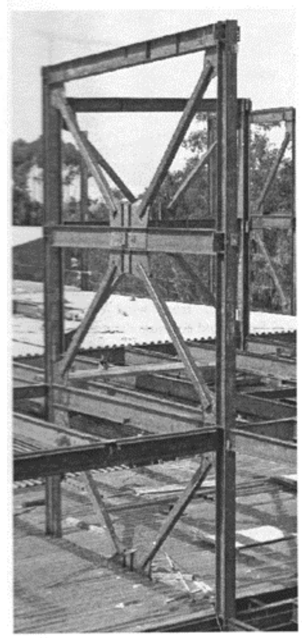


**Photo 2.3:** The cross-bracing, together with the attached columns and horizontal floor beams in the plane of the bracing, forms a deep continuous, vertical truss that extends the full height of the building (from foundation to roof) and produces a stiff, lightweight structural element for transmitting lateral wind and earthquake forces into the foundation.

Note that wind loading can occur from any direction. Dynamic interaction between wind and the structure can occur for flexible buildings, especially high-rise buildings. Wind tunnel testing is commonly done for tall buildings to determine how the building responds to wind loading. Models are typically mounted on a platform that can be rotated to study effects of directionality. In addition to studying dynamic response, wind tunnel testing also enables measurement of local pressures, which is useful for designing the cladding for the building.

## 10.3. ASCE 7 DIRECTIONAL PROCEDURE FOR DETERMINING WIND DESIGN FORCES

ASCE 7 contains a variety of procedures for determining the wind forces for design of the *main wind force-resisting system (MWFRS)*. In this text we use only the directional procedure. The *directional procedure* uses the traditional method of determining wind pressures that act on each building surface. This text considers only walls of rigid, enclosed, Risk Category II buildings. Roofs, partially enclosed or open buildings, and buildings in other risk categories are not included. See ASCE 7 for those other building types and elements.

The ASCE 7 procedure can be broken into a series of seven steps.

**Step 1**: Establish the building *risk category*. Risk categories are from Table 1.5.1 of ASCE 7-10, which is reproduced below as Table 10.1.

**Step 2**: Determine the basic wind speed, $V$, for the applicable risk category. See Figure 10.1. Note that this text only provides the wind speed for Risk Category II buildings. See ASCE 7 for other risk categories. For Risk Category II buildings in California, $V = 110$ mph.

**Step 3**: Determine wind load parameters:

- *Exposure category*. Three exposure categories are defined. See Figure 10.2. The following lines describe these categories in greater detail:
  - B: Urban and suburban, or wooded areas with low structures;
  - C: Open terrain with scattered obstructions generally less than 30 ft high;
  - D: Flat, unobstructed areas exposed to wind flowing over open water for a distance of at least 5000 ft or 20 times the building height, whichever is greater.
- *Wind directionality factor*, $K_d$ – This factor accounts for the reduced probability of maximum winds coming from any given direction and for the reduced probability of the maximum pressure developing for any given wind direction. See Table 10.2.
- *Topographic factor*, $K_{zt}$. This factor accounts for effects such as wind speed-up over hills, ridges, and escarpments. This text considers only flat topography, for which $K_{zt} = 1.0$.
- *Gust Effect Factor*, $G$ - This factor accounts for the loading effects in the along-wind direction due to wind turbulence–structure interaction. For rigid structures such as those considered in this text (period of vibration $T \leq 1$s), $G = 0.85$.
- *Enclosure classification*, as follows:
  - Open building – a building having each wall at least 80% open
  - Partially enclosed building – a building having (a) total area of openings in a wall that receives positive external pressure exceeding the sum of areas of openings in the balance of the building envelope by more than 10%, AND (b) the total area of openings in a wall that receives positive pressure exceeds 4 ft$^2$.
  - Enclosed building – a building that is neither open or partially enclosed. This text only considers enclosed buildings.
- *Internal pressure coefficient*, $(GC_{pi})$ – This factor accounts for the change in internal pressure than can occur in enclosed and partially enclosed buildings. For enclosed buildings, $(GC_{pi}) = \pm 0.18$. Note that this pressure acts internally on all of the walls. As such, the pressures cancel when considering the main wind force-resisting system.

**Step 4**: Determine *velocity pressure exposure coefficient*, $K_z$ or $K_h$. These coefficients account for both the height above grade and the exposure category. See Table 10.3.

**Step 5:** Determine velocity pressure $q_z$ or $q_h$. See Eq. (10.2).

$$q_z = 0.00256 K_z K_{zt} K_d V^2 \ (psf) \ US\ Customary\ Units$$
$$q_z = 0.613 K_z K_{zt} K_d V^2 \ \ (N/m^2) \ SI\ Units \tag{10.2}$$

Note that $q_z$ refers to pressure varying as a function of elevation z above the base, whereas $q_h$ is used to define the pressure at the mean roof height $h$ of the building. The latter is used to define pressure on the side and leeward walls, where pressures are constant over height, with the value depending on the mean roof height $h$.

**Step 6**: Determine external pressure coefficients, $C_p$ or $C_N$. The coefficients vary with the windward, leeward, and side walls as shown in Figure 10.4. The coefficients shown apply only to enclosed and partially enclosed buildings. ASCE 7 also contains coefficients for different types of roofs and for open buildings, but these are not covered in this text.

**Step 7:** Calculate wind pressure, $p$, on each building surface, in accordance with Eq. (10.3).

$$p = qGC_p - q_i(GC_{pi}) \tag{10.3}$$

in which

$q = q_z$ for windward walls evaluated at height $z$ above the ground;
$q = q_h$ for leeward walls, side walls, and roofs, evaluated at height $h$;
$q_i = q_h$ for windward walls, side walls, leeward walls, and roofs of enclosed buildings;
$G$ = gust-effect factor;
$C_p$ = external pressure coefficient from Figure 10.4;
$(GC_{pi})$ = internal pressure coefficient, which for enclosed buildings is ±0.18.

Note that the pressure varies with height on the windward wall. On the leeward and side walls, however, the pressure is determined at the mean roof height $h$ and is constant over building height. See Figure 10.4.

Note that $(GC_{pi})$ is applied in two combinations, (a) with all surfaces subjected to negative pressure and (b) with all surfaces subjected to positive pressure. Thus, in a typical building the forces on one side of the building will cancel those on the other, so there is no effect on the main wind force-resisting system. Consequently, it will not be included in calculations for the main wind force-resisting system in this text.

Table 10.1 Risk Category of Buildings and Other Structures for Flood, Wind, Snow, Earthquake, and Ice Loads

| **Use or Occupancy of Buildings and Structures** | **Risk Category** |
|---|---|
| Buildings and other structures that represent a low risk to human life in the event of failure | I |
| All buildings and other structures except those listed in Risk Categories I, III, and IV | II |
| Buildings and other structures, the failure of which could pose a substantial risk to human life.<br>Buildings and other structures, not included in Risk Category IV, with potential to cause a substantial economic impact and/or mass disruption of day-to-day civilian life in the event of failure.<br>Buildings and other structures not included in Risk Category IV (including, but not limited to, facilities that manufacture, process, handle, store, use, or dispose of such substances as hazardous fuels, hazardous chemicals, hazardous waste, or explosives) containing toxic or explosive substances where the quantity of the material exceeds a threshold quantity established by the authority having jurisdiction and is sufficient to pose a threat to the public if released.[a] | III |
| Buildings and other structures designated as essential facilities.<br>Buildings and other structures, the failure of which could pose a substantial hazard to the community.<br>Buildings and other structures (including, but not limited to, facilities that manufacture, process, handle, store, use, or dispose of such substances as hazardous fuels, hazardous chemicals, or hazardous waste) containing sufficient quantities of highly toxic substances where the quantity of the material exceeds a threshold quantity established by the authority having jurisdiction and is sufficient to pose a threat to the public if released.[a]<br>Buildings and other structures required to maintain the functionality of other Risk Category IV structures. | IV |

[a] Buildings and other structures containing toxic, highly toxic, or explosive substances shall be eligible for classification to a lower Risk Category if it can be demonstrated to the satisfaction of the authority having jurisdiction by a hazard assessment as described in Section 1.5.3 that a release of the substances is commensurate with the risk associated with that Risk Category.

Table 10.2 Wind directionality factor, $K_d$.

| Structure Type | Directionality Factor $K_d$* |
|---|---|
| **Buildings** | |
| Main Wind Force Resisting System | 0.85 |
| Components and Cladding | 0.85 |
| **Arched Roofs** | 0.85 |
| **Chimneys, Tanks, and Similar Structures** | |
| Square | 0.90 |
| Hexagonal | 0.95 |
| Round | 0.95 |
| **Solid Freestanding Walls and Solid Freestanding and Attached Signs** | 0.85 |
| **Open Signs and Lattice Framework** | 0.85 |
| **Trussed Towers** | |
| Triangular, square, rectangular | 0.85 |
| All other cross sections | 0.95 |

Table 10.3 Velocity pressure exposure coefficient, $K_z$.

| Height above ground level, $z$ | | Exposure | | |
|---|---|---|---|---|
| ft | (m) | B | C | D |
| 0-15 | (0-4.6) | 0.57 | 0.85 | 1.03 |
| 20 | (6.1) | 0.62 | 0.90 | 1.08 |
| 25 | (7.6) | 0.66 | 0.94 | 1.12 |
| 30 | (9.1) | 0.70 | 0.98 | 1.16 |
| 40 | (12.2) | 0.76 | 1.04 | 1.22 |
| 50 | (15.2) | 0.81 | 1.09 | 1.27 |
| 60 | (18) | 0.85 | 1.13 | 1.31 |
| 70 | (21.3) | 0.89 | 1.17 | 1.34 |
| 80 | (24.4) | 0.93 | 1.21 | 1.38 |
| 90 | (27.4) | 0.96 | 1.24 | 1.40 |
| 100 | (30.5) | 0.99 | 1.26 | 1.43 |
| 120 | (36.6) | 1.04 | 1.31 | 1.48 |
| 140 | (42.7) | 1.09 | 1.36 | 1.52 |
| 160 | (48.8) | 1.13 | 1.39 | 1.55 |
| 180 | (54.9) | 1.17 | 1.43 | 1.58 |
| 200 | (61.0) | 1.20 | 1.46 | 1.61 |

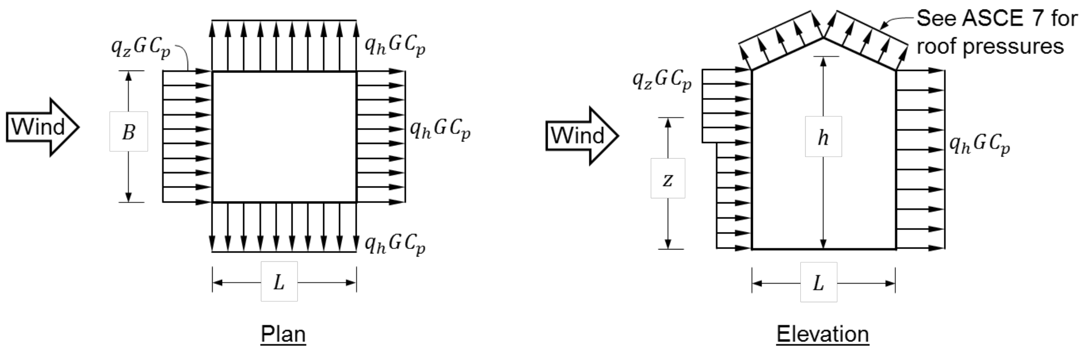



Figure 10.4 Illustration of external pressure coefficients acting on plan and elevation of building.

| Table 10.4 **Wall Pressure Coefficients, $C_p$.** (Interpolation is permitted.) | | | |
|---|---|---|---|
| **Surface** | **L/B** | $C_p$ | **Use With** |
| Windward Wall | All values | 0.8 | $q_z$ |
| Leeward Wall | 0-1 | -0.5 | $q_h$ |
| | 2 | -0.3 | |
| | ≥4 | -0.2 | |
| Side Wall | All values | -0.7 | $q_h$ |

**Example 1**: For the purpose of determining the forces on the Main Wind Force-Resisting System (MWFRS), determine the wind pressure distribution on the four sides of an eight-story office building shown in Figure 10.5. The building measures 60 ft by 60 ft in plan, with total height of 99 ft. Floor elevations are in Table 10.5. The building is located in Walnut Creek, California. Use the ASCE 7-10 Directional Procedure.

**Step 1**: The building is Risk Category II (Table 10.1).

**Step 2**: The basic wind speed is $V = 110$ mph.

**Step 3**: Determine wind load parameters:

- *Exposure category*: B
- *Wind directionality factor*: From Table 10.2, $K_d = 0.85$.
- *Topographic factor*: $K_{zt} = 1.0$.
- *Gust Effect Factor*: $G = 0.85$.
- *Enclosure classification*: This is an enclosed building.
- *Internal pressure coefficient*: For enclosed buildings, $(GC_{pi}) = \pm 0.18$. However, the internal pressures cancel for the purpose of determining the total force on the MWFRS. Therefore, ignore $(GC_{pi})$.

**Step 4**: Determine *velocity pressure exposure coefficient*: Values of $K_z$ can be read from Table 10.3. For this example, we will interpolate between values in that table to determine values at the centerline of each floor level, so that we can readily use that pressure along with tributary area concepts to determine the total loads.

**Step 5:** Determine velocity pressure $q_z$ or $q_h$. See Eq. (10.2).

**Step 6**: Determine external pressure coefficients: Values of $C_p$ are shown in Figure 10.4 and Table 10.4.

**Step 7:** Calculate wind pressure, $p$, on each building surface, in accordance with Eq. (10.3). We will ignore ($GC_{pi}$) because it cancels out for the purpose of finding the total building lateral force. The pressure distribution on the windward face is presented in the table below. Note that $K_z$ values are interpolated from Table 10.3. For example, level 8 is at elevation 99 ft, giving an interpolated value of $K_z$ = [(99 ft – 90 ft)/(100 ft – 90 ft)](0.99 - 0.96) + 0.96 = 0.99.

Table 10.5 Example 1.

| Level | Elevation, $z$, ft | $K_z$ | $q_z = 0.00256K_zK_{zt}K_dV^2$, psf | $p = q_zGC_p$, psf |
|---|---|---|---|---|
| 8 | 99 | 0.99 | $(0.00256)(0.99)(1)(0.85)(110)^2 = 26.1$ | (25.3)(0.85)(0.8) = 17.7 |
| 7 | 87 | 0.95 | 25.0 | 17.0 |
| 6 | 75 | 0.91 | 24.0 | 16.3 |
| 5 | 63 | 0.86 | 22.6 | 15.4 |
| 4 | 51 | 0.81 | 21.3 | 14.5 |
| 3 | 39 | 0.75 | 19.7 | 13.4 |
| 2 | 27 | 0.68 | 17.9 | 12.2 |
| 1 | 15 | 0.57 | 15.0 | 10.2 |

The pressure on the leeward face is evaluated at the mean roof height, or 99 ft. Therefore, the pressure on the leeward face is $p = q_hGC_p$ = (26.1 psf)(0.85)(-0.5) = -11.1 psf. The pressure on the side walls is also evaluated at the mean roof height, or 99 ft. Therefore, the pressure on the side walls is $p = q_hGC_p$ = (26.1 psf)(0.85)(-0.7) = -15.5 psf.

Figure 10.5 presents the solution for Example 1.

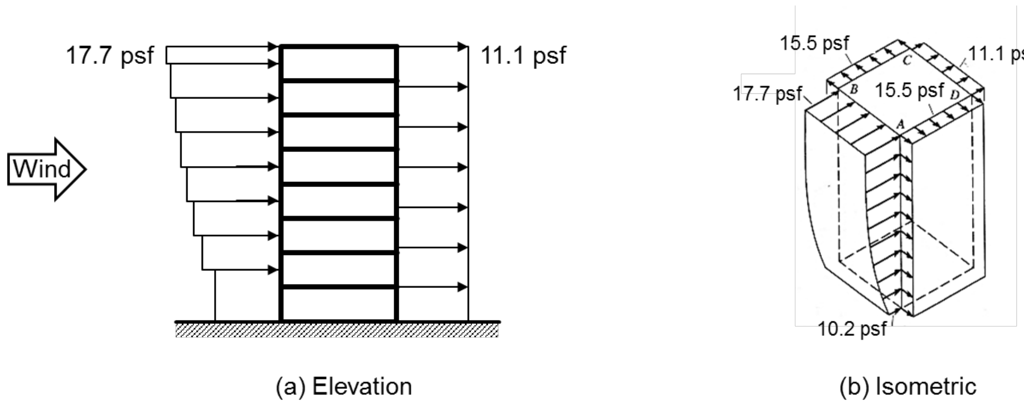



Figure 10.5 Solution to Example 1 (Building not to scale).

## 10.4. TRANSFER OF WIND FORCES THROUGH A BUILDING

Wind loads are applied mainly as pressure acting on the exterior surface of a building (some additional internal pressure may also apply). Usually, the building is clad with a façade that is supported by the floor slabs/beams (Figure 10.6). Therefore, for the purpose of determining the wind forces, we can assume that the façade spans vertically from one floor to another. The tributary area method can be used to determine the force per floor.

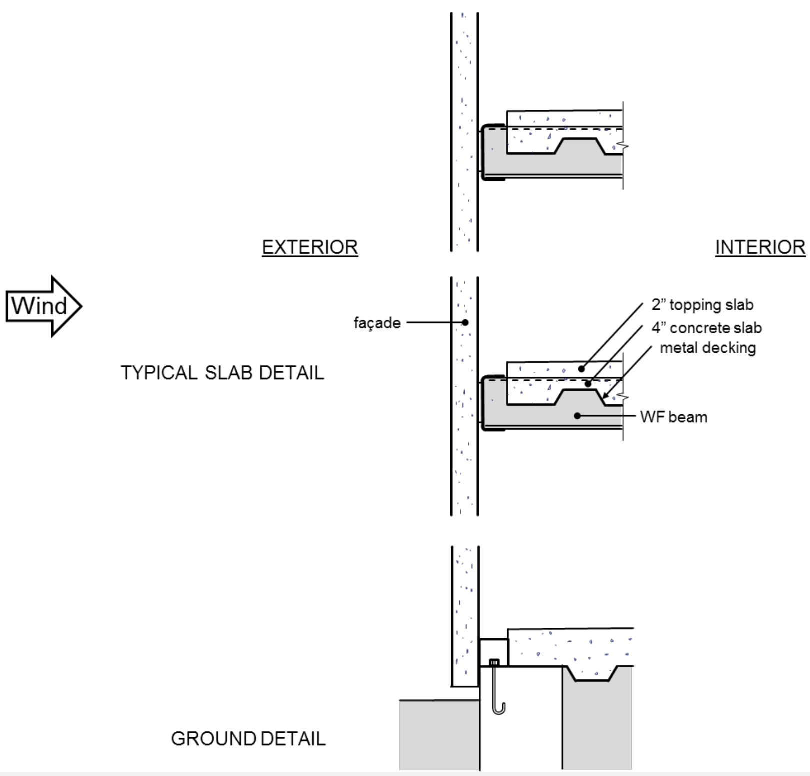



Figure 10.6 Sample façade details.

Figure 10.7 illustrates the load path for wind forces applied to the windward side of a building. Similar effects occur on the other sides. Considering the top story only, wind pressure *p* actions on cladding panel abcd. Panel abcd is supported by the roof slab along line ab and the floor slab below that along line cd. The roof and floor slabs act as *diaphragms* to connect the framing system together and transfer lateral forces to the *vertical elements* of the *lateral-force-resisting system (or main wind force-resisting sysem)*. In this example, structural walls (sometimes referred to as shear walls) are the vertical elements of the lateral-force-resisting system. Therefore, the diaphragm spans between the shear walls. The walls resist lateral forces from each of the floor diaphragms in a similar manner, and transfer these forces and the resulting moments down to the foundation.

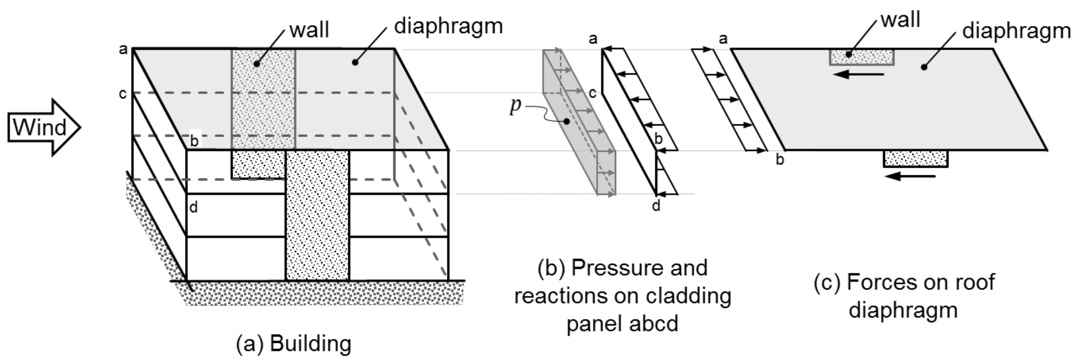



Figure 10.7 Load path for wind forces.

Diaphragms commonly are idealized as beams spanning horizontally between the vertical elements of the lateral-force-resisting system. Thus, as shown in Figure 10.8, we can calculate shears and moments in the diaphragm.

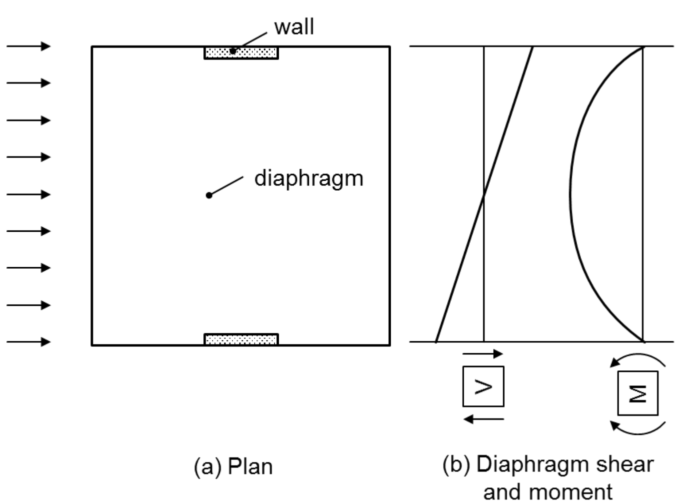



Figure 10.8 Diaphragm shear and moment.

Moment is usually resisted by tension and compression chords concentrated near the boundaries of the diaphragm (Figure 10.9). Given the concentrated resistance to moment, equilibrium requires shear flow in the diaphragm to be constant, similar to the nearly constant shear flow in an I-beam. Figure 10.9 illustrates the uniformly distributed shear stresses in the diaphragm near the vertical elements of the lateral-force-resisting system. The shear stress located far from the vertical element needs to be collected and dragged back to the vertical element. A *collector* is the element provided to collect the distributed diaphragm shear and drag it to the vertical element. In Figure 10.9, collector element ghij acts in compression along gh and in tension along ij.

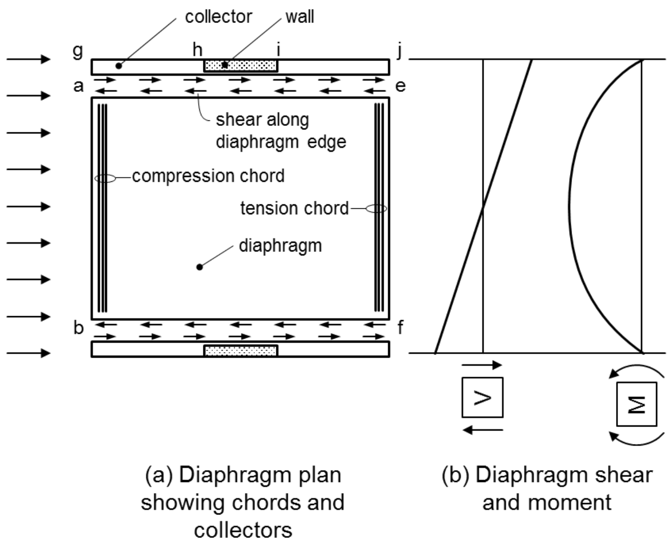



Figure 10.9 Collector and chord elements of a diaphragm.

**Example 2**: A four-story building is located near the eight-story building of Example 1. The building has a light frame and stiff structural walls (Figure 10.10). Calculate the force per foot of length acting on each diaphragm; the forces, shears, and moments on one the walls; and the internal forces on the first-story diaphragm.

**Solution**: Figure 10.10 shows the lateral pressures acting on windward and leeward walls. Note that the pressures on the windward wall are the same as the pressures on the lower four stories of eight-story tall building, but the pressures on the leeward and side walls are different because the building height is less. The pressures on the side wall do not contribute to the diaphragm and wall forces for the direction under consideration, and are not shown. In the table under the building drawing, we list the height $\Delta h_i$ of each story, the total height $h_i$ of the level above the base, the tributary height $h_{trib}$ for each diaphragm, the sum of pressures on the windward and leeward walls, the distributed force on each diaphragm ($f_i = p_i \times h_{trib}$), and the total force $F_i$ on the diaphragm at each level.

The next table summarizes the forces on a single wall (each wall resists half the forces $F_i$ determined in the previous step). Forces are summed to obtain wall shears, and multiplied by heights to obtain wall moments. These are plotted at the bottom of the sheet.

The next sheet continues with forces on the first-level diaphragm. Shears and moments are calculated and shear and moment diagrams are plotted. Shear stress at the edge of the diaphragm is calculated assuming uniform shear flow. The stress acts along lengths of the collectors, enabling determination of the collector compressive and tensile forces.

***Wind pressures***

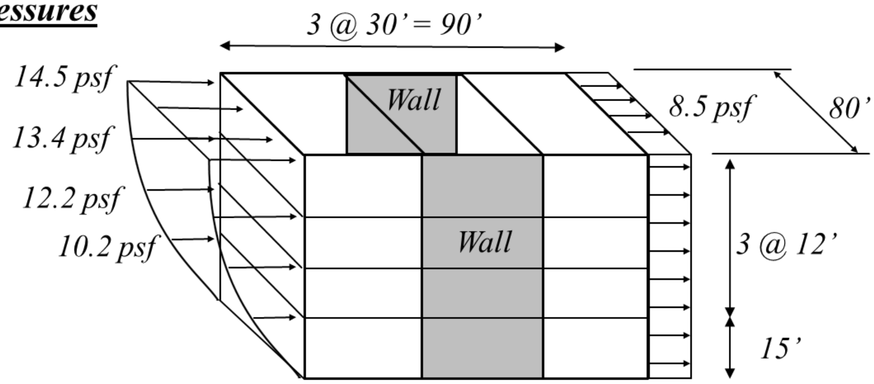



***Diagram Forces***

| Level | $\Delta h_i$ | $h_i$ | $h_{trib}$ | $\Sigma p_i$, psf | $f_i$ , plf | $F_i$, kip |
|---|---|---|---|---|---|---|
| 4 | 12' | 51' | 6' | 23.0 | 138 | 11.0 |
| 3 | 12' | 39' | 12' | 21.9 | 263 | 21.0 |
| 2 | 12' | 27' | 12' | 20.7 | 248 | 19.8 |
| 1 | 15' | 15' | 13.5' | 18.7 | 252 | 20.2 |
| 0 | | 0 | | 18.7 | - | - |

***Wall Forces***

| $\Delta h_i$ , ft | $V_i$ | $M_i = V_i \Delta h_i + M_{i+1}$ |
|---|---|---|
| 12 | 5.5 k | 66.0 k-ft |
| 12 | 16.0 k | 258 k-ft |
| 12 | 25.9 k | 569 k-ft |
| 15 | 36.0 k | 1110 k -ft |

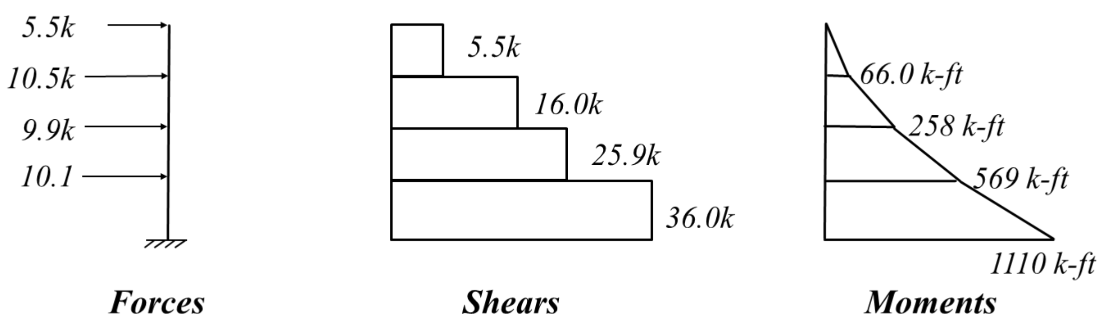
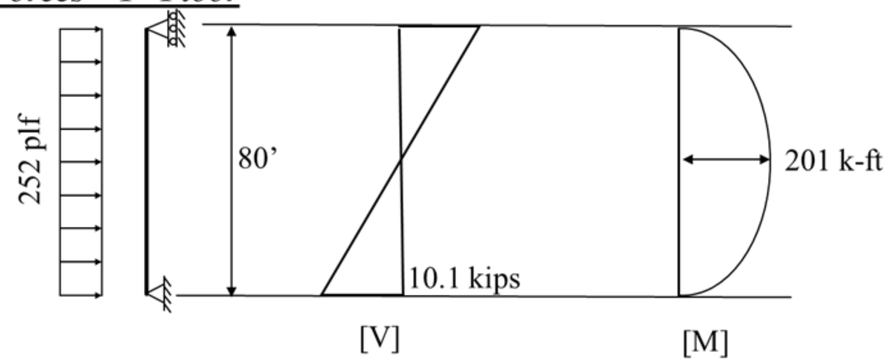



Figure 10.10 Example 2.

***Diagram Forces – 1st Floor***



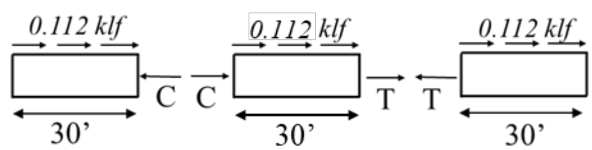

$$V_{max} = \frac{wl}{2} = \frac{252plf*80'}{2} = 10.1\ kips$$

$$M_{max} = \frac{wl^2}{8} = \frac{252plf*80^2}{8} = 201\ k\text{-}ft$$

***Collector Forces – 1st Floor***

Assume shear uniformly distributed along cdgc

$$v = \frac{V}{l} = \frac{10.1\ kips}{90'} = 0.112klf$$




$C = (0.112klf)*(30') = 3.37\ kips$

$T = (0.112klf)*(30') = 3.37\ kips$

**Figure 10.10: Example 2 (continued)**

## 10.5. DISTRIBUTION OF DIAPHRAGM FORCES – RIGID VERSUS FLEXIBLE DIAPHRAGMS

In the previous examples the diaphragm was supported by only two vertical elements of the lateral-force-resisting system, such that the distribution of forces to the vertical elements could be determined by equilibrium alone. Where multiple vertical elements support the diaphragm, structural analysis using computer software is sometimes done to determine the distribution of forces. However, two idealized cases are often considered to simplify the analysis problem, as follows:

- *Rigid diaphragm*: If the diaphragm is very stiff compared with the stiffness of the vertical elements, it is common to idealize the diaphragm as being rigid. Examples include concrete diaphragms or concrete fill on metal deck diaphragms supported by frames or by wood construction. In this case, assuming the diaphragm to translate without rotation, the distribution of forces to the various elements will be in proportion with the relative stiffness of the elements (Figure 10.11a). Pure translation without rotation occurs where the center of resistance of the vertical elements coincides with the centroid of the applied lateral forces. Where this does not occur, the analysis needs to include effects of diaphragm plan rotation. This can be solved by hand calculations, but it is more commonly done by computer software.

- *Flexible diaphragm*: If the diaphragm is very flexible compared with the stiffness of the vertical elements, it is common to idealize the diaphragm as being completely flexible. Examples include wood diaphragms supported by concrete or masonry walls, or by steel braced frames. In this case, the diaphragm is idealized as having zero stiffness (similar to a cable), in which the reactions to the vertical elements can be determined by tributary area method (Figure 10.11b).

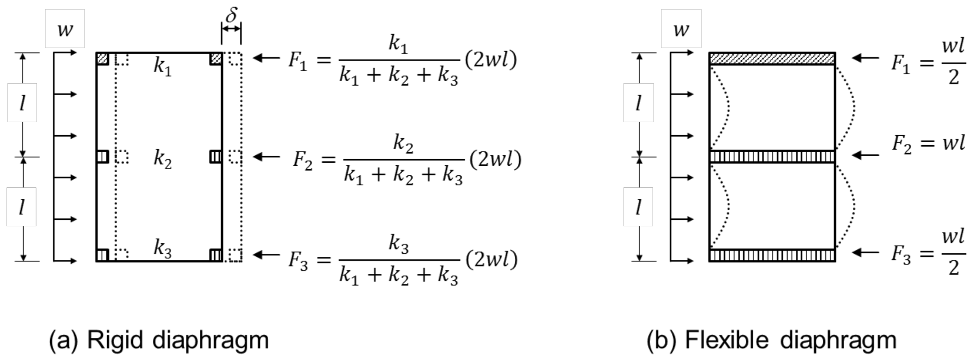



Figure 10.11 Rigid versus flexible diaphragm idealizations.

## 10.6. LATERAL-FORCE-RESISTING SYSTEMS

Lateral-force-resisting systems are considered to comprise vertical elements (such as shear walls) and horizontal elements (diaphragms). Figure 10.12 shows the main types of vertical elements. Shear walls (also called structural walls) can be made of reinforced concrete, reinforced masonry, or plywood. Braced frames are usually made of steel construction. Rigid frames can be either reinforced concrete or structural steel. Rigid frames with fixed supports are somewhat unusual in modern construction.

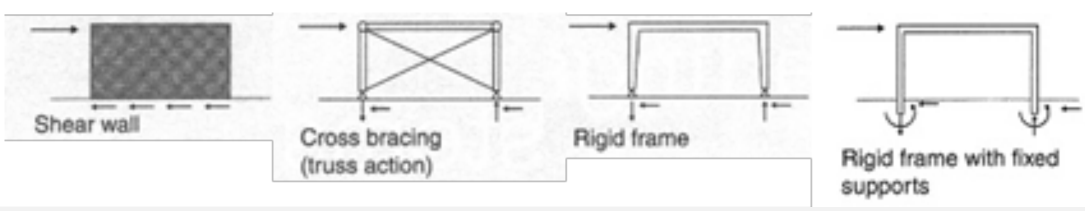



Figure 10.12 Vertical elements of lateral-force-resisting systems. Figures from Schodek and Bechthold, 2014[3]).

The vertical elements of the lateral-force-resisting system should be placed so that the center of resistance is close to the centroid of the applied lateral forces, thereby minimizing torsion.

[3] Structures, D.L. Schodek and M. Bechthold, 7th edition, Pearson Education, 2014.

Figure 10.13 shows braced frames, shear walls, and moment-resisting frames in a one-story structure.

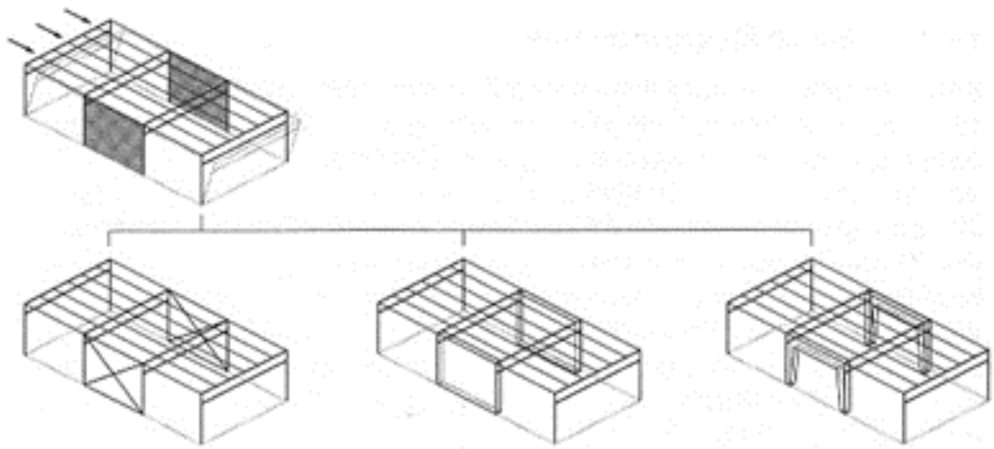


Figure 10.13 Positioning of vertical elements of the lateral-force-resisting system. Figures from Schodek and Bechthold, 2014[4]).

To be effective, lateral-force-resisting systems need to have diaphragms capable of transferring horizontal forces to the vertical elements of the lateral-force-resisting system. Figure 10.14 shows the two main options: (a) In steel structures the diaphragm can be composed of horizontal trusses within the plane of the diaphragm. (b) In wood, steel, concrete, or masonry structures, a solid diaphragm composed of wood, reinforced concrete, or reinforced concrete on metal deck can serve as the diaphragm.

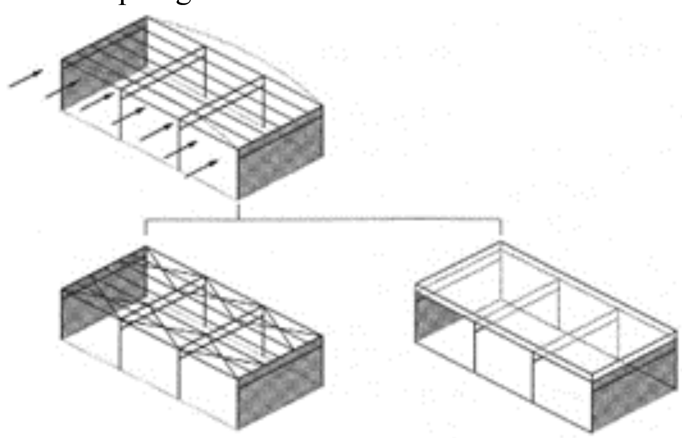


Figure 10.14 Diaphragms of lateral-force-resisting systems. Figures from Schodek and Bechthold, 2014).

Lateral forces can act in any direction. Therefore, lateral bracing is required along both principal directions of the structural system. Figure 10.15b shows beam-column framing in each

[4] Structures, D.L. Schodek and M. Bechthold, 7th edition, Pearson Education, 2014.

bay of a structure. Figure 10.15c shows the option of providing bracing only around the perimeter of the structure. Figure 10.15d shows a plan with distributed walls, including a centrally located core wall. Figure 10.15e shows a plan with end bracing. Figure 10.15f shows a plan in which some wings are inadequately braced.

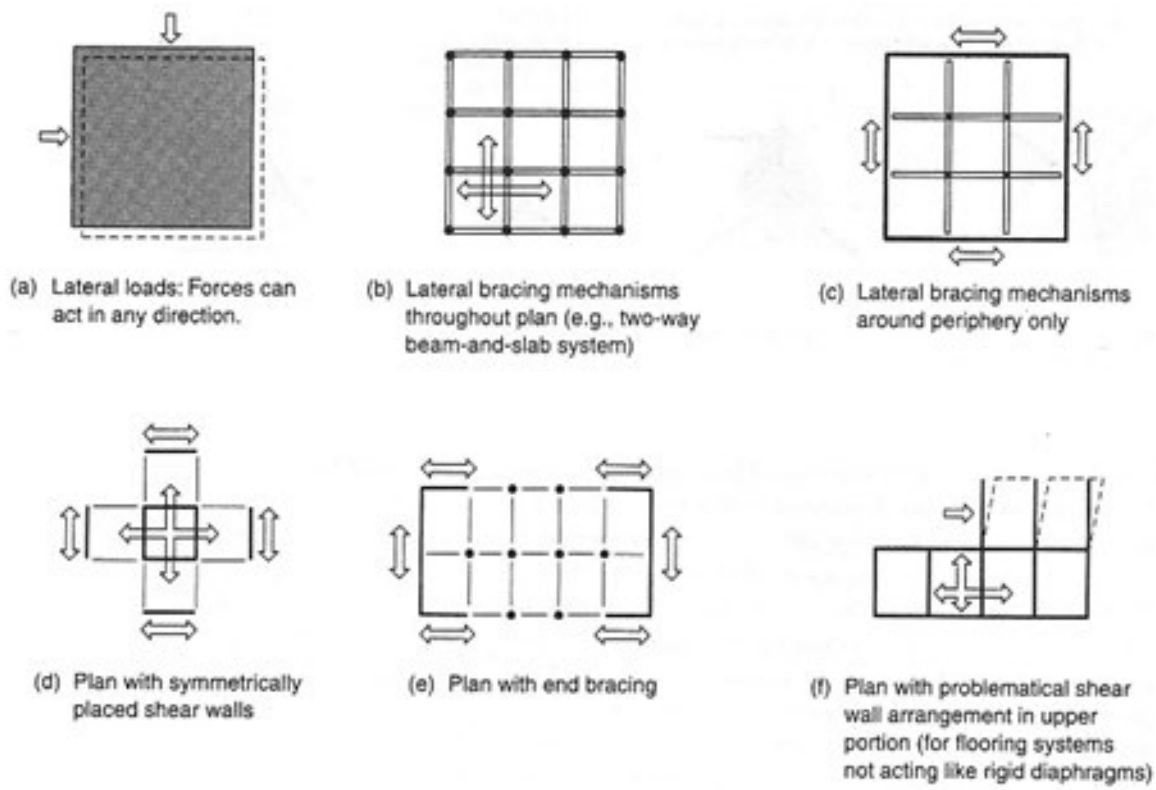



Figure 10.15 Location of vertical elements within the floor plan. Figures from Schodek and Bechthold, 2014).

Figure 10.16 shows a typical structural steel system. Diaphragms are composed of reinforced concrete on metal deck. The vertical elements in one direction are moment-resisting frames, while those in the orthogonal direction are steel braced frames.

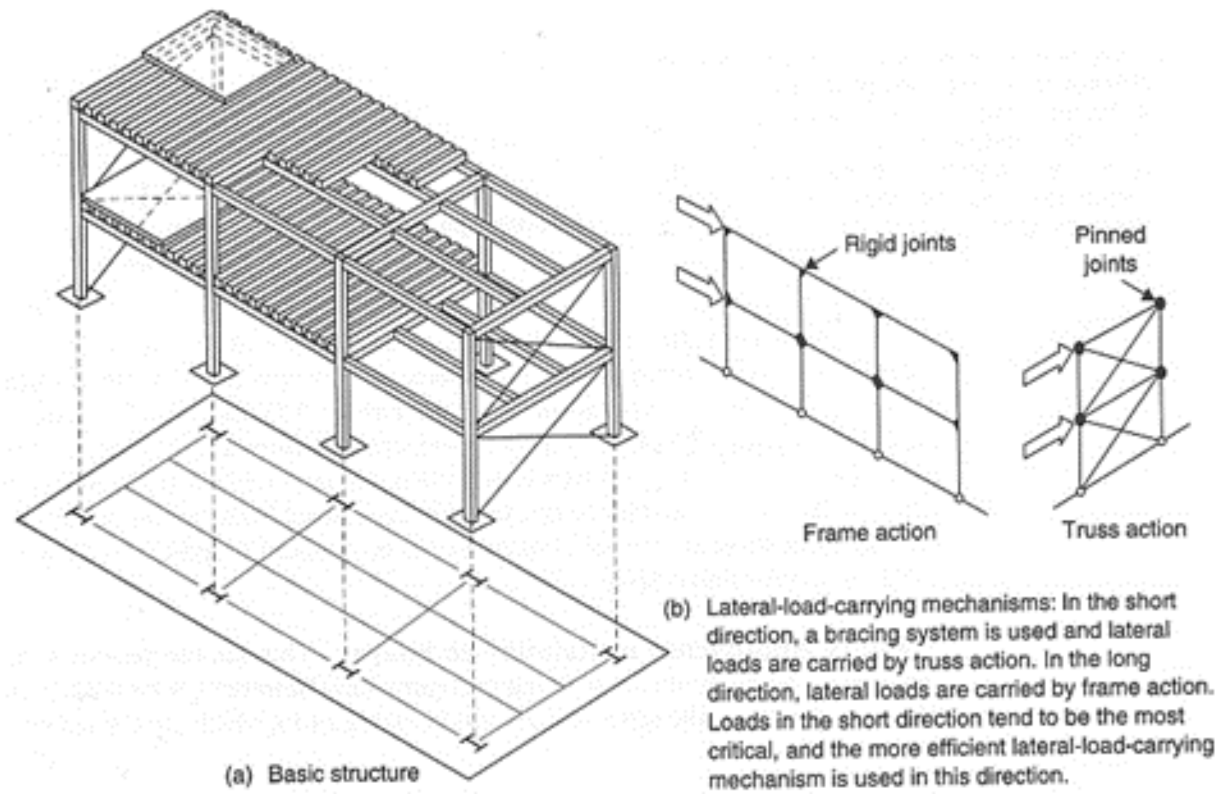



Figure 10.16 Lateral-force-resisting system in a steel building. Figures from Schodek and Bechthold, 2014).

Tall buildings require an efficient system of elements to resist lateral forces. In some buildings the entire perimeter of the building is designed to act as a tube to resist shear and overturning moment (Figure 10.17).

Figure 10.18 illustrates some options for lateral-force-resisting systems of tall buildings. Figure 10.18a illustrates a moment frame system. Figure 10.18b illustrates a dual system involving a moment frame and core wall. In many modern buildings, the core wall acts alone to resist lateral forces. The outrigger system of Figure 10.18c can improve the efficiency of very tall buildings by engaging the axial stiffness of the exterior columns as outriggers of the core wall. Figure 10.18d depicts a tube system in which a closely spaced grid of beams and columns provides a stiff box around the perimeter of the building. Figure 10.18e and f depict steel braced frame options.

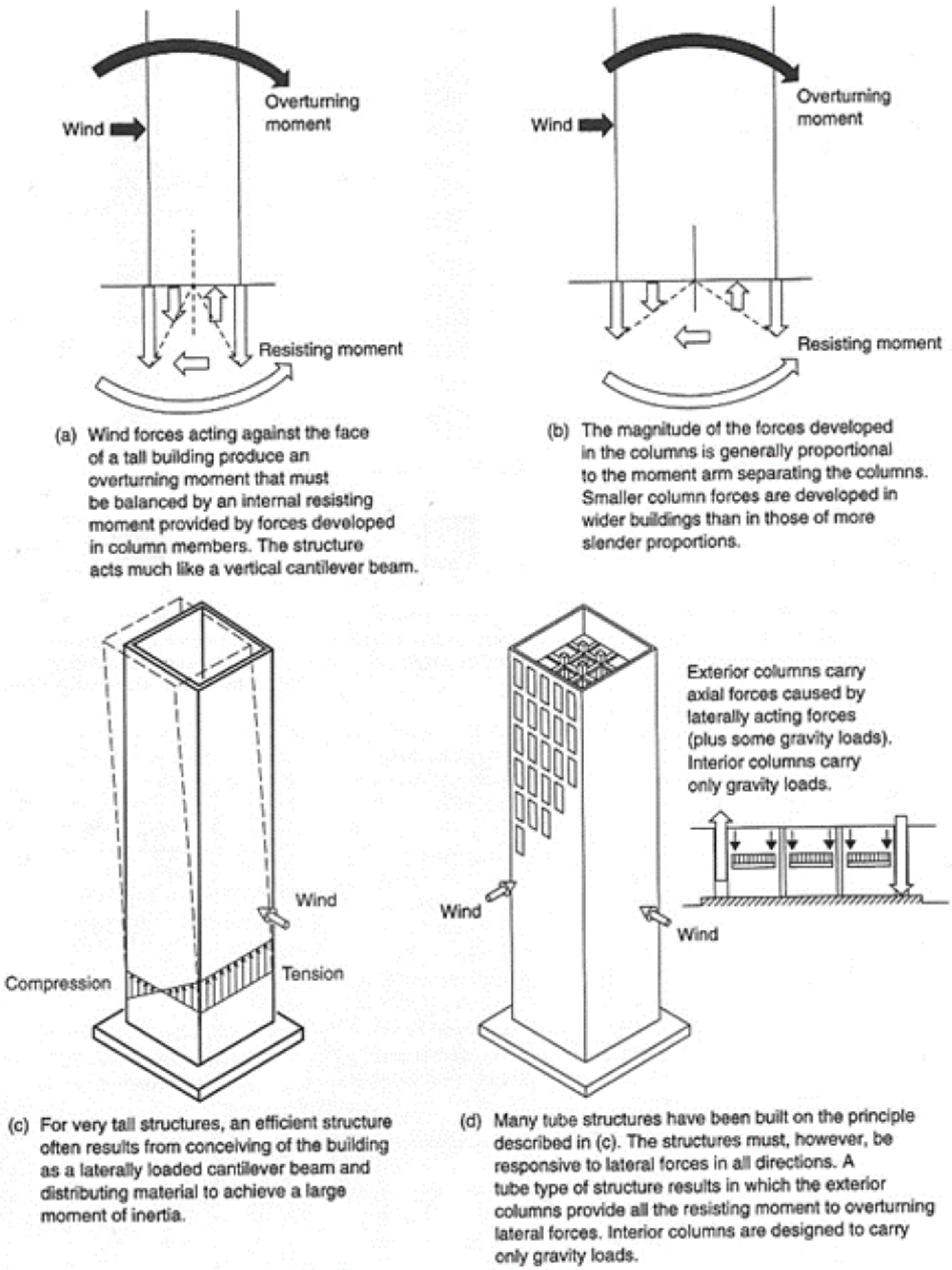



Figure 10.17 Tube structures to resist overturning moments in tall buildings. Figures from Schodek and Bechthold, 2014).

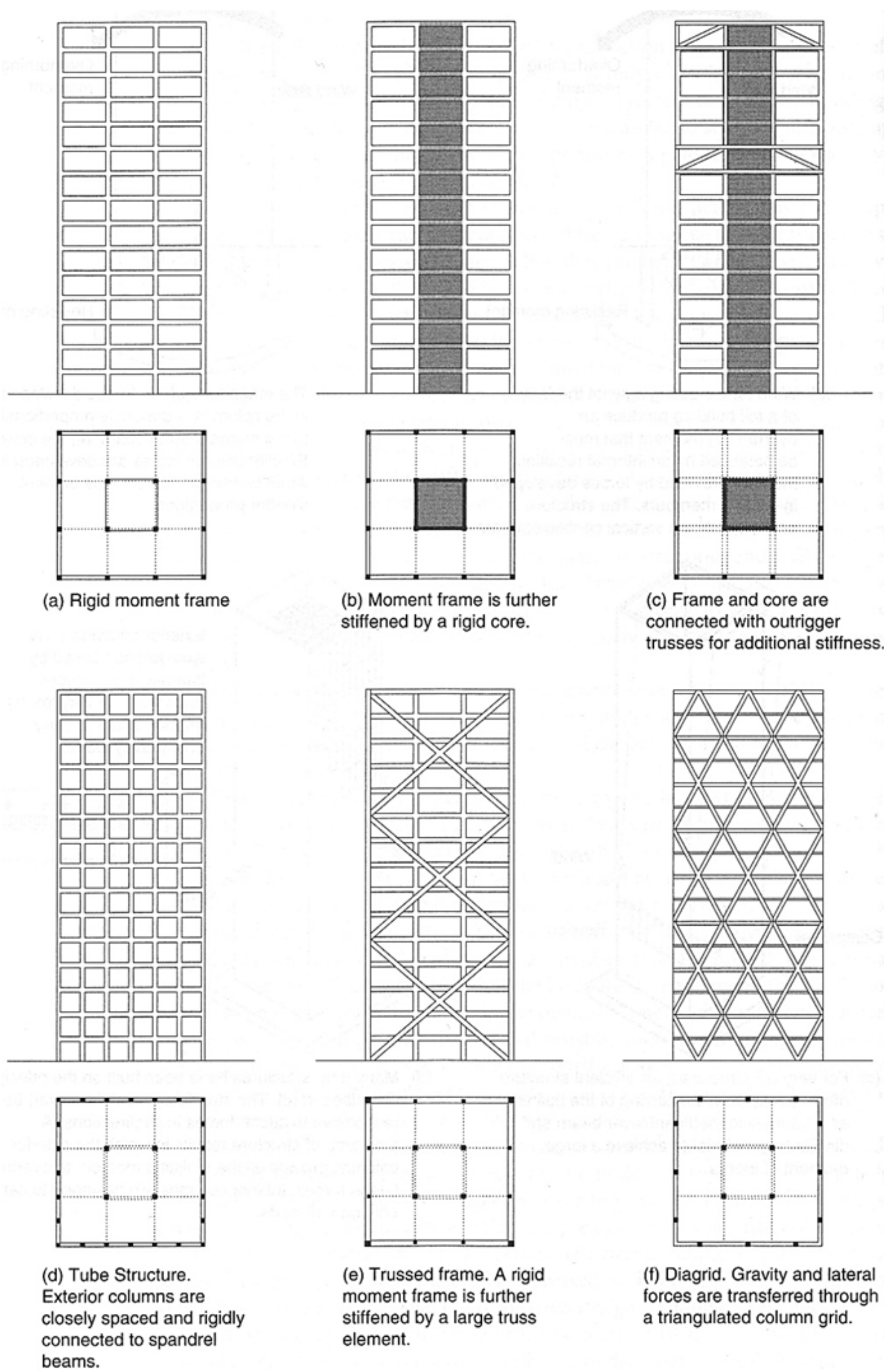



Figure 10.18 Different framing systems for tall buildings. Figures from Schodek and Bechthold, 2014).

# 11. Earthquake Loads and Load Paths

## 11.1. INTRODUCTION

This chapter introduces how a structural engineer determines the earthquake loads for design of a structure. The concept of a load path for earthquake loading is introduced and used to identify how a structure resists lateral loads. Examples of lateral-force-resisting systems are shown. The loads and methods generally follow the procedures of ASCE 7-10.[1]

## 11.2. GENERAL NATURE OF EARTHQUAKE LOADS

### 11.2.1. Earthquakes and Earthquake Hazards

The term earthquake refers generally to any event that generates seismic waves. Earthquakes can be due to natural or man-made causes. From an earthquake engineering perspective, however, the earthquakes of greatest interest are due to rupture along geologic faults. The vast majority of such earthquakes occur near boundaries of tectonic plates, and are the result of energy release and fault rupture as plate boundaries slip past one another along a zone of geologic faults. Intra-plate earthquakes, occurring distant from the tectonic plate boundaries, can also be important, as in the New Madrid earthquake fault zone in the Central United States, or the volcanic-origin earthquakes of Hawaii.

Ground shaking is the main cause of earthquake damage to buildings. For this reason, earthquake ground shaking hazard is the main focus of most seismic designs and performance assessments. Other seismic hazards that may cause damage to buildings include surface fault rupture, liquefaction and associated settlement and lateral spreading, differential settlement of foundation material, landsliding, and tsunami. These latter effects should be included in assessment and design where they may occur.

Earthquake ground shaking has been recorded using strong motion instruments since the 1933 Long Beach, California earthquake. Instruments typically record the acceleration at a point in two horizontal directions and the vertical direction. Acceleration recordings can be processed to remove errors and noises, and then integrated to obtain velocity and displacement records. Figure 11.1 shows examples of recorded accelerations and derived velocities from several earthquakes.

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[1] ASCE 7-10, Minimum Design Loads for Buildings and Other Structures (ASCE/SEI 7-10), American Society of Civil Engineers, 2010,

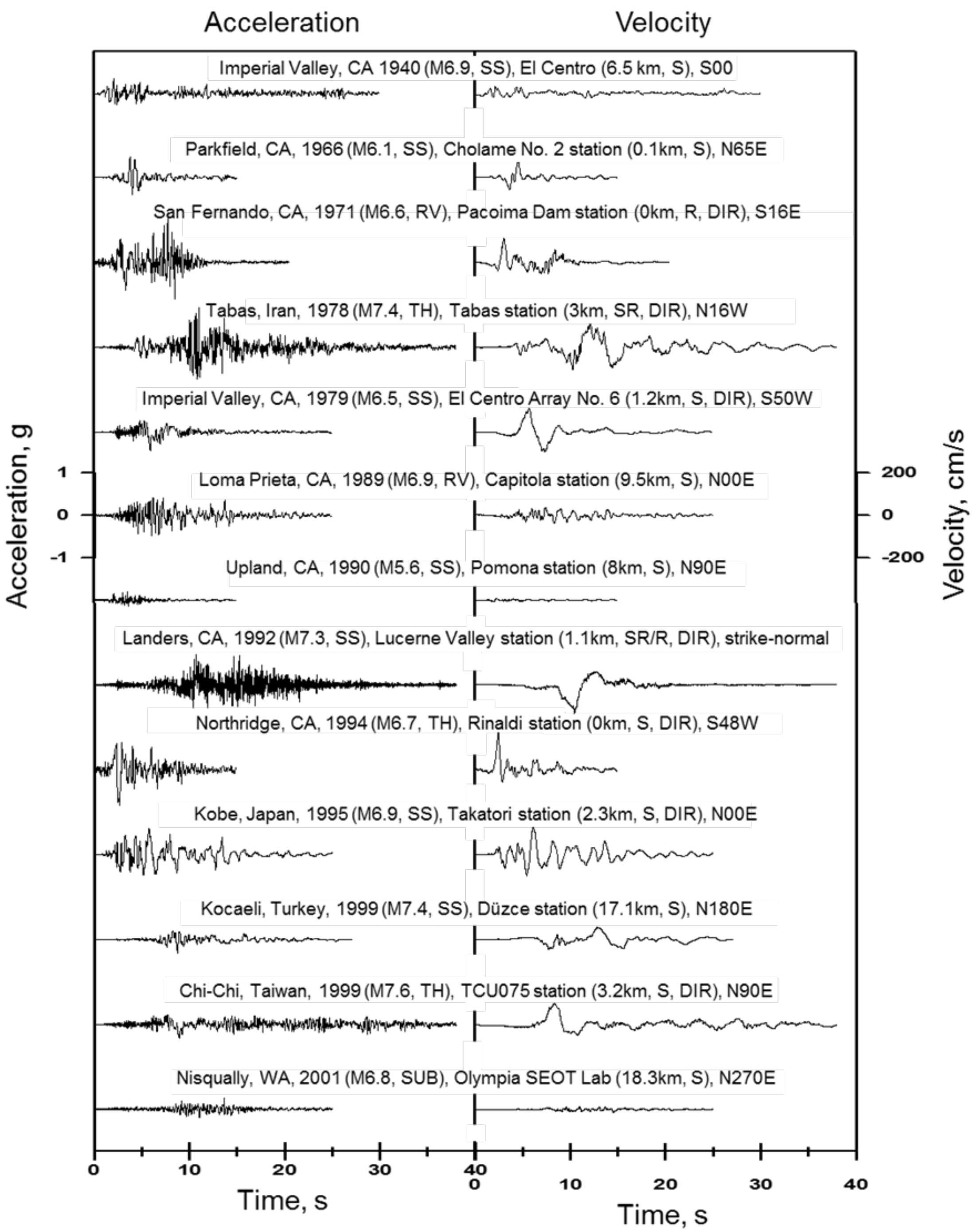



Figure 11.1 Selected recorded ground accelerations and corresponding ground velocities (plotted at the same scale). SS = strike-slip faulting; RV = reverse faulting; TH = thrust faulting; SUB = subduction intraslab earthquake; S = soil site; R = rock site; SR = soft rock site; DIR = record includes fault rupture directivity effects. Distance measure is from the recording site to surface projection of fault rupture plane (epicentral distance for the Nisqually earthquake) (after Bozorgnia and Campbell, 2004).

The nature of ground shaking at a building site is a function of several factors, including earthquake magnitude, style of faulting, depth to top of fault rupture, source-to-site distance, site location on hanging wall or footwall of dipping faults, near-surface soil response, sedimentary basin depth/depth to basement rock, and other effects related to the three-dimensional wave propagation from the source to the site. The following paragraphs describe some of the main effects.

*Earthquake magnitude* is a measure of the energy released in an earthquake, and therefore relates to the rupture area and displacement along the fault. Larger-magnitude earthquakes generally have longer fault rupture lengths. Because it takes time for rupture to extend along a fault, we should anticipate that larger-magnitude earthquakes generally have a potential for longer shaking duration at a site. The larger energy release also creates a potential for higher shaking intensity at a site, especially for longer period ground motions. A given fault can be capable of generating earthquakes with magnitudes ranging from the low end to some upper bound constrained by the length of the fault. Generally, smaller-magnitude earthquakes occur more frequently, with larger-magnitude earthquakes occurring less frequently. Some faults and fault segments, however, tend to repeatedly generate *characteristic earthquakes* of comparable magnitude.

Earthquake hazard experts develop and use empirical or simulation-based ground motion attenuation models (or ground motion prediction equations) to estimate how ground motion intensity varies with magnitude and distance from the fault. Figure 11.2 shows the median attenuation for peak horizontal ground acceleration from Campbell and Bozorgnia [2014]. The model shows that peak ground acceleration at short distances is nearly independent of magnitude for moment magnitude greater than about M 6.5, and that ground motion only slightly attenuates within approximately 5 km of the fault. For additional discussion on attenuation models, see Bozorgnia et al. (2014).

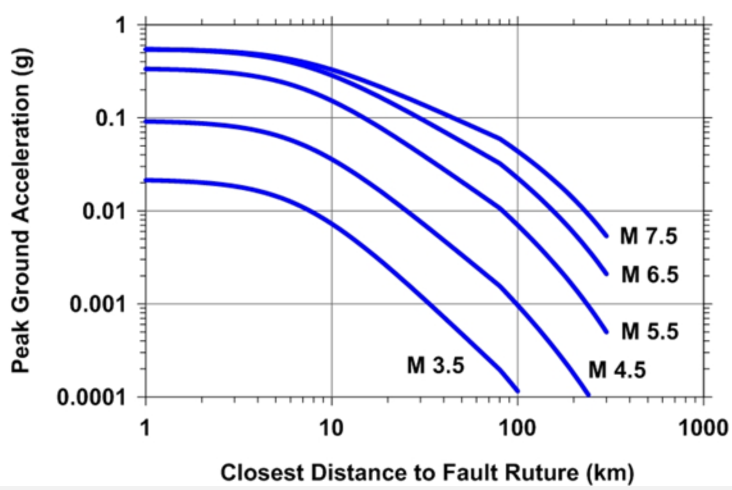



Figure 11.2 Expected attenuation of ground motion with rupture distance showing its dependence on moment magnitude (M) (after Campbell and Bozorgnia, 2014).

Ground motions in close proximity to the seismic source (within approximately 10 km) can be significantly influenced by near-fault effects referred to as *rupture directivity*. Fault rupture releases energy in the form of waves that propagate from the rupture source. Because the velocity at which rupture propagates and the velocity at which resulting waves propagate are usually similar, the progression of rupture along the fault results in a buildup of energy in the direction of rupture. Earthquake rupture toward a site tends to produce strongly impulsive ground motions, an effect referred to as *forward-directivity*. The impulsive motion may be especially strong in the fault-normal direction. In contrast, earthquake rupture away from a site, referred to as either *neutral-directivity* or *backward-directivity*, produces waves that are continually sent toward the site from increasing distance, and therefore tends to produce longer-duration motion of relatively lower amplitude. Figure 11.3 illustrates an example of the effect of rupture directivity on earthquake ground motion.

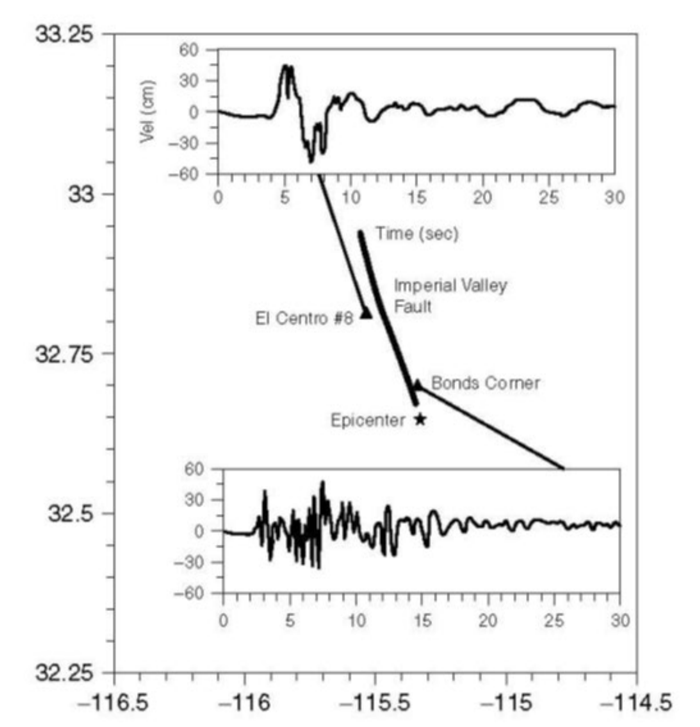



Figure 11.3 Velocity records from the 1979 Imperial Valley, California, earthquake at the Bonds Corner and El Centro Differential Array strong ground motion recording sites. Note the shorter duration, impulsive motion at El Centro #8 (forward-directivity) and longer duration lower amplitude motion at Bonds Corner (backward-directivity). (Bolt, 2004).

Ground shaking at a site also is affected by near-surface soil flexibility. Soft soil deposits tend to amplify earthquake ground motions, especially for longer periods. Complicating the problem, nonlinear response of soft sites may result in de-amplification as the intensity of input motions increases. Building codes have developed site amplification factors based on a geotechnical site classification system and the intensity of input motions. As an alternative to building code site amplification factors, geotechnical engineers sometimes use one-dimensional and three-dimensional modeling procedures to estimate site amplification effects.

Ground shaking at a site is influenced by other geologic factors. Thrust faults, especially if the rupture does not reach the surface, generate higher than average ground motions especially at shorter periods. Furthermore, shaking tends to be higher on the hanging wall (that is, the portion of the earth's crust above the fault plane) for thrust faults. As earthquake waves travel from the source to the site, complex geologic structures can reflect and refract earthquake ground motions, resulting in focusing of earthquake energy at some sites. On the other hand, normal faults, generally produce ground motions either comparable or less than those generated by strike-slip faults.

While some of these effects can be anticipated based on knowledge of the geologic setting, others depend on details of the faulting mechanism and source-to-site path, which cannot be known before an earthquake. Thus, quantification of uncertainty in forward estimation of ground shaking is an important topic in earthquake engineering, and earthquake ground motion commonly is described in probabilistic terms that enable general statements about the expected shaking and the variability about that expectation.

### 11.2.2. Dynamic Response of Structures

Consider the planar, one-story structure shown in Figure 11.4a. Assume the axial stiffness of the columns and the overall stiffness of the supported mass are infinite, such that displacement can occur only due to flexural and shear deformations of the columns. Under these conditions, the only possible movement of the mass is lateral sway. Thus, we say this structure has a *single degree of freedom* (SDOF).

When an earthquake occurs, seismic waves travel from the source to the site, causing displacement of the supports (Figure 11.4b). If we take the structure at rest, and suddenly impose a lateral ground displacement $u_g$, internal shear forces $V$ and moments $M$ will develop in the supporting columns (Figure 11.4c). Equilibrium requires that the mass $m$ accelerate such that an inertial force $ma$ is developed to equilibrate the sum of the column shears. Thus, ground motion causes the structure to respond dynamically.

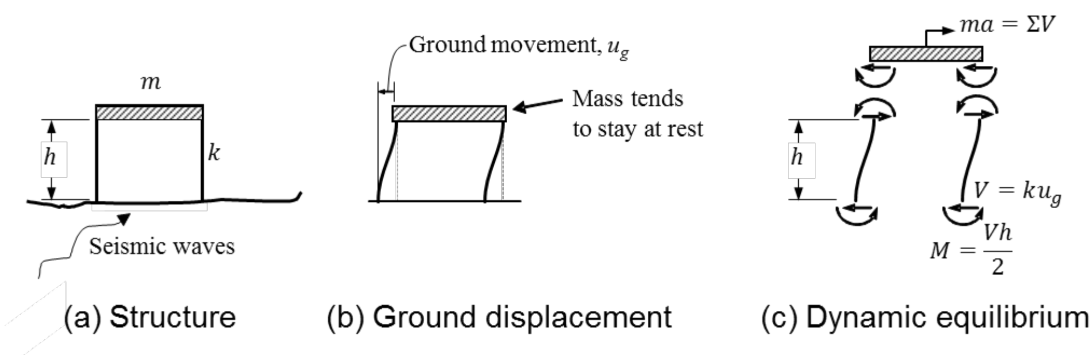



Figure 11.4 Dynamic equilibrium of a SDOF structure.

Structural analysis methods, usually employed in computer software, enable calculation of the dynamic response of a SDOF structure given an input ground motion. Figure 11.5 illustrates examples of dynamic response calculations for SDOF structures having different vibration periods, these being either $T$ = 0.5s, 1.0s, or 2.0s. The input ground motion is one recorded at El Centro, CA in 1940. For the structure having $T$ = 0.5s, the maximum absolute value of the displacement response is 2.48 in. For the structures having $T$ = 1.0s and 2.0s, the maximum displacements (6.61 in. and 8.84 in.) occur at different times. We could repeat similar calculations for different vibration periods. If we plotted the absolute values of the peak responses for all periods of interest, we would obtain the plot shown at the bottom of Figure 11.5. We call this a displacement *response spectrum*, that is, it is a plot of the maximum absolute values of the relative displacement $u$ as a function of the vibration period $T$ of the structure. Building codes commonly substitute the variable $S_d$ for $u$, where $S_d$ = maximum displacement of the oscillator relative to the ground.

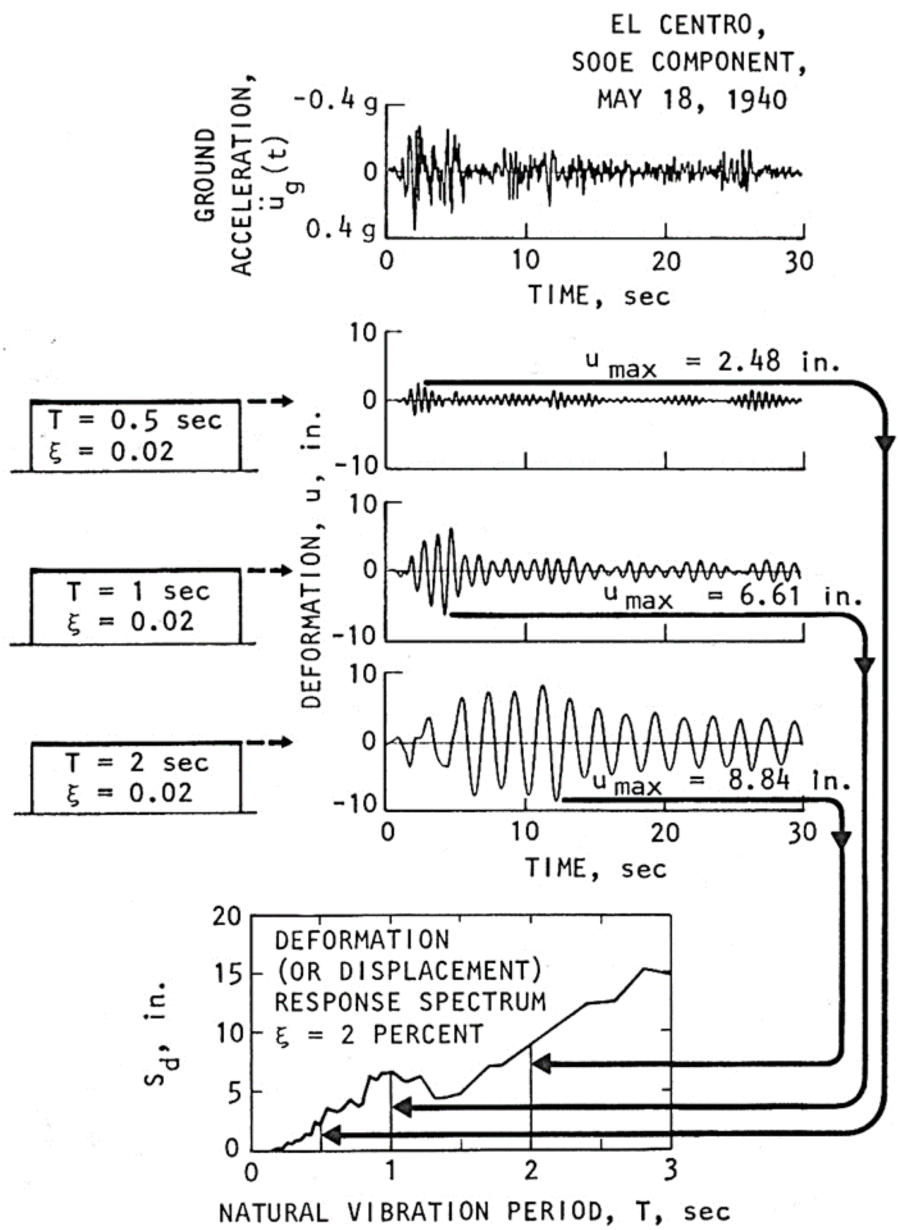



Figure 11.5 Construction of displacement response spectrum, for viscous damping equal to 2% of critical damping. (After Chopra, 1980)

Similarly, we could plot maximum velocity and maximum acceleration response spectra. As an alternative, it is common to define pseudo-velocity $S_v = \omega S_d$ ($\omega$ = circular frequency = $2\pi/T$) and pseudo-acceleration $S_a = \omega^2 S_d$. The pseudo-velocity and pseudo-acceleration values are approximately the same as the absolute velocity and absolute acceleration values. Figure 11.6 presents examples of $S_a$, $S_v$, and $S_d$ response spectra for an oscillator having damping equal to 5% of the critical value subjected to the three components of an earthquake ground motion record. From these response spectra, we can determine peak values of response to this earthquake record. For example, for a vibration period $T$ = 1s, the maximum response in the S48W direction is relative displacement of $S_d$ = 1.8 in. (45 cm), pseudo-acceleration of $S_a$ = 1.82g. Knowing the pseudo-acceleration, we can write the base shear as $V = ma = S_a\ W/g$, where $W$ is weight and $g$ is gravity acceleration.

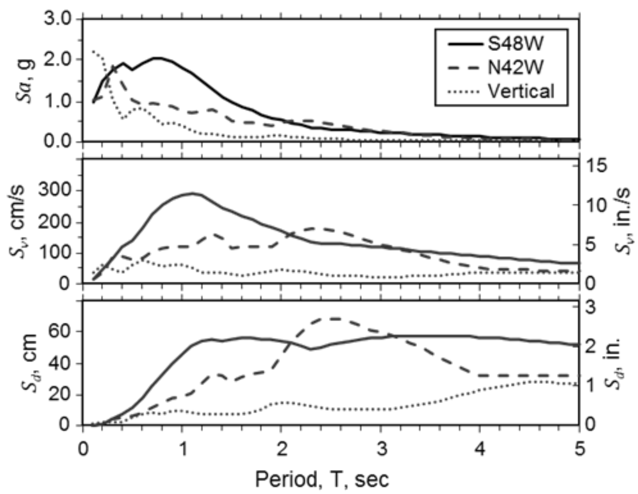



Figure 11.6 Linear elastic pseudo-acceleration ($S_a$), pseudo-velocity ($S_v$), and relative displacement ($S_d$) response spectra for 5% damping for the ground motion recorded at the Rinaldi Receiving Station during the 1994 Northridge, CA earthquake.

**11.2.3. Design Response Spectra in U.S. Building Codes**

Building codes in the United States use the USGS seismic hazard analysis resources along with site amplification factors and a standard response spectrum shape to determine response spectra for seismic design. For most of the United States, the spectral values are approximately equal to pseudo-acceleration response values having 2% probability of exceedance in 50 years. The spectral values were adjusted up or down so that the probability of collapse for an individual facility was equal to approximately 1% in 50 years. Near known active faults with significant slip rates and characteristic earthquakes with magnitudes in excess of about 6.0, the design values are limited by 1.8 times median response spectral values associated with a characteristic earthquake on the fault. The resulting spectral response values are referred to as the *Risk-Targeted Maximum Considered Earthquake*, designated $MCE_R$ level. For design, the *Design Earthquake* level is set at DE = ⅔$MCE_R$, in anticipation of structural safety margin factor of 1.5 inherent in the design procedures.

The specific procedure for establishing the design response spectra is as follows:

**Step 1**: Use the USGS resources (http://earthquake.usgs.gov/designmaps) to determine the 5%-damped spectral response pseudo-accelerations at short-period, $S_S$, and 1-s period, $S_1$, at the Risk-Targeted Maximum Considered Earthquake ($MCE_R$) shaking level.

**Step 2**: Adjust the values for effects of geotechnical site class, as $S_{MS} = F_a S_S$ and $S_{M1} = F_v S_1$. For this purpose, first determine the geotechnical site class in accordance with Table 11.1. (A geotechnical engineer usually provides this information.) Then use Table 11.2 and Table 11.3 to obtain site coefficients $F_a$ and $F_v$.

**Step 3**: Adjust the values to the design level using Eqs. (11.1) and (11.2).

$$S_{DS} = \frac{2}{3} S_{MS} = \frac{2}{3} F_a S_S \tag{11.1}$$

$$S_{D1} = \frac{2}{3} S_{M1} = \frac{2}{3} F_v S_1 \tag{11.2}$$

**Step 4**: Given values of $S_{DS}$ and $S_{D1}$, the design response spectrum is defined using the standard spectrum shape shown in Figure 11.7.

Note that the USGS site has a tool that will calculate all the quantities in these four steps.

**Table 11.1** Geotechnical site class definitions (after ASCE 7, 2010).

| Site Class | Soil shear wave velocity, $\bar{v}_s$, ft/s (m/s) | Standard penetration resistance, $\overline{N}$ | Soil undrained shear strength, $\bar{s}_u$, psf (Pa) |
|---|---|---|---|
| A. Hard rock | > 5000 (1520) | N/A | N/A |
| B. Rock | 2500 (760) to 5000 (1520) | N/A | N/A |
| C. Very dense soil and soft rock | 1200 (370) to 2500 (760) | > 50 | > 2000 (0.096) |
| D. Stiff soil | 600 (180) to 1200 (370) | 15 to 50 | 1000 (0.048) to 2000 (0.096) |
| E. Soft clay soil | < $\bar{v}_s$ 600 (180) | < 15 | < 1000 (0.048) |
| | Any profile with more than 10 ft (3 m) of soil having the following characteristics:<br>- Plasticity index PI > 20<br>- Moisture content $w \geq$ 40%, and<br>- Undrained shear strength $\bar{s}_u$ < 500 psf (0.024 Pa) | | |
| F. Soils requiring site response analysis | Soils vulnerable to failure or collapse under seismic loading, peats and/or highly organic clays, very high plasticity clays, or very thick soft/medium stiff clay.* | | |

*Simplified description. See ASCE 7 for complete description.

**Table 11.2** Site coefficient $F_a$ to modify $S_S$ values (after ASCE 7, 2010)

| Site Class | Mapped Maximum Considered Earthquake Spectral Response Acceleration Parameter at Short Period | | | | |
|---|---|---|---|---|---|
| | $S_S \le 0.25$ | $S_S = 0.5$ | $S_S = 0.75$ | $S_S = 1.0$ | $S_S \ge 1.25$ |
| A | 0.8 | 0.8 | 0.8 | 0.8 | 0.8 |
| B | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| C | 1.2 | 1.2 | 1.1 | 1.0 | 1.0 |
| D | 1.6 | 1.4 | 1.2 | 1.1 | 1.0 |
| E | 2.5 | 1.7 | 1.2 | 0.9 | 0.9 |
| F | Site-specific analysis required. | | | | |

Note: Use straight-line interpolation for intermediate values of $S_S$.

**Table 11.3** Site coefficient $F_v$ to modify $S_1$ values (after ASCE 7, 2010)

| Site Class | Mapped Maximum Considered Earthquake Spectral Response Acceleration Parameter at Short Period | | | | |
|---|---|---|---|---|---|
| | $S_1 \le 0.1$ | $S_1 = 0.2$ | $S_1 = 0.3$ | $S_1 = 0.4$ | $S_1 \ge 0.5$ |
| A | 0.8 | 0.8 | 0.8 | 0.8 | 0.8 |
| B | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| C | 1.7 | 1.6 | 1.5 | 1.4 | 1.3 |
| D | 2.4 | 2.0 | 1.8 | 1.6 | 1.5 |
| E | 3.5 | 3.2 | 2.8 | 2.4 | 2.4 |
| F | Site-specific analysis required. | | | | |

Note: Use straight-line interpolation for intermediate values of $S_S$.

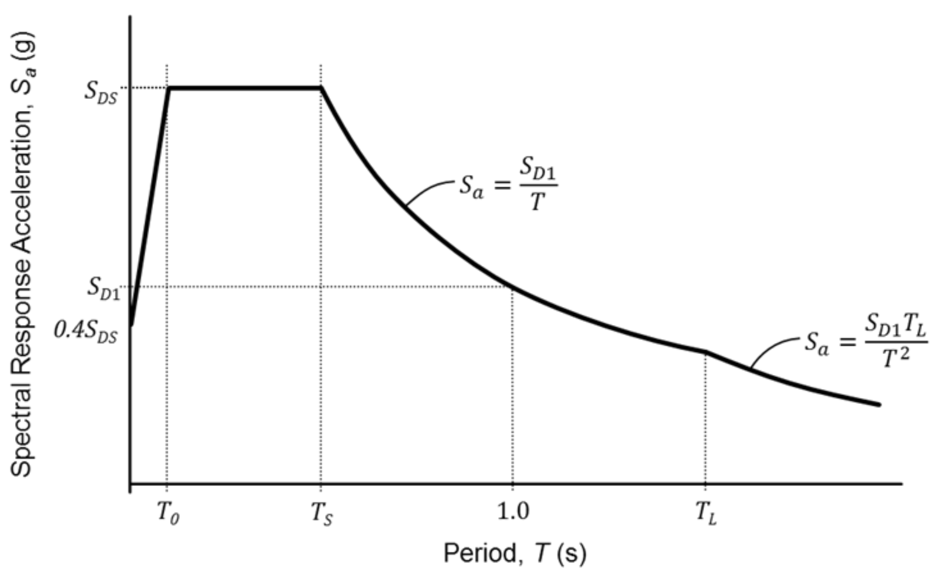



Figure 11.7 Design response spectrum (after ASCE 7).

In this figure,

$T$ = fundamental period of the structure, in seconds (s)

$$T_S = \frac{S_{D1}}{S_{DS}}$$

$T_0 = 0.2T_S$

$T_L$ = long-period transition period. Values for $T_L$ are 4s or greater, which is beyond the range of interest for this text. See ASCE 7 for details.

**Example 1**: Construct the pseudo-acceleration ($S_a$) design response spectrum and the corresponding relative displacement ($S_d$) response spectrum for a site at site with latitude 34.045 and longitude -118.264 in the City of Los Angeles. The site is determined to be site class C.

**Solution**: Using the USGS resources, $S_S$ = 2.320g, $S_1$ = 0.815g, and $T_L$ = 8s. From Table 11.2 and Table 11.3, $F_a$ = 1.0 and $F_v$ = 1.3. From Eqs. (11.1) and (11.2), $S_{DS}$ = 1.547g and $S_{D1}$ = 0.707g. Using the standard spectrum shape in Figure 11.7, the pseudo-acceleration design response spectrum is as shown in Figure 11.8(a). The corresponding displacement design response spectrum (Figure 11.8b) is derived from the pseudo-acceleration design response spectrum using the relation $S_d = S_a / \omega^2$.

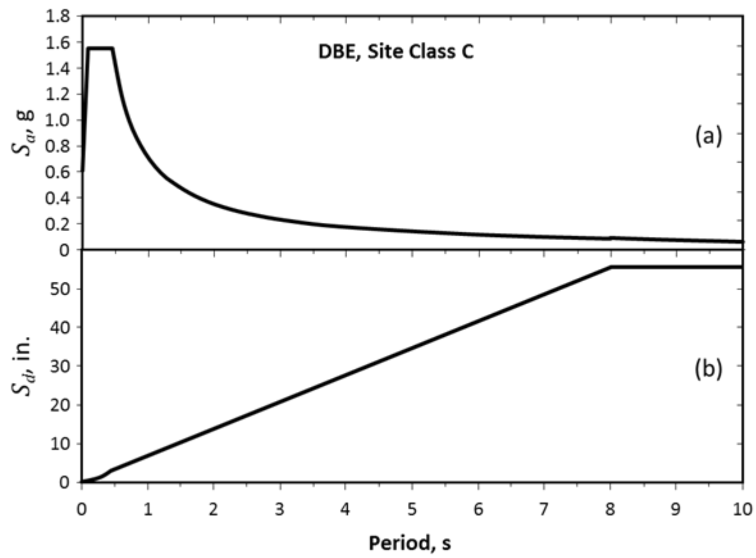



Figure 11.8 Pseudo-acceleration and displacement response spectra for the Design Earthquake (DE), site class C, for a site in the City of Los Angeles.

### 11.2.4. Distribution of Response Over Building Height

Vibration of a building results in acceleration of the building mass, which produces inertial forces that are the product of acceleration and mass. In practical building analysis, it is common to lump the building mass (or weight) at the floor levels, where most of the mass (weight) is actually located. For example, in Figure 11.9(a), the roof mass (weight) can be approximated as the mass of the floor itself plus the mass of the vertical elements including cladding over half the story height below the roof level. For the third level, the mass would be the mass of the floor itself plus the mass of the vertical elements including cladding over half the story height above and below that level. If we can approximate the individual floor level accelerations as the building vibrations back and forth (Figure 11.9c), then the design forces can be obtained as the product of mass and acceleration, that is, $F_i = a_i m_i = a_i w_i / g$, as shown in Figure 11.9(d). As we shall see later in this chapter, building codes have expressions to approximate these force distributions.

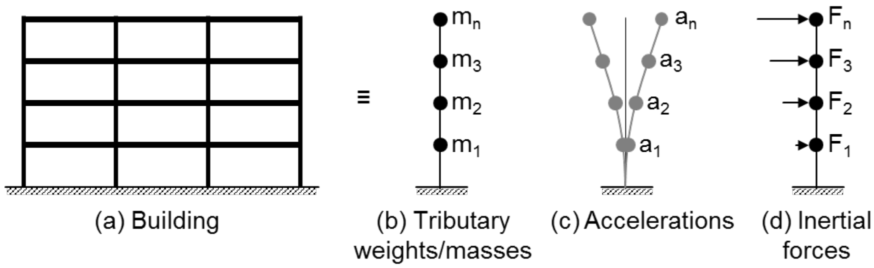



Figure 11.9 Distribution of lateral forces over building height.

## 11.3. EARTHQUAKE DEMANDS ON BUILDING STRUCTURES

### 11.3.1. Linear-Elastic Response of Structures

Earthquake demands on buildings will vary from earthquake to earthquake, and maximum expected demands will vary from region to region. For buildings located in regions of high seismicity, the maximum expected earthquake shaking levels may produce lateral displacements of several inches relative to the ground, with lateral forces for linear-elastic systems approaching or even exceeding the weight of the building. Except for very special structures, it will not be economically feasible to design buildings with conventional structural systems to respond linearly to such strong shaking. Some nonlinear response may have to be accepted.

We can demonstrate this for sites in the highly seismic western United States using the Design Earthquake (DE) response spectrum presented in Example 1. Adopting the rule of thumb that vibration period of a building is $T \approx N/10$, where $N$ = number of stories, we can estimate that a five-story building has a vibration period around 0.5s. From Figure 11.8, for $T$ = 0.5s, $S_a$ = 1.4g and $S_d$ = 3.5 in. (89 mm). The corresponding base shear can be estimated to be $V_b = S_a W/g$ = 1.4$W$.

While it is possible to design a structure to remain linear-elastic under the forces determined above, doing so would require considerable expense. It might also require use of a massive structural system that would interfere with the functional purpose of the building. Neither the expense nor the functional disruption can be justified in most building projects, especially considering the rarity of design-level earthquake shaking and the limited resources available for building construction. Therefore, ASCE 7 allows for inelastic response of a building during strong shaking. The next section provides a brief overview of inelastic response to earthquake ground motions.

### 11.3.2. Inelastic Response of Structures

Consider the single-degree-of-freedom oscillator shown in Figure 11.10(a). The mass $M$ (= $W/g$) is set equal to 1 kip-s$^2$/in. and the stiffness $K$ is set equal to 39.5 kip/in., resulting in linear period

$T = 1$s. Damping is modeled as viscous with damping ratio equal to 5% of the critical value. The force-displacement relation (Figure 11.10b) has stiffness-degrading behavior that approximates behavior of reinforced concrete construction. By setting the yield force $V_y$ sufficiently high, the response will be linear-elastic. For strength lower than the linear-elastic force demand, behavior follows the relation shown in Figure 11.10(b).

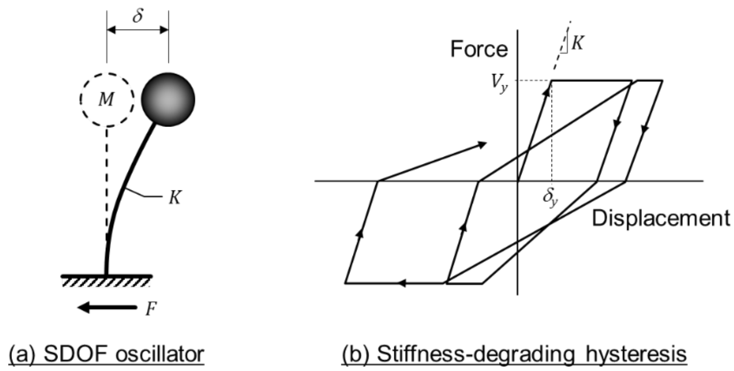



Figure 11.10 (a) Single-degree-of-freedom (SDOF) oscillator, (b) force-displacement response.

Response of the oscillator is calculated using the software BISPEC (2009). Figure 11.11 plots the displacement response history for linear response and for moderately nonlinear response. For linear response (Figure 11.11a), the maximum displacement is 17.9 in. (455 mm), consistent with the spectral displacement $S_d$ that can be read from the linear response spectrum at $T = 1$s (Figure 11.6). The maximum restoring force in the spring can be obtained as $V_e = K \times S_d =$ 39.5 kip/in. × 17.9 in. = 705 kips (3140 kN).

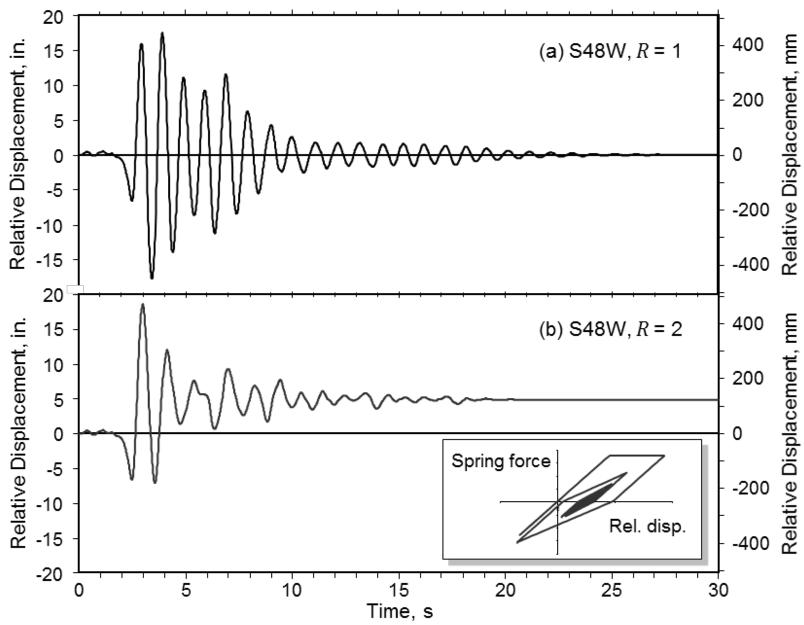



Figure 11.11 Calculated response of an oscillator having initial period $T$ = 1s and 5% damping subjected to the ground motion recorded at the Rinaldi Receiving Station during the 1994 Northridge, CA earthquake. (a) $R$ = 1 corresponds to linear-elastic response, (b) $R$ = 2 corresponds to inelastic response for the oscillator having yield strength equal to half the elastic strength demand. The hysteretic relation between spring force and relative displacement is shown in the inset.

We now introduce the response modification coefficient, $R$, defined by

$$R = V_e / V_y \tag{11.3}$$

in which $V_e$ = the force developed in the spring for linear-elastic response and $V_y$ = the yield force of the spring. Linear-elastic response requires $R$ = 1 (or less). $R$ = 2 corresponds to an oscillator having a yield base shear equal to half the elastic shear force for a linear-elastic oscillator. The response modification coefficient is used to define design strength requirements for seismic designs complying with ASCE 7, as will be discussed subsequently.

The nonlinear oscillator whose response is plotted in Figure 11.11(b) was defined to have $R$ = 2, that is, yield strength equal to 705 kips / 2 = 353 kips. The effects of yielding are apparent in two characteristics of the response history. First, the apparent vibration period is elongated relative to the initial period of $T$ = 1s; this is because yielding results in effective stiffness degradation in the load-displacement relation. Second, nonlinear response is apparent in the permanent offset of 4.9 in. (120 mm).

An important observation from Figure 11.11 is that the peak displacements for linear response (17.9 in.) and nonlinear response with $R$ = 2 (18.6 in.) are nearly equal. If we were to further investigate this observation for $R$ = 3, 4, and 6 we would find peak displacements of 21.0 in., 19.5 in., and 16.4 in., respectively. Apparently, for this structure and this ground motion record, the peak displacement is relatively insensitive to the strength within this range of strengths.

The observation that the peak displacement for nonlinear response is approximately equal to the peak displacement for linear response is known as the *equal displacement rule*. We should note that it is not a mathematically derived result, but instead is an empirical observation that holds on average for many structures and many ground motions.

## 11.4. ASCE 7 EQUIVALENT LATERAL FORCE PROCEDURE FOR DETERMINING EARTHQUAKE DESIGN FORCES

ASCE 7 provides several different methods for establishing earthquake design forces for buildings. Here we emphasize the *Equivalent Lateral Force* procedure, illustrated using a series of steps that can generally be followed when using the procedure.

**Steps 1 through 4**: See Section 11.2.3.

**Step 5**: Determine *Importance Factor* and *Risk Category* (ASCE 7 Section 11.5).

a. Risk category is defined in ASCE 7 Table 1.5-2. In this text, see Table 11.4. Risk Category II is typical.

b. Read Seismic Importance Factor from ASCE 7 Table 1.5-2. In this text, see Table 11.4. I = 1.0 is typical

**Step 6**: Determine *Seismic Design Category* (ASCE 7 Section 11.6). In this text, see Table 11.5 and Table 11.6.

**Step 7**: Determine the values of $R$, $C_d$, and $\Omega_o$ based on the vertical elements of the seismic force resisting system. These are in ASCE 7 Table 12.2-1, which is partially reproduced as Table 11.7 in this text. Factor $R$, previously defined by Eq. (11.3), reflects the inherent capability of the framing system to respond in the inelastic range. Factor $C_d$ is used to adjust the calculated displacement to a value similar to that obtained by the equal displacement rule (see Section 11.3.2). Factor $\Omega_o$ is a system overstrength factor that is used to estimate how large some critical design forces can be if the structure yields during the earthquake and develops its actual strength.

(Sections 12.4.2 through 12.6 of ASCE 7 cover a range of subjects including vertical seismic loading, factors for low redundancy, overstrength effects, and effects of loading in two orthogonal directions. In CE 120, we will not consider these effects.)

**Step 8**: Determine the effective seismic weight, $W$, of the building. The effective seismic weight includes the dead load; a minimum of 25 percent of the floor live in areas used for storage; weight of partitions; total operating weight of permanent equipment; snow loads; and weight of landscaping and other materials at roof gardens and similar areas. The weights associated with each floor level are defined to include the tributary weight of slabs, beams, columns, walls, cladding, etc. For this purpose, the tributary weight of elements spanning between floors is taken as the weight within half story height above and below the floor under consideration.

**Step 9**: Calculate the fundamental period of the structure, $T$, in the direction under consideration. As an alternative to performing an analysis to determine the fundamental period, $T$, it is permitted to use the approximate building period, $T_a$, calculated in accordance with Eq. .

$$T_a = C_t h_n^x \tag{11.4}$$

where

$h_n$ is the structural height defined as the vertical distance from the base to the highest level of the seismic force-resisting system of the structure. For pitched or sloped roofs, the structural height is from the base to the average height of the roof.
$C_t$ and $x$ are determined from Table 11.8. ASCE 7 contains some alternative equations not presented here.

**Step 10**: Calculate the seismic base shear $V$ in accordance with the following:

$$V = C_s W \tag{11.5}$$

where

$C_s$ = the seismic response coefficient, defined below
$W$ = the effective seismic weight

The seismic response coefficient is defined by the following five expressions:

$$C_s = \frac{S_{DS}}{\left(\frac{R}{I_e}\right)} \tag{11.6}$$

where

$S_{DS}$ = the design spectral response acceleration parameter in the short period range as determined in Step 3.
$R$ = the response modification factor from Table 11.7
$I_e$ = the importance factor from Table 11.4.

The value of $C_s$ computed in accordance with Eq. (11.6) need not exceed the following:

$$C_s = \frac{S_{D1}}{T\left(\frac{R}{I_e}\right)} \text{ for } T \leq T_L \tag{11.7}$$

$$C_s = \frac{S_{D1} T_L}{T^2\left(\frac{R}{I_e}\right)} \text{ for } T > T_L \tag{11.8}$$

Furthermore, $C_s$ shall not be less than

$$C_s = 0.044 S_{DS} I_e \geq 0.01 \tag{11.9}$$

In addition, for structures located where $S_1 \geq 0.6$g, $C_s$ shall not be less than

$$C_s = 0.5S_1/(R/I_e) \tag{11.10}$$

**Step 11**: Determine the vertical distribution of seismic forces. The lateral seismic force ($F_x$) (kip or kN) induced at any level shall be determined from the following equations:

$$F_x = C_{vx}V \tag{11.11}$$

$$C_{vx} = \frac{w_x h_x^k}{\sum_{i=1}^{n} w_i h_i^k} \tag{11.12}$$

where

$C_{vx}$ = vertical distribution factor
$V$ = total design lateral force or shear at the base of the structure (kip or kN)
$w_i$ and $w_x$ = the portion of the total effective seismic weight of the structure ($W$) located or assigned to Level $i$ or $x$
$h_i$ and $h_x$ = the height (ft or m) from the base to Level $i$ or $x$
$k$ = an exponent related to the structure period as follows:
For structures having $T \leq 0.5$ s, $k = 1$
For structures having $T \geq 2.5$ s, $k = 2$
For other structures, $k$ shall be 2 or shall be determined by linear interpolation between 1 and 2.

**Step 12**: Determine the horizontal distribution of forces, if required. In general, the force at every level is distributed in proportion with the mass distribution of that level. ASCE 7 also contains provisions for determination of inherent and accidental torsion, but we will not consider torsion in this text.

**Step 13**: Determine the story drifts. The deflection at Level $x$ ($\delta_x$) (in. or mm) is calculated in accordance with Equation (11.13):

$$\delta_x = \frac{C_d \delta_{xe}}{I_e} \tag{11.13}$$

where

$C_d$ = the deflection amplification factor in Table 11.7
$\delta_{xe}$ = the deflection determined by an elastic analysis.
$I_e$ = the importance factor in Table 11.4.

Note that the deflection $\delta_{xe}$ is calculated for seismic forces that have been reduced by divisor ($R/I_e$) in Step 10. Thus, the deflection $\delta_x$ from Eq. (11.13) is the deflection for unreduced forces factored by $C_d/R$, which is approximately 1.0. This, therefore, brings the deflections effectively back to values expected based on the equal displacement rule.

Table 11.4 Risk Category of Buildings and Other Structures for Flood, Wind, Snow, Earthquake, and Ice Loads, and Seismic Importance Factor

| **Use or Occupancy of Buildings and Structures** | **Risk Category** | **Seismic Importance Factor, $I_e$** |
|---|---|---|
| Buildings and other structures that represent a low risk to human life in the event of failure | I | 1.0 |
| All buildings and other structures except those listed in Risk Categories I, III, and IV | II | 1.0 |
| Buildings and other structures, the failure of which could pose a substantial risk to human life.<br><br>Buildings and other structures, not included in Risk Category IV, with potential to cause a substantial economic impact and/or mass disruption of day-to-day civilian life in the event of failure.<br><br>Buildings and other structures not included in Risk Category IV (including, but not limited to, facilities that manufacture, process, handle, store, use, or dispose of such substances as hazardous fuels, hazardous chemicals, hazardous waste, or explosives) containing toxic or explosive substances where the quantity of the material exceeds a threshold quantity established by the authority having jurisdiction and is sufficient to pose a threat to the public if released.[a] | III | 1.25 |
| Buildings and other structures designated as essential facilities.<br><br>Buildings and other structures, the failure of which could pose a substantial hazard to the community.<br><br>Buildings and other structures (including, but not limited to, facilities that manufacture, process, handle, store, use, or dispose of such substances as hazardous fuels, hazardous chemicals, or hazardous waste) containing sufficient quantities of highly toxic substances where the quantity of the material exceeds a threshold quantity established by the authority having jurisdiction and is sufficient to pose a threat to the public if released.[a]<br><br>Buildings and other structures required to maintain the functionality of other Risk Category IV structures. | IV | 1.5 |

[a] Buildings and other structures containing toxic, highly toxic, or explosive substances shall be eligible for classification to a lower Risk Category if it can be demonstrated to the satisfaction of the authority having jurisdiction by a hazard assessment as described in Section 1.5.3 that a release of the substances is commensurate with the risk associated with that Risk Category.

**Table 11.5** Seismic Design Category based on short-period response acceleration parameter, $S_{DS}$ (after ASCE 7)

| Value of $S_{DS}$ | Risk Category I or II | III | IV |
|---|---|---|---|
| $S_{DS} < 0.167$ | A | A | A |
| $0.167 \le S_{DS} < 0.33$ | B | B | C |
| $0.33 \le S_{DS} < 0.50$ | C | C | D |
| $0.50 \le S_{DS}$ | D | D | D |

**Table 11.6** Seismic Design Category based on 1 s period response acceleration parameter, $S_{D1}$ (after ASCE 7)[a]

| Value of $S_{D1}$ | Risk Category I or II | III | IV |
|---|---|---|---|
| $S_{D1} < 0.067$ | A | A | A |
| $0.067 \le S_{D1} < 0.133$ | B | B | C |
| $0.133 \le S_{D1} < 0.20$ | C | C | D |
| $0.20 \le S_{D1}$ | D | D | D |

[a] Risk Category I, II, or III structures located where $S_1 \ge 0.75$ shall be assigned to Seismic Design Category E. Additional requirements apply for Risk Category IV.

**Table 11.7** Design coefficients and factors for seismic force-resisting systems (after ASCE 7)[a]

| Seismic Force-Resisting System | Response Modification Coefficient, $R$ | System Overstrength Factor, $\Omega_0$ | Deflection Amplification Factor, $C_d$ | Structural System Limitations and Building Height (ft) Limit[b] Seismic Design Category B | C | D | E | F |
|---|---|---|---|---|---|---|---|---|
| **A. BEARING WALL SYSTEMS** | | | | | | | | |
| Special reinforced concrete shear walls | 5 | 2.5 | 5 | NL | NL | 160 | 160 | 100 |
| Light-frame (wood) walls sheathed with wood structural panels rated for shear resistance | 6½ | 3 | 4 | NL | NL | 65 | 65 | 65 |
| | | | | | | | | |
| **B. BUILDING FRAME SYSTEMS** | | | | | | | | |
| Steel special concentrically braced frames | 6 | 2 | 5 | NL | NL | 160 | 160 | 160 |
| Special reinforced concrete shear walls | 6 | 2.5 | 5 | NL | NL | 160 | 160 | 100 |
| Steel buckling-restrained braced frames | 8 | 2.5 | 5 | NL | NL | 160 | 160 | 100 |
| | | | | | | | | |
| **C. MOMENT-RESISTING FRAME SYSTEMS** | | | | | | | | |
| Steel special moment frames | 8 | 3 | 5.5 | NL | NL | NL | NL | NL |
| Special reinforced concrete moment frames | 8 | 3 | 5.5 | NL | NL | NL | NL | NL |
| | | | | | | | | |
| **D. DUAL SYSTEMS WITH SPECIAL MOMENT FRAMES CAPABLE OF RESISTING AT LEAST 25% OF PRESCRIBED SEISMIC FORCES** | | | | | | | | |
| Steel special concentrically braced frames | 7 | 2.5 | 5.5 | NL | NL | NL | NL | NL |
| Special reinforced concrete shear walls | 7 | 2.5 | 5.5 | NL | NL | NL | NL | NL |
| | | | | | | | | |

[a]Partial presentation. See ASCE-7 for complete description.

[b]NL = Not Limited; NP = Not Permitted. 40 ft = 12.2 m, 100 ft = 30.5m, 160 ft = 48.4 m. Heights are measured from the base of the structure.

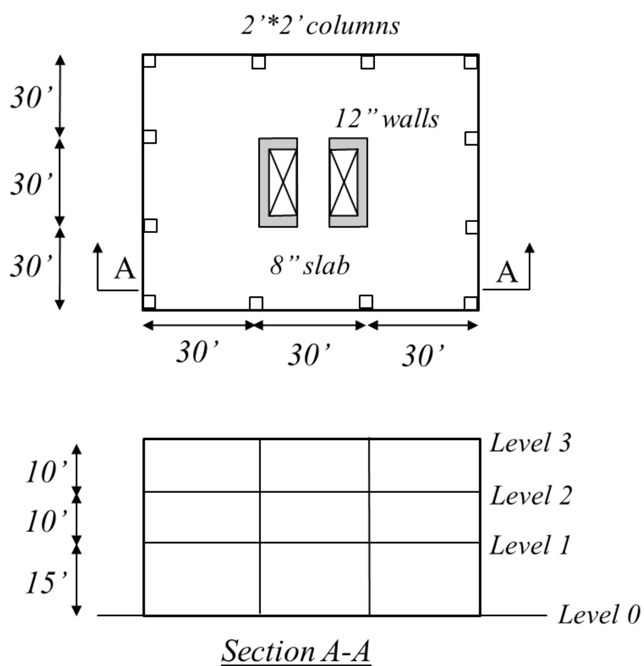
**Table 11.8** Values of approximate period parameters $C_t$ and $x$

| Structure Type | $C_t$ | $x$ |
|---|---|---|
| Steel moment-resisting frames | 0.028 (0.0724)[a] | 0.8 |
| Concrete moment-resisting frames | 0.016 (0.0466)[a] | 0.9 |
| Steel eccentrically braced frames | 0.03 (0.0731)[a] | 0.75 |
| Steel buckling-restrained frames | 0.03 (0.0731)[a] | 0.75 |
| All other structural systems | 0.02 (0.0488)[a] | 0.75 |

[a]Metric equivalents are shown in parentheses.

**Example 2**: A 3-story tall concrete office building is located in Los Angeles, CA (Lat 34.045, Long -118.264). The site is site class C. The structural system comprises a special reinforced concrete bearing wall as a core wall surrounded by gravity framing. The structural system and various design loads are summarized in the sketch below. Find the design base shear, the distribution of lateral forces over height, and the wall shears and moments. You may treat the core wall as a single solid wall for this example.

**Solution**: The sketch below shows the structural system and design loads. Steps 1 through 4 are taken from Example 1. The remainder of the solution is in Steps 5-11.




**Loads:**

Dead: Slab (8/12)*(150) = 100 psf
Additional per floor = 16 psf
Cladding = 15 psf
Columns: 2*2*150 = 600 plf
Walls: 1'*120'*150 = 18 klf

Live: Offices = 50 psf
Partitions = 15 psf

Steps 1-4: Example 1
Step 5: Risk Category II ; $I_e = 1$
Step 6: Seismic Design Category (SDC)
$S_{DS} = 1.547g$, $S_{D1} = 0.707g$ → From tables 11.5 & 11.6, SDC = D.
However, $S_1 = 0.815g$ (see Example 1), → see footnote of Table 11.6 → SDC = E

**Figure 11.12 Example 2**

Step 7: Given SDC D, this wall will be required to be a special reinforced concrete wall.
$R = 5$, $\Omega o = 2{,}5$ , $Cd = 5$

partitions

Step 8: Determine W

| | | |
|---|---|---|
| Level 3: | Distributed: $(100 + 16 + 15)(90)^2$ | = 1060 kips |
| | Tributary ; [(15)(4*90')+(12)(600)+18000]5' | = 153 kips |
| Level 2: | $(131)(90)^2 + (30.6)(10')$ | = 1370 kips |
| Level 1: | $(131)(90)^2 + (30.6)(12.5')$ | = 1440 kips |

<u>Total W = 4020 kips</u>

Step 9: $T_n = C_t h_n{}^x = (0.02)(35)^{0.75} = 0.29s$

Step 10: $V = C_s W$

$$C_s = \frac{S_{DS}}{K/Ie} = 0.31 \;\le\; \frac{S_{D1}}{T(\frac{R}{Ie})} = 0.49$$
$$\ge 0.044\, S_{DS}\, Ie = 0.068$$
$$\ge 0.01$$
$$\ge 0.5\, \frac{S_1}{R/Ie} = 0.082$$

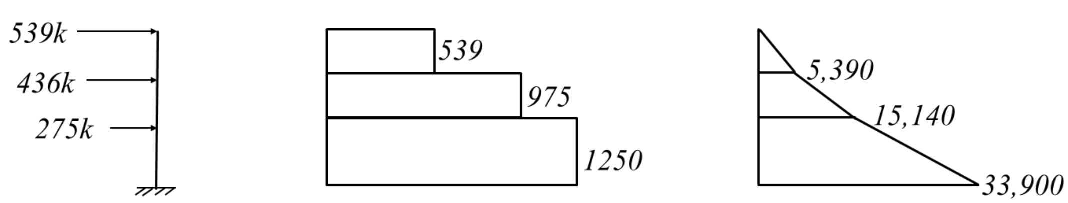
so $V = C_s W = 0.31 * 4020\text{kips} = 1250$ kips

Step 11:

| Level | $h_x$ | $h_x{}^k$ | $W_x$ | $h_x{}^k W_x$ | $C_{vx}$ | $F_x = C_{vx}V$ |
|---|---|---|---|---|---|---|
| 3 | 35' | 35 | 1210 kips | 42400 | 0.431 | 539 |
| 2 | 25' | 25 | 1370 kips | 34300 | 0.349 | 436 |
| 1 | 15' | 15 | 1440 kips | 21600 | 0.220 | 275 |
| | | $\Sigma$ | 4020 | 98300 | 1 | 1250k |

<u>Vertical Distribution of Seismic Forces, Shears, Moments</u>

*Assume all lateral force is resisted by the core wall*




*Lateral Force* *Shear, kips* *Moment, k-ft*

Figure 11.12 Example 2 (continued)

## 11.5. TRANSFER OF SEISMIC FORCES THROUGH A BUILDING

Seismic forces are generally assumed to act only at the floor levels because the mass of the floors usually comprises the majority of the building mass. The forces are generated by the mass of the floor system undergoing accelerations. Thus, the forces should be distributed horizontally across the floor in proportion with the distribution of seismic mass. Small irregularities caused by minor openings or by distribution of cladding, columns, and walls, are commonly ignored. Figure 11.13 illustrates a basic structural framing system and illustrates how inertial forces are distributed across the floor for loading toward the north. Once the lateral force distribution is identified, the transfer of forces through the building follows the general procedures presented in Chapter 10 for wind loading.

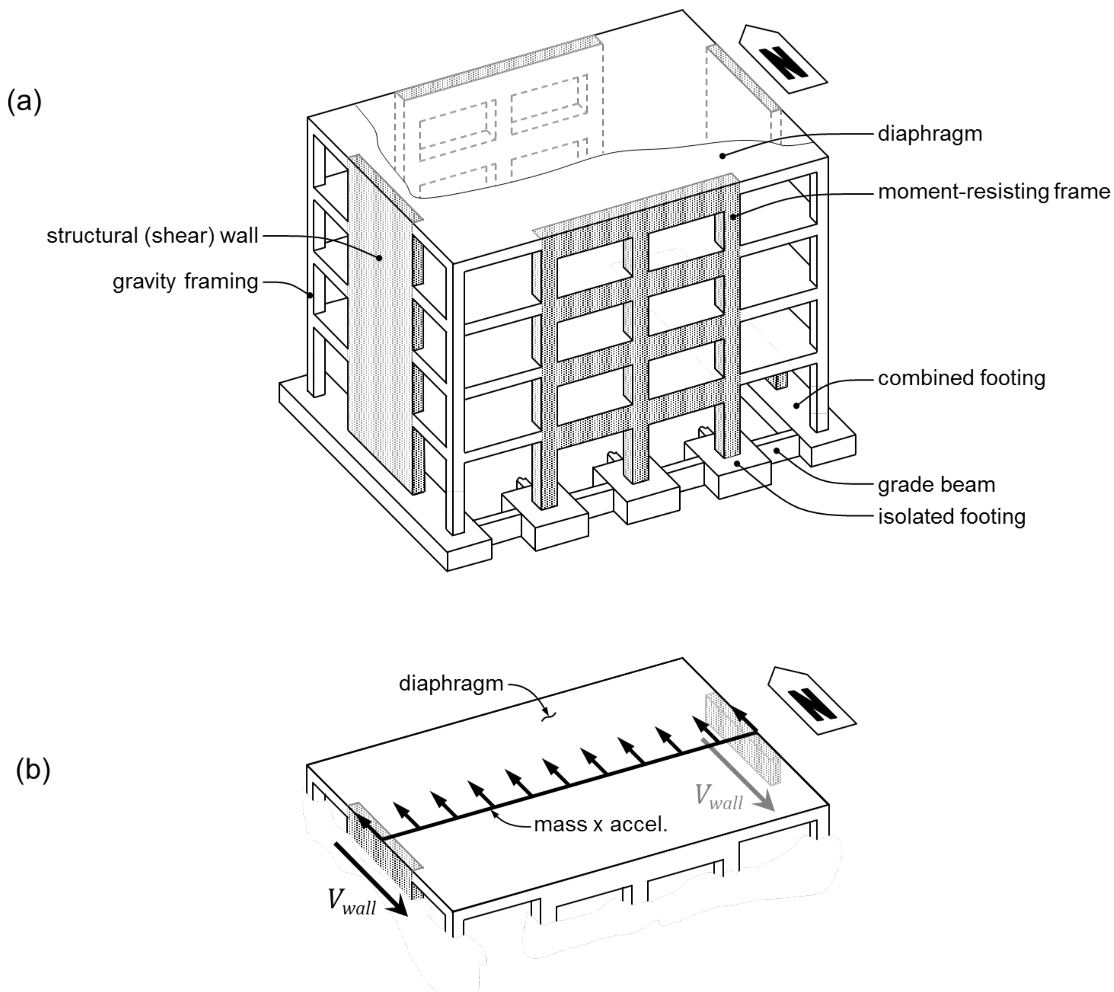



Figure 11.13 (a) Basic structural framing and (b) transfer of inertial loads in a diaphragm.

## 11.6. DISTRIBUTION OF DIAPHRAGM FORCES – RIGID VERSUS FLEXIBLE DIAPHRAGMS

Concepts for rigid and flexible diaphragms in earthquake-resisting buildings are the same as those for wind-resisting buildings. See discussion in Chapter 10.

## 11.7. LATERAL-FORCE-RESISTING SYSTEMS

As with wind-resisting buildings, the lateral-force-resisting systems for earthquake resistance are considered to comprise vertical elements (such as shear walls) and horizontal elements (diaphragms). The discussion of these elements for wind design in Chapter 10 is largely applicable for earthquake resistance. Please refer to Chapter 10 for more information.

One important distinction between main-wind-force-resisting systems and seismic-force-resisting systems lies in the performance expectations and, consequently, the required detailing of the structural system. Buildings are expected to resist wind forces in the effectively linear-elastic range of response, without yielding of the structural system. In contrast, as discussed in Section 11.2.4, buildings resisting strong earthquake shaking may need to be capable of responding well beyond the linear range of response. Consequently, the structure needs to be configured, detailed, and constructed so that it can perform adequately even though it is yielding during the design earthquake.

The International Building Code, ASCE 7, and the materials codes impose restrictions on the types of structural systems that can be used for earthquake resistance. Those codes also specify strict requirements for proportioning, detailing, and inspecting the construction of these buildings so that they can perform properly. Table 11.7 identifies some of the types of structural systems that are permitted in regions of highest seismicity, including the West Coast of the United States. The different structural systems are defined as:

- A *structural wall* is a wall that is designed to resist lateral (and perhaps vertical) loads within the plane of the wall. A structural wall is also sometimes referred to as a *shear wall*. A *special structural wall* is one that is proportioned and detailed to enable it to meet the performance requirements in regions of highest seismicity. Structural walls (shear walls) can be either of reinforced concrete (Figure 11.14) or light-frame (wood) sheathed with plywood (Figure 11.15).
- A *moment-resisting frame* is a vertical element in which beams, columns, and beam-column joints are connected to compose a rigid framework that resists lateral and vertical forces through moment, shear, and axial forces in the members (Figure 11.16). A *special moment frame* is one that is proportioned and detailed to enable it to meet the performance requirements in regions of highest seismicity.
- A *concentrically braced frame* is a vertical truss in which beams, columns, and diagonal bracing members are arranged such that their axes intersect at joints (Figure 11.17). (An *eccentric braced frame* is one in which the diagonals are intentionally offset from the joints.) Braced frames are almost exclusively made of steel members. A *steel special concentrically braced frame* is one that is proportioned and detailed to enable it to meet the performance requirements in regions of highest seismicity. Diagonals of steel braced frames have a tendency to buckle under compressive loading. A *steel buckling-restrained braced frame* is one that has specially detailed diagonals that are restrained from buckling (Figure 11.18).

- A *bearing wall system* is a structural system with structural walls providing support for all or major portions of the vertical loads, in addition to providing earthquake force resistance. Bearing walls can be either of (a) special reinforced concrete shear walls or (b) Light-frame (wood) walls sheathed with wood structural panels (plywood) rated for shear resistance.
- A *building frame system* is a structural system with an essentially complete space frame providing support for vertical loads. Seismic force resistance is provided by shear walls or braced frames.
- A *dual system* is a combination of moment-resisting frames and either structural walls or braced frames, proportioned to resist the design earthquake loads in proportion with their respective rigidities, except the moment-resisting frames must be capable of resisting at least 25% of the prescribed earthquake forces.

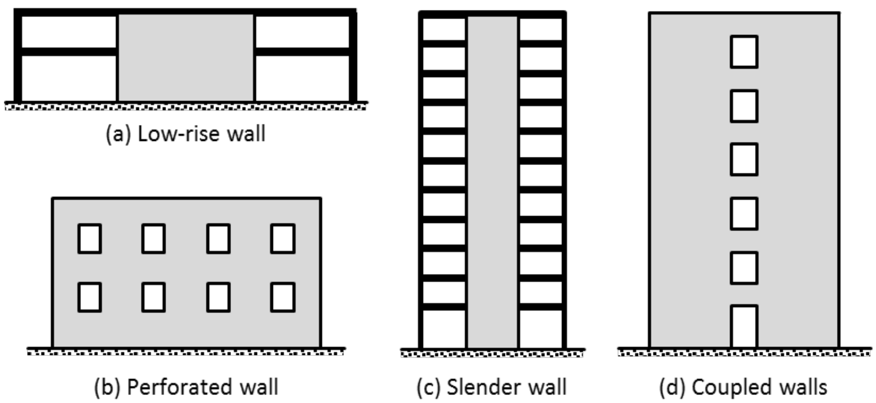



Figure 11.14 Elevations of various types of reinforced concrete structural walls. (Moehle, McGraw-Hill, 2014)

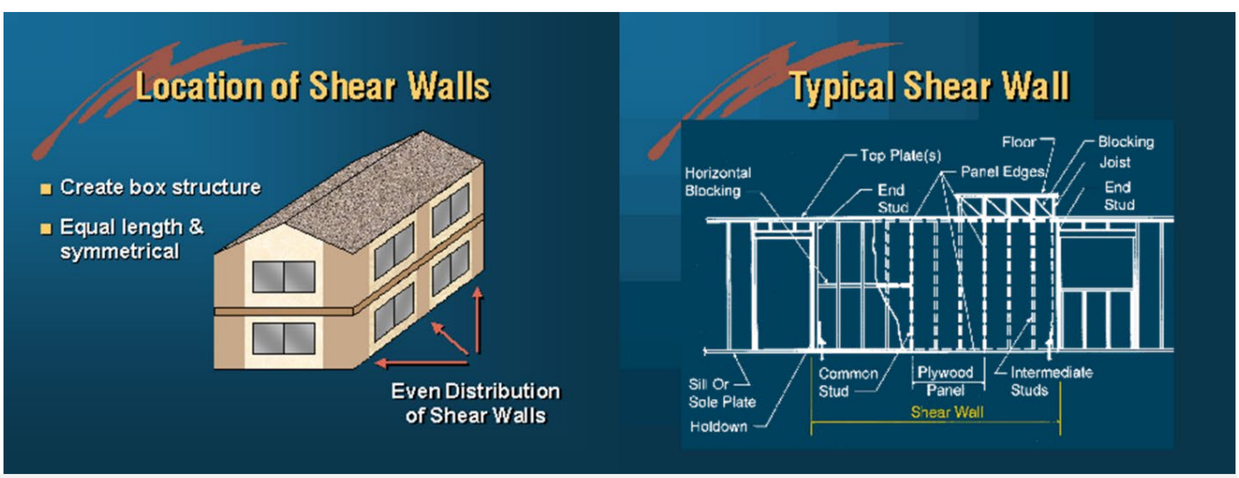



Figure 11.15 Locations and details of wood shear walls (ABAG)


Figure 11.16 Reinforced concrete moment-resisting frame under construction (Cary Kopczynski & Company, Seattle).

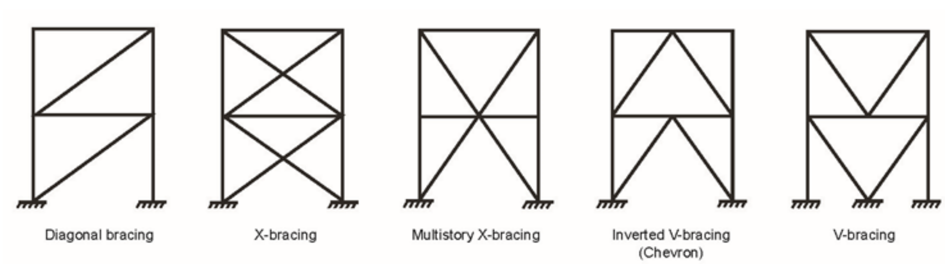



Figure 11.17 Elevations of various types of steel concentrically braced frames. (NIST GCR 13-917-24)

Figure 11.18 Buckling-restrained braced frame in Stanley Hall, UC Berkeley.

In a typical building the majority of the mass is in the floor system. An important consideration in laying out the seismic force-resisting system is to locate the vertical elements such that the center of resistance is close to the center of mass, thereby reducing plan torsional effects. The fundamental problem is illustrated in Figure 11.19, in which severe eccentricity between the centers of mass and resistance results in torsion that needs to be considered in design. In a building with rectangular plan, an ideal location for the vertical elements of the seismic force-resisting system is around the building perimeter, such that the centers of mass and resistance coincide and high torsional resistance is provided. This location for vertical elements may not be ideal from the perspective of overturning resistance or building function.

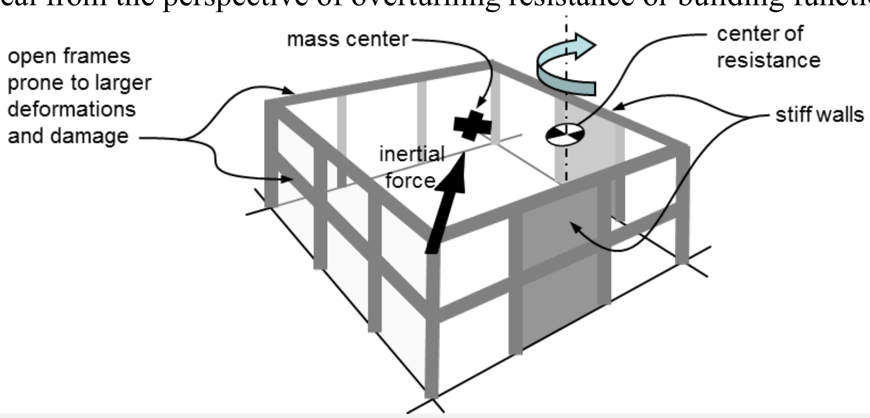



Figure 11.19 Building in which location of the vertical elements of the seismic force-resisting system results in severe torsion problem.

The vertical elements of the seismic force-resisting system are required to transmit the accumulated seismic forces to the foundation system. Generally, it is preferable for the vertical elements to be continuous over height. Figure 11.20 illustrates two examples of discontinuous vertical elements. Figure 11.20(a) shows a structural wall in the first story that terminates at the first elevated level; the sudden discontinuity in stiffness and strength can result in distress in the story immediately above the wall cutoff. In Figure 11.20(b), a wall in the upper stories is discontinuous in the first story. The resulting discontinuity in stiffness and strength can result in a weak first story that is highly vulnerable to earthquake effects. The condition is exacerbated by the overturning forces from the wall, which must be resisting in the first story by the supporting columns. Seismic building codes may prohibit the type of discontinuity shown in Figure 11.20(b) for buildings in highly seismic zones because it is known to result in poor performance.

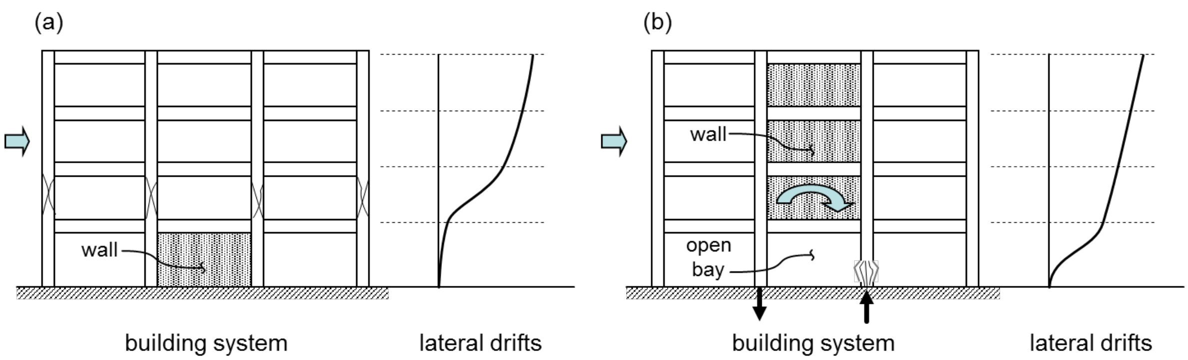



Figure 11.20 Buildings with discontinuous vertical elements of the seismic force-resisting system.

# 12. Design Methods

## 12.1. INTRODUCTION

Previous chapters have addressed design loads and analysis for reactions, internal forces, and deflections under applied loads. The next step in design is to proportion the structure such that it will meet performance expectations with an appropriate degree of reliability. In general, a structure must be designed such that is will be both serviceable under normal loads and safe under unusually severe loads. This chapter introduces design methods that are commonly used to check serviceability and safety. Subsequent chapters will illustrate use of these methods for wood, steel, and reinforced concrete structures.

## 12.2. DESIGN AND BEHAVIOR OF STRUCTURES

The design of any structure should consider both serviceability under expected loads and safety under extreme loads. Figure 12.1 illustrates the general nature of the problem.

- Under service loads, which include calculated dead loads plus the tabulated live loads from ASCE 7, the structure should perform in a serviceable manner. Specific serviceability requirements might include (a) a limit on the maximum deflection and (b) a restriction on maximum stresses so as to control cracking and crack width.
- Because of the life safety consequences of structural failure, there should be only a small probability that the load effects will exceed the ultimate ability of the structure to resist those loads over a specified life of the structure. In general, there exists some variability in the loads and in the ability of a structure to resist those loads (Figure 12.2). To ensure a small probability of failure, the expected resistance of the structure will need to exceed the expected load effects by some margin. This will be provided either through (a) a factor of safety in the design or (b) a load-and-resistance factor design method that amplifies the loads and decreases the resistance in consideration of the variability. Both methods will be described later in this chapter.
- In addition to providing a margin of safety between the expected load effects and the expected resistance, we normally design structures to provide some warning of impending failure in the event that the loads approach the capacity. For example, this might be achieved by designing the structure so that it will yield and deform in a ductile manner prior to its final failure, as illustrated for the beam in Figure 12.1(c). We may also introduce a capacity design method intended to control the mechanism by which the structure fails.

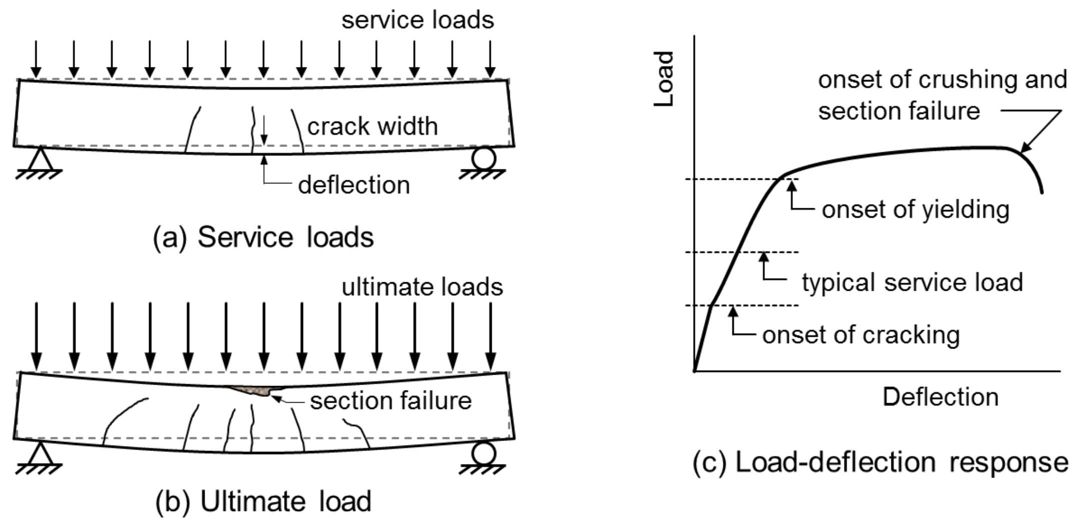



Figure 12.1 Load-deflection response of a reinforced concrete beam.

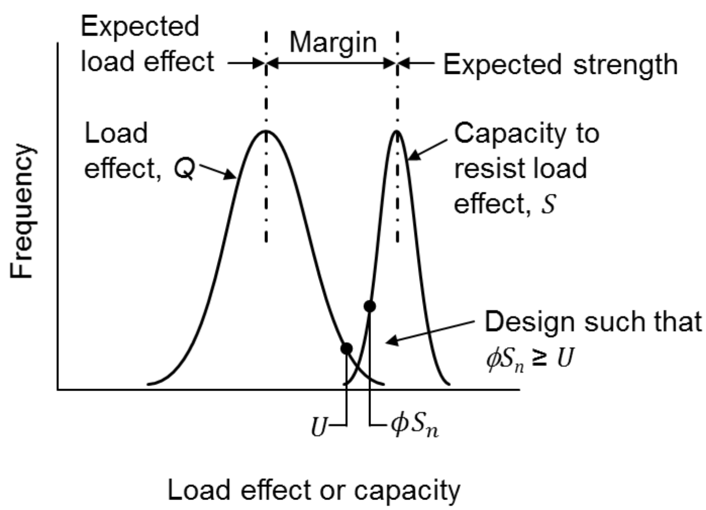



Figure 12.2 Controlling the risk of failure.

## 12.3. LIMIT STATE DESIGN

A *limit state* is a condition of a structural member (or structural system) beyond which the structural member (or system) no longer satisfies a performance requirement. In relation to the discussion in Section 12.2, the two most commonly limit states considered in design are the *serviceability limit state* and the *ultimate limit state*. *Limit states design* is a process by which the various limit states are identified and then the design is carried out to ensure that the limit state is not exceeded. The methods described in the remainder of this chapter are used as part of the limit state design procedure.

## 12.4. DEFLECTION LIMITS

According to the International Building Code (IBC), structural systems and members thereof shall be designed to have adequate stiffness to limit deflections and lateral drift, as defined in the following subsections.

### 12.4.1. Deflections Under Vertical Loads

The deflections of structural members are not to exceed the values of Table 12.1. In that table, $l$ = the member span, $L$ refers to the deflection calculated due to live load only, $S$ refers to the deflection calculated due to snow load only, and $KD + L$ refers to the deflection calculated due to combined dead plus live load that occurs after the member is constructed, that is, it is the deflection due to live load plus any long-term deflection under dead loads. For example, wood and concrete members creep under sustained dead loads, and therefore, $K \neq 0$. Steel does not creep, therefore, $K = 0$. See The deflections are calculated using service loads without load factors.

**Table 12.1 Deflection limits**

| Construction | $L$ | $S$ | $KD + L$ |
|---|---|---|---|
| Roof members: | | | |
| Supporting plaster ceiling | $l/360$ | $l/360$ | $l/240$ |
| Supporting non-plaster ceiling | $l/240$ | $l/240$ | $l/180$ |
| Not supporting ceiling | $l/180$ | $l/180$ | $l/120$ |
| Floor members | $l/360$ | – | $l/240$ |
| Exterior walls and interior partitions: | | | |
| With brittle finishes | – | $l/240$ | – |
| With flexible finishes | – | $l/120$ | – |

**Table 12.2 Values of K for creep deflection**

| Wood | | Reinforced Concrete | Structural Steel |
|---|---|---|---|
| Unseasoned | Seasoned | | |
| 1.0 | 0.5 | Take CE 123 | 0 |

### 12.4.2. Deflections Under Wind Loads

Here we are concerned with the lateral drift or sway of a building under design wind loads. Building codes traditionally have not contained limits on lateral drift under wind loading. However, it is traditional to limit the lateral drift of the entire building to $h_n/500$ and the lateral drift of individual stories to $h_i/400$, where $h_n$ = height from base to roof level and $h_i$ = individual story height. These limits generally are sufficient to avoid damage to cladding and nonstructural walls and partitions. These limits traditionally have been associated with 50-year wind speeds (some designers use shorter or longer periods). Most designers consider the 700-year wind speeds currently in ASCE 7 to be excessively conservative for checking lateral drift under wind loading. The 700-year wind loads can be converted approximately to 50-year wind loads by dividing by 1.6.

Wind tunnel testing can be used to study the dynamic response of tall buildings under wind loading. From such tests, an engineer can determine the floor accelerations as a function of vibration frequency and wind return period. It is common to design a building such that occupants will not feel discomfort for wind return periods in the range of 1 to 10 years. Studies of occupant perception show that the acceptable floor accelerations are a function of the vibration frequency.

### 12.4.3. Deflections Under Earthquake Loads

ASCE 7 specifies allowable story drifts under design earthquake loads. Recall that design earthquake loads are modified from the linear-elastic loads by dividing by factor $R/I_e$, where $R$ = response modification factor and $I_e$ = importance factor. To estimate the lateral drifts for design, the drifts calculated with these $R/I_e$ reduced forces need to be adjusted by factor $C_d/I_e$, where $C_d$ = deflection amplification factor. See Chapter 11 for the various factors. The calculated story drifts are not to exceed values listed in Table 12.3. Note that these drifts are much larger than those permitted for wind design. The smaller drifts for wind design are because the design basis is for linear-elastic response with minimal damage to structural and nonstructural elements, whereas the larger drifts for seismic design are because the design basis is for nonlinear response that accepts the occurrence of some damage to structural and nonstructural elements in the design earthquake.

**Table 12.3 Allowable story drift, $\Delta_a$**

| | Risk Category | | |
|---|---|---|---|
| | I or II | III | IV |
| Structures, other than masonry shear wall structures, 4 stories or less above the base, with interior walls, partitions, ceilings, and exterior wall systems that have been designed to accommodate the story drifts. | $0.025h_{sx}$ | $0.020h_{sx}$ | $0.015h_{sx}$ |
| Masonry cantilever shear wall structures | $0.010h_{sx}$ | $0.010h_{sx}$ | $0.010h_{sx}$ |
| Other masonry shear wall structures | $0.007h_{sx}$ | $0.007h_{sx}$ | $0.007h_{sx}$ |
| All other structures | $0.020h_{sx}$ | $0.015h_{sx}$ | $0.015h_{sx}$ |

## 12.5. ALLOWABLE (WORKING) STRESS DESIGN METHOD

The *allowable stress design* (*ASD*) method, alternatively known as the *working stress design* (*WSD*) method, is one of two commonly used methods to design for the ultimate limit state. (The other method is the load and resistance factor method, described in Section 12.6.) The basic approach for the ASD method is to keep the stresses due to service loads well below the stress capacities of the materials, thereby providing a margin of safety. The specific steps in the ASD method are as follows:

Step 1: Apply the service loads to a linear-elastic model of the structure, and calculate the internal forces such as axial force ($P$), shear ($V$), and moment ($M$).

Step 2: Calculate the internal stresses due to the internal forces $P$, $V$, and $M$ using methods of linear-elastic structural mechanics. Specifically, shear stress is calculated as $v = VQ/Ib$ and normal stress due to combined axial force and moment is calculated as $\sigma = P/A \pm Mc/I$.

Step 3: Check that the stresses from Step 2 do not exceed allowable stresses. The allowable stresses are specified by the applicable building code.

The ASD method is widely used for wood design, as discussed in Chapter 13. It is also used for checking soil bearing pressures for foundation design. It was formerly used for steel, reinforced concrete, and reinforced masonry design, but current practice for those materials generally uses the LRFD method.

## 12.6. LOAD AND RESISTANCE FACTOR DESIGN METHOD (LRFD)

The *load and resistance factor design* (*LRFD*) method is the preferred method (compared with ASD) to design for the ultimate limit state.

**General approach**

The LRFD method can be expressed generically through Eq. (12.1).

$$\phi S_n \geq U \tag{12.1}$$

in which $\phi S_n$ is referred to as the design strength, $\phi$ = strength reduction factor, $S_n$ = nominal strength, and $U$ = factored load effect. In practice, Eq. (12.1) is applied to internal member forces such as shear and moment, as in

$$\phi V_n \geq V_u \tag{12.2}$$

$$\phi M_n \geq M_u \tag{12.3}$$

in which $V_n$ = nominal shear strength, $M_n$ = nominal moment strength, $V_u$ = shear due to factored loads, and $M_u$ = moment due to factored loads. Nominal strengths are strengths that are calculated using methods specified in the building codes.

Although the LRFD method refers to an ultimate limit state approaching the failure or collapse state, structural analysis for the limit state is usually done using assumptions of linear-elastic behavior. Thus, the ultimate limit state for the structural system as a whole is presumed to be reached for the loading that first causes a member cross section to reach the design strength $\phi S_n$.

Load and resistance factors for the LRFD method are established considering variability and uncertainty in different load effects and material properties, the accuracy and variability of nominal strengths, the brittleness of different failure modes, and the consequences of failure. For buildings assigned to Risk Category II of ASCE 7, the intended annual probabilities of failure for load conditions that do not include earthquake are 3 x $10^{-5}$/yr for failure that is not sudden and does not lead to wide-spread progression of damage, 3 x $10^{-6}$/yr for failure that is either sudden or leads to widespread progression of damage, and 7 x $10^{-7}$/yr for failure that is sudden and results in widespread progression of damage (ASCE 7).

**ASCE 7 factored load combinations**

The factored load effect is represented by $U$ in Eq. (12.1). In practice, the quantity $U$ is the maximum (or minimum) load effect determined through a series of load combinations. Each load combination considers one or more load cases, whose load factors have been adjusted to achieve approximately uniform reliability.
The main load cases are listed below, and refer to the load itself or to its effect on internal moments and forces:

$D$ = dead load
$E$ = earthquake load
$F$ = load due to fluids with well-defined pressures and maximum heights
H = load due to lateral earth pressure, ground water pressure, or pressure of bulk materials
$L$ = live load
$L_r$ = roof live load
$S$ = snow load
$W$ = wind load

The basic *load combinations* consider different combinations of the load cases, as follows:

1. $1.4D$
2. $1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$
3. $1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (\alpha_L L \text{ or } 0.5W)$
4. $1.2D + 1.0W + \alpha_L L + 0.5(L_r \text{ or } S \text{ or } R)$
5. $1.2D + 1.0E + \alpha_L L + 0.2S$
6. $0.9D + 1.0W$
7. $0.9D + 1.0E$

In combinations 3, 4, and 5, the factor $\alpha_L$ applied to $L$ is equal to 1.0 for garages, for areas occupied as places of public assembly, and for any occupancies in which $L > 100$ psf (4.8 kPa). Otherwise, $\alpha_L = 0.5$.

Where fluid loads $F$ are present, they are to be included with the same load factor as dead load $D$ in combinations 1 through 5 and 7.

Where loads $H$ are present, they are to be included as follows:

1. Where the effect of $H$ adds to the primary variable load effect, include $H$ with a load factor of 1.6;
2. Where the effect of $H$ resists the primary variable load effect, include $H$ with a load factor of 0.9 where the load is permanent or a load factor of 0.0 for all other conditions.

In any of the load combinations, effects of one or more loads not acting, or effects of loads acting in the opposite direction (where possible) are to be investigated. The most unfavorable effects from both wind and earthquake loads are to be investigated, where appropriate, but they need not be considered to act simultaneously. Additional effects of flood, atmospheric ice loads, and self-restraining loads are not covered in this reader. See ASCE 7 for additional details.

For earthquake-resistant design, the engineer must consider the effects of earthquake directionality. In general, this includes effects of earthquake loads in two principal horizontal directions plus vertical earthquake shaking effects. Effects of overstrength on design loads must also be considered in some special cases. These details are not covered in this text.

Figure 12.3 illustrates the application of the load combinations for a planar system considering the load cases *D*, *L*, and *E*. Basic load combinations 1 and 2 consider only *D* and combined *D* and *L*. In this illustration, both *D* and *L* are taken at their full intensities. To obtain the worst shear at beam mid-span, however, *L* should be placed on only half of the beam span. The building code requires that this latter loading case also be considered.

Diagrams 5a and 5b in Figure 12.3 illustrate ASCE 7 load combination 5; note that *E* must be considered both from left to right and from right to left. Illustrations 7a and 7b in Figure 12.3 illustrate load combination 7. In a typical structure, load combination 5 results in higher axial compression in columns while load combination 7 results in higher axial tension in columns. Both load combinations must be considered in design. Not shown in these diagrams is the effect of vertical earthquake loads, which must be considered in accordance with ASCE 7.

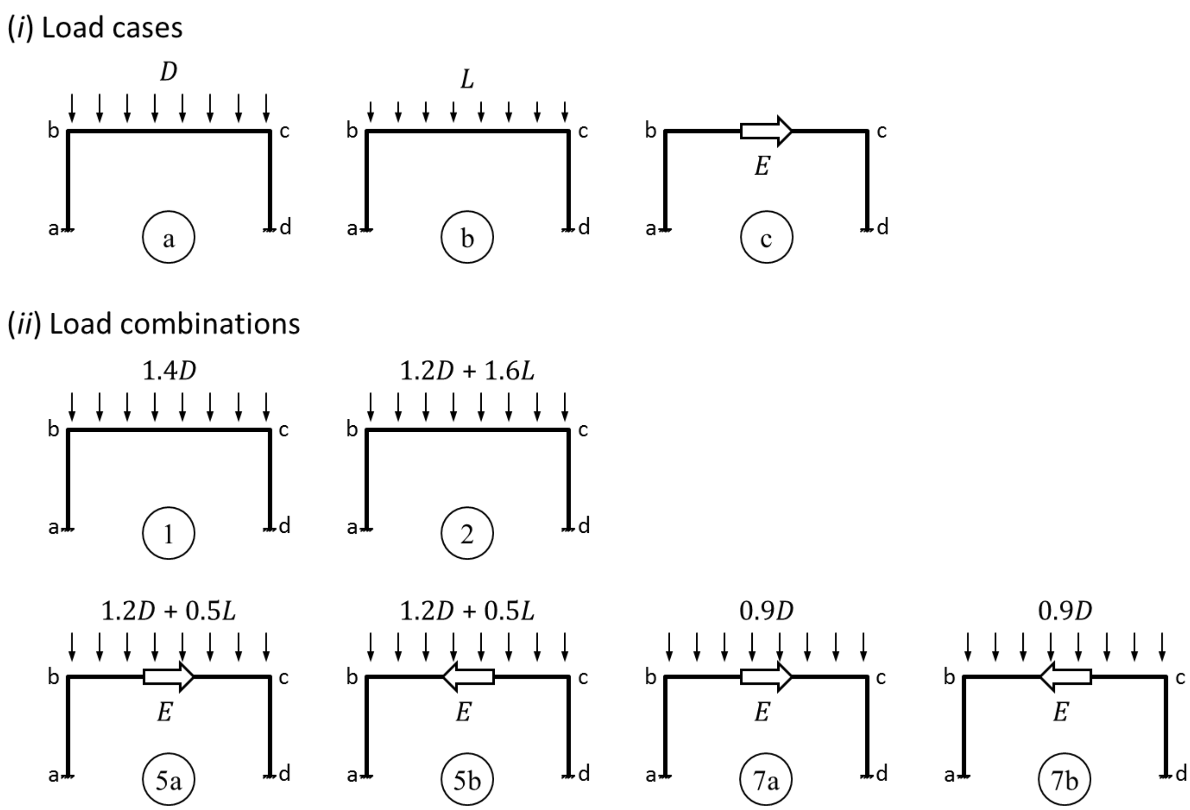



Figure 12.3 Load cases and load combinations in load and resistance factor design.

**Resistance factors, $\phi$**

In Eq. (12.1), the term $\phi S_n$ is referred to as the design strength, which is the product of strength reduction factor $\phi$ and nominal strength $S_n$. Nominal strength is determined using nominal strength equations (which are covered in later chapters of this reader). The strength reduction factors have numerical values less than 1.0, and are provided (1) to allow for the possibility of under-strength members due to variations in material strengths and dimensions, (2) to allow for inaccuracies in the design equations, (3) to reflect the available ductility and required reliability of the member under the load effects being considered, and (4) to reflect the importance of the member in the structure. See later chapters on steel and reinforced concrete design to find the $\phi$ factors applicable to those materials.

**Example 1**: A weightless, one-bay, one-story frame has configuration and loading shown in Figure 12.4. Dead load $D$ is 3 klf (44 kN/m), live load $L$ is 1.8 klf (26 kN/m), and earthquake load $E$ is 45 kips (200 kN). Use the LRFD method to determine the required beam moment strengths at the faces of the beams (Sections 1 and 2).

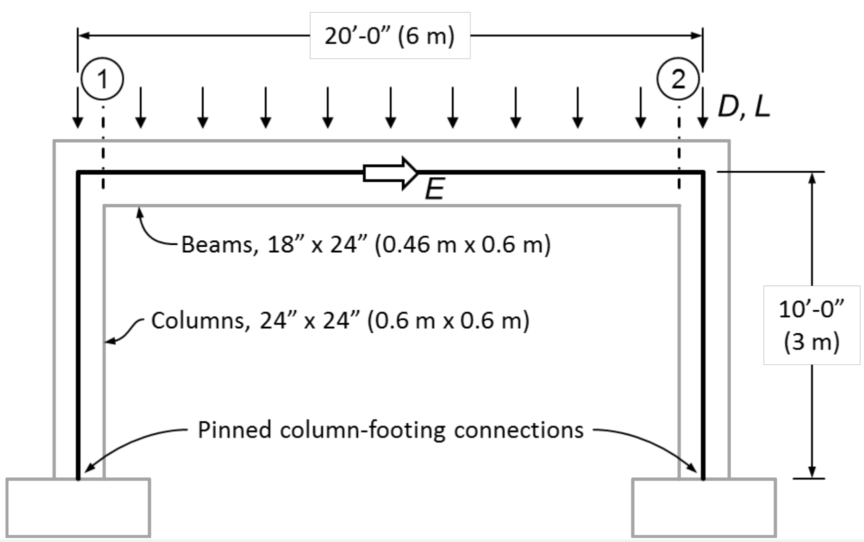



Figure 12.4 Example 1.

**Solution**: The load cases and load combinations are shown in Figure 12.3. The structure is modeled using flexural stiffness equal to $0.3EI_g$ for beams and columns and analyzed for the load cases using computer software for structural analysis. The results of the load cases are then combined using the load combinations. Calculated moments at sections 1 and 2 are tabulated below.

| | **Moments, k-ft (kN-m)** | |
|---|---|---|
| **Load case** | Section 1 | Section 2 |
| $D$ | -51.6 (-70.2) | -51.6 (-70.2) |
| $L$ | -31.0 (-42.1) | -31.0 (-42.1) |
| $E$ | 203 (275) | -203 (-275) |

| Load Combination | | |
|---|---|---|
| 1.4D | -72.2 (-98.2) | -72.2 (-98.2) |
| 1.2D + 1.6L | -111 (-152) | -111 (-152) |
| 1.2D + 0.5L + E | 125 (170) | -280 (-381) |
| 1.2D + 0.5L - E | -280 (-381) | 125 (170) |
| 0.9D + E | 156 (212) | -249 (-339) |
| 0.9D - E | -249 (-339) | 156 (212) |
| **Minimum** | **-280 (-381)** | **-280 (-381)** |
| **Maximum** | **156 (212)** | **156 (212)** |

**Example 2**: Determine the required nominal moment strengths of the beam at sections 1 and 2 considering the loading of Example 1. Assume the strength reduction factor is $\phi = 0.9$ for beam moment strength.

**Solution**: From Example 1, the required moment strengths are $M_u$ = -280 k-ft and +156 k-ft. Thus, the required nominal moment strengths are $M_n = M_u/\phi$ = -311 k ft and +173 k-ft. The beams would need to be designed to provide at least these nominal strengths.

## 12.7. CAPACITY DESIGN

Capacity design is a design method for controlling the yielding mechanism of a structure that is expected to respond inelastically to a design loading or an overload. A common application is in design of earthquake-resistant structures. The capacity design method involves the following steps:

**Step 1**: Select a target yielding mechanism for the structural system, identifying all the member sections that are intended to yield. The selected mechanism should be one that can be detailed for ductile response.

**Step 2**: Apply the design loads to the structural system, and proportion the selected yielding sections for required strength.

**Step 3**: Determine the internal forces that will develop within the structure when the structure, as designed in Step 2, forms the intended mechanism with each yielding section developing the expected member strength.

**Step 4**: Design the yielding regions for ductile response. Design the remainder of the structure to have strength necessary to resist the internal forces determined in Step 3.

**Example 3**: A steel rod supports a service live load of 100 kips. The steel rod has yield stress of 60 ksi and ultimate strength of 90 ksi. The strength reduction factor for axial tension is $\phi = 0.9$. Use capacity design to select an appropriate yield mechanism, design for that mechanism, and then design the rest of the structure to ensure it will not fail in any other mechanism in the event of an overload.

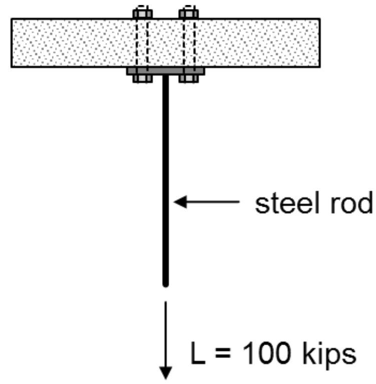



Figure 12.5 Example 3.

**Solution**: It is preferable to have the rod yield in tension rather than to have the connecting bolts yield in tension. Therefore, use the steps in capacity design, as follows:

Step 1: The target yielding mechanism is yielding of the steel rod.

Step 2: The controlling load combination is $U = 1.2D + 1.6L = 1.6 \text{ x } 100$ kips = 160 kips. Therefore, the steel rod requires tensile strength $T_u$ = 160 kips. The required nominal tensile strength is $T_n = T_u/\phi = 160/0.9 = 178$ kips. The yield strength is 60 ksi. Therefore, the required area of the steel rod is 178 kips / 60 ksi = 2.96 in.$^2$ Select a rod having cross-sectional area $A = 3$ in.$^2$

Step 3: Under a severe overload, the steel rod can develop tensile strength of 90 ksi x 3 in.$^2$ = 270 kips. Thus, the design load for the bolts is $T_u$ = 270 kips.

Step 4: Design the bolts. Assuming the bolts have tensile capacity of 50 ksi, with $\phi = 0.9$, the required bolt nominal strength is $T_n = T_u/\phi$ = 270 kips / 0.9 = 300 kips. The required bolt area is A = 300 kips / 50 ksi = 6 in.$^2$

# 13. Design of Wood Structures

## 13.1. INTRODUCTION

This chapter introduces wood as a structural material and describes design of simple elements of wood construction. Wood design can be carried out using either allowable stress design or Load and Resistance Factor Design (LRFD). In this chapter we only consider allowable stress design. At UC Berkeley, CE 124 provides more in depth treatment of the design of wood structures.

## 13.2. WOOD CHARACTERISTICS

Wood is a natural, renewable material that is processed from trees. Trees have evolved to be able to resist normal stresses due to axial forces and bending moments acting either along the axis of the trunk or the branches (Figure 13.1).

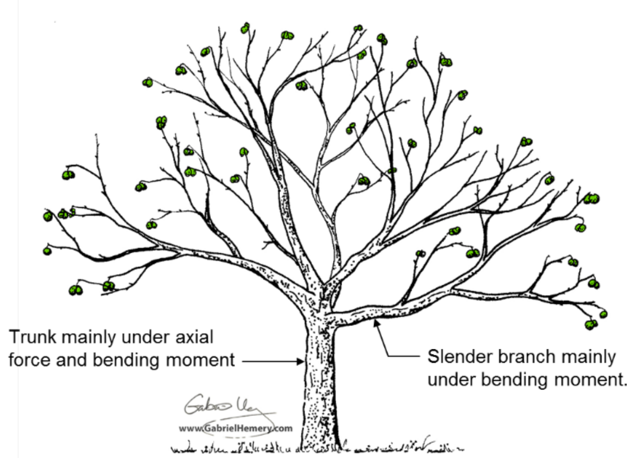



Figure 13.1 Primary actions in the trunk and branches of a tree.

Figure 13.2 illustrates the structure of a tree trunk. The trunk grows from the inside outward, putting on a new layer of growth in each growing season. Growing seasons generally repeat once annually, hence, a tree has annual rings, the number of rings identifying the age of the three in years. The cells are oriented primarily vertically. This affects the physical properties of lumber sawn from a tree. The fibers are stronger in the direction parallel to the cells than perpendicular to the cells. Thus, we say that wood is stronger parallel to the grain than perpendicular to the grain. As wood dries out, water is lost from the cells. This results in shrinkage perpendicular to the grain of the wood, with significantly less shrinkage parallel to the grain.

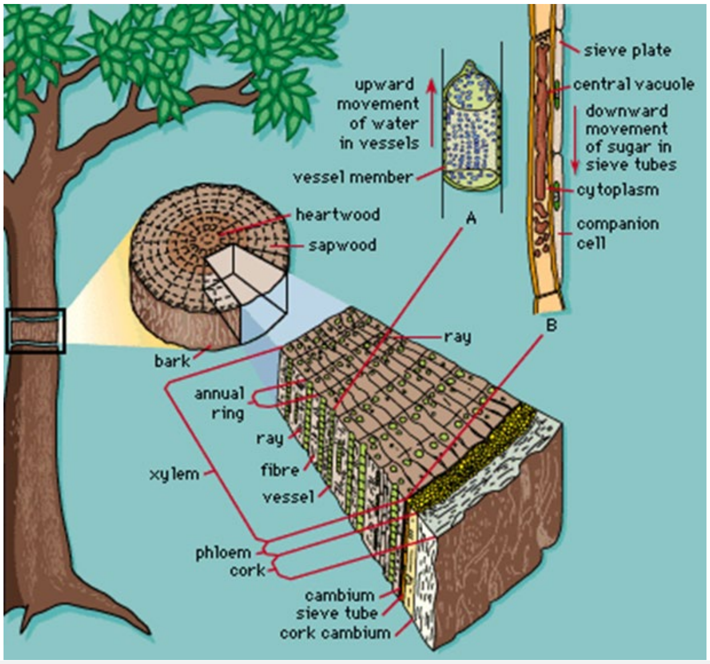



Figure 13.2 Structure of a tree trunk. (www.quora.com)

## 13.3. TYPES OF LUMBER

**Sawn lumber**

*Sawn lumber* refers to boards that are sawn directly from a tree. Figure 13.3 illustrates different boards that might be obtained from a log using the plain sawn method. This method refers to cutting the log in a single plane and perpendicular to that plane, as shown.

Figure 13.3 Illustration of boards sawn using the plain sawn method. Other methods are the quarter sawn and rift sawn methods. (http://www.forestryforum.com)

*Dimensional lumber* is a term used for lumber that is cut to standardized width and depth. In the United States, the nominal dimensions refer to the dimensions in inches of the original sawn piece. *Rough-sawn lumber* refers to lumber that is sawn and then shrinks somewhat from the original dimensions as drying occurs. Lumber used in most construction is sanded after it is sawn, which reduces the dimensions, and shrinkage further reduces the dimensions, resulting in what is referred to as the *dressed size*. The dressed size is smaller than the nominal size by which the lumber is specified.

## **Plywood**

*Plywood* is manufactured from sheets of cross-laminated veneer that are bonded under heat and pressure with adhesives (Figure 13.4). By alternating the grain direction of the veneers from layer to layer, or "cross-orienting," panel strength and stiffness in both directions are maximized. Plywood is used as flooring and to form shear walls in wood-frame construction. In the United States, plywood comes in 4 ft by 8 ft sheets.

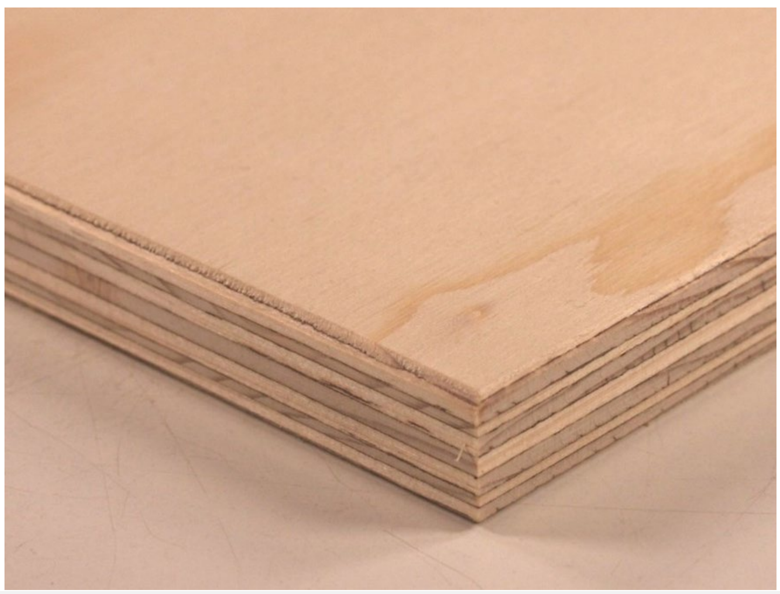


Figure 13.4 Plywood

## **Glued laminated timber**

*Glued laminated timber*, also known as *glulam*, is composed of several layers of dimensional timber glued together with adhesives, creating structural member that can be used as vertical columns or horizontal beams (Figure 13.5). Whereas dimensional sawn lumber is limited in size because of limitations in log sizes and because of splitting that occurs as a result of restrained shrinkage of sawn wood, glulams can be made in large sizes with different pieces of dimensional lumber placed optimally to achieve desired member properties. Glulam can also be produced in curved shapes, offering extensive design flexibility.

Figure 13.5 Glulams

**Parallams**

*Parallel strand lumber* (PSL), or the brand name *parallam*, is made from clipped veneer strands aligned and bonded with adhesive (Figure 13.6). It is used for beams, headers, columns, posts, and other uses. Parallams generally have higher strength, and can be constructed in large sizes, than dimensional sawn lumber.

Figure 13.6 Photograph of a parallam.

## I-Joists

I-joists are "I"-shaped structural members designed for use in floor and roof construction. An I-joist consists of top and bottom flanges united with webs. The flanges resist bending stresses and the web resists shear stresses. Figure 13.7 shows floor I-joists framing into glulam girders.

**Figure 13.7 I-Joists framing into a glulam. (APA – The Engineered Wood Association)**

## Roof trusses

Roof and floor trusses are prefabricated trusses made of wood, typically with metal gussets to connect the truss members. Figure 13.8 shows a typical configuration for a roof truss in residential construction.

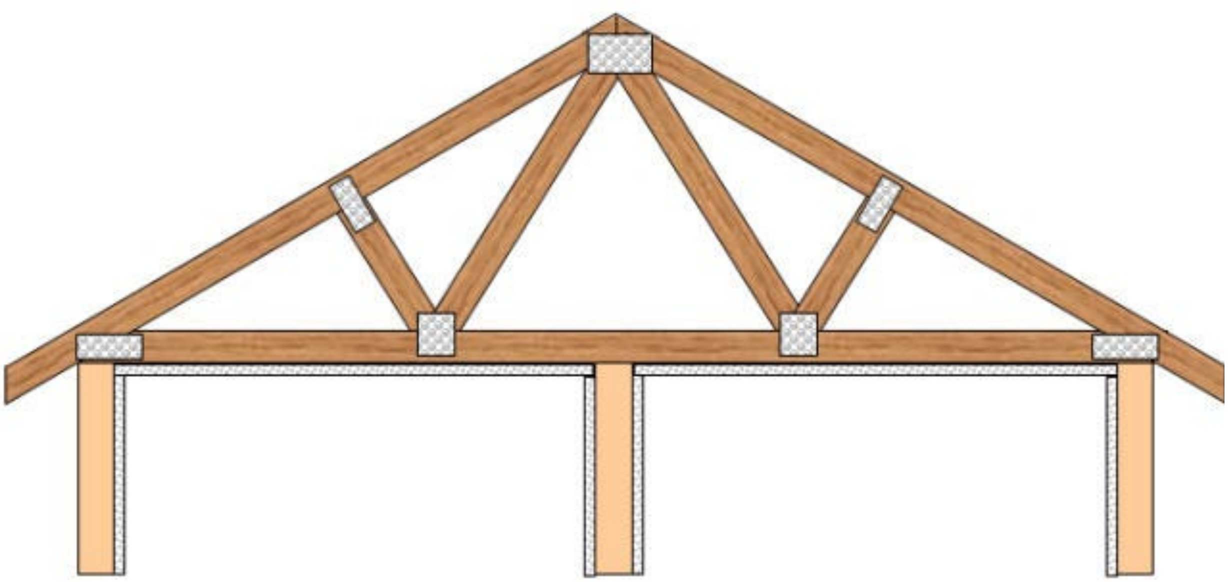


Figure 13.8 Roof trusses (http://www.renovation-headquarters.com/roof-truss-uplift.htm)

## 13.4. MECHANICAL PROPERTIES OF WOOD[1]

### 13.4.1. Basic properties

Lumber strengths depend on the direction of the stress relative to the wood grain. The allowable stresses in building codes are identified with capital letters (as opposed to the actual stresses due to applied loads, which will be identified with lower case letters). The properties of interest are: allowable fiber stress in bending ($F_b$), allowable tension parallel-to-grain ($F_t$), allowable horizontal shear ($F_v$), allowable compression parallel-to-grain ($F_c$), and allowable compression perpendicular-to-grain ($F_{c\perp}$ ). The modulus of elasticity ($E$) is also of interest.

Values for the allowable stresses vary depending on wood species, the grade (quality) of the wood within the species, and the type of member in which the wood is used. These aspects are covered in classes that are devoted to the subject of wood design. For CE 120, we will assume the properties in Table 13.1 for wood (based on Douglas Fir, 2" to 4" thick, 5" and wider).

**Table 13.1 Basic allowable unit stresses for structural lumber**

| Species and Commercial Grade | Allowable Unit Stresses in psi | | | | | | |
|---|---|---|---|---|---|---|---|
| | Extreme Fiber in Bending, $F_b$ | | Tension Parallel to Grain, $F_t$ | Horizontal Shear Stress, $F_v$ | Compression Perpendicular to Grain, $F_{c\perp}$ | Compression Parallel to Grain, $F_c$ | Modulus of Elasticity, $E$ |
| | Single-member Uses | Repetitive-member Uses* | | | | | |
| Douglas Fir – Larch (North) | | | | | | | |
| Dense Select Structural | 2100 | 2400 | 1400 | 95 | 730 | 1650 | 1,900,000 |
| Select Structural | 1800 | 2050 | 1200 | 95 | 625 | 1400 | 1,800,000 |
| Dense No. 1 | 1800 | 2050 | 1200 | 95 | 730 | 1450 | 1,900,000 |
| No. 1 | 1500 | 1750 | 1000 | 95 | 625 | 1250 | 1,800,000 |
| Dense No. 2 | 1450 | 1700 | 775 | 95 | 730 | 1250 | 1,700,000 |
| No. 2 | 1250 | 1450 | 650 | 95 | 625 | 1050 | 1,700,000 |
| No. 3 and Stud | 725 | 850 | 375 | 95 | 625 | 675 | 1,500,000 |
| Appearance | 1500 | 1750 | 1000 | 95 | 625 | 1500 | 1,800,000 |

* Spacing ≤ 24 in.

### 13.4.2. Adjustment Factors

The basic allowable unit stresses must be adjusted for various factors, as noted below:

**Size factor, $C_F$**

Large members may split because of restrained shrinkage as the member dries. The size factor $C_F$ accounts for this effect (Figure 13.9).

[1] http://www.wwpa.org/TECHGUIDE/DesignValues/tabid/855/Default.aspx

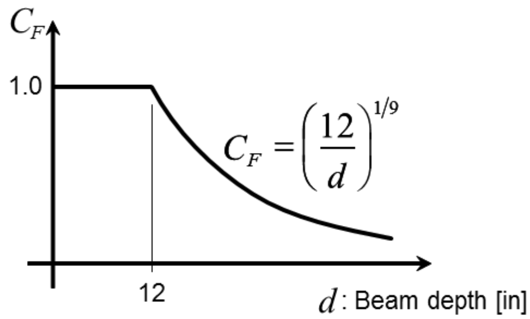



Figure 13.9 Size factor $C_F$.

**Form factor, $C_f$**

This factor adjusts for different cross-sectional shapes. In CE 120, use $C_f = 1.0$.

**Load duration factor, *LDF***

Strength of wood is a function of the loading rate. The basic allowable stresses are intended for use under 10-year loadings, corresponding to the case of dead plus live load (D + L). For other durations, a load duration factor applies

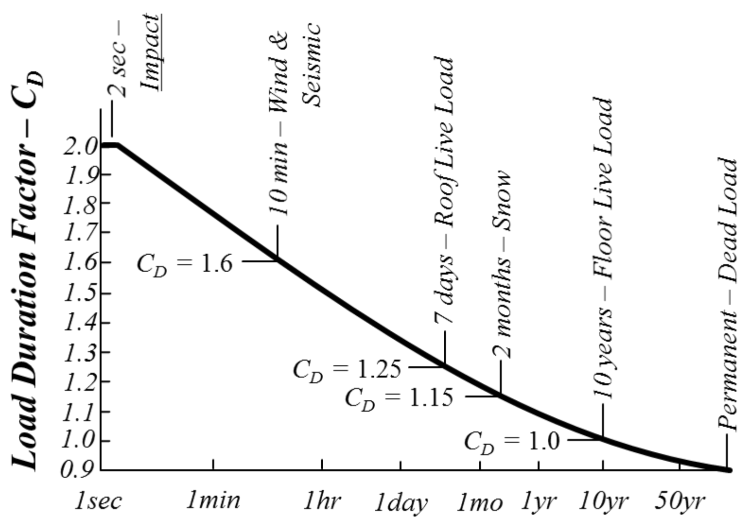
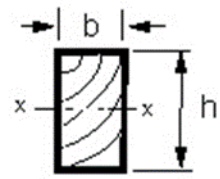



Figure 13.10 Load duration factor, *LDF*.

## 13.5. DIMENSIONAL PROPERTIES OF SAWN LUMBER

As noted previously, lumber is cut to standard dimensions, is sanded, and then shrinks as it dries. Consequently, the nominal dimensions by which a member is designated are different from the actual dimensions. Table 13.2 lists section properties of dressed dimensional lumber.

Table 13.2 Section properties of dimensional lumber




| Nominal $b$ x $h$, inches | Surfaced Size, $b$ x $h$, inches | Area $A = bh$, in.$^2$ | Section Modulus, $S = \frac{bh^2}{6}$, in.$^3$ | Moment of Inertia, $I = \frac{bh^3}{12}$, in.$^4$ | Board Feet per Lineal Foot of Piece |
|---|---|---|---|---|---|
| 2 × 2 | 1.5 × 1.5 | 2.25 | 0.562 | 0.422 | 0.33 |
| 2 × 3 | 1.5 × 2.5 | 3.75 | 1.56 | 1.95 | 0.50 |
| 2 × 4 | 1.5 × 3.5 | 5.25 | 3.06 | 5.36 | 0.67 |
| 2 × 6 | 1.5 × 5.5 | 8.25 | 7.56 | 20.80 | 1.00 |
| 2 × 8 | 1.5 × 7.25 | 10.88 | 13.14 | 47.63 | 1.33 |
| 2 × 10 | 1.5 × 9.25 | 13.88 | 21.39 | 98.93 | 1.67 |
| 2 × 12 | 1.5 × 11.25 | 16.88 | 31.64 | 177.98 | 2.00 |
| 2 × 14 | 1.5 × 13.25 | 19.88 | 43.89 | 290.78 | 2.33 |
| 3 × 3 | 2.5 × 2.5 | 6.25 | 2.60 | 3.26 | 0.75 |
| 3 × 4 | 2.5 × 3.5 | 8.75 | 5.10 | 8.93 | 1.00 |
| 3 × 6 | 2.5 × 5.5 | 13.75 | 12.60 | 34.66 | 1.50 |
| 3 × 8 | 2.5 × 7.25 | 18.12 | 21.90 | 79.39 | 2.00 |
| 3 × 10 | 2.5 × 9.25 | 23.12 | 35.65 | 164.89 | 2.50 |
| 3 × 12 | 2.5 × 11.25 | 28.12 | 52.73 | 296.63 | 3.00 |
| 3 × 14 | 2.5 × 13.25 | 33.12 | 73.15 | 484.63 | 3.50 |
| 3 × 16 | 2.5 × 15.25 | 38.12 | 96.90 | 738.87 | 4.00 |
| 4 × 4 | 3.5 × 3.5 | 12.25 | 7.15 | 12.51 | 1.33 |
| 4 × 6 | 3.5 × 5.5 | 19.25 | 17.65 | 48.53 | 2.00 |
| 4 × 8 | 3.5 × 7.25 | 25.38 | 30.66 | 111.15 | 2.67 |
| 4 × 10 | 3.5 × 9.25 | 32.38 | 49.91 | 230.84 | 3.33 |
| 4 × 12 | 3.5 × 11.25 | 39.38 | 73.83 | 415.28 | 4.00 |
| 4 × 14 | 3.5 × 13.25 | 46.38 | 102.41 | 678.48 | 4.67 |
| 4 × 16 | 3.5 × 15.25 | 53.38 | 135.66 | 1034.42 | 5.33 |
| 6 × 6 | 5.5 × 5.5 | 30.25 | 27.73 | 76.26 | 3.00 |
| 6 × 8 | 5.5 × 7.5 | 41.25 | 51.56 | 193.36 | 4.00 |
| 6 × 10 | 5.5 × 9.5 | 52.25 | 82.73 | 392.96 | 5.00 |
| 6 × 12 | 5.5 × 11.5 | 63.25 | 121.23 | 697.07 | 6.00 |
| 6 × 14 | 5.5 × 13.5 | 74.25 | 167.06 | 1127.67 | 7.00 |
| 6 × 16 | 5.5 × 15.5 | 85.25 | 220.23 | 1706.78 | 8.00 |
| 6 × 18 | 5.5 × 17.5 | 96.25 | 280.73 | 2456.38 | 9.00 |
| 6 × 20 | 5.5 × 19.5 | 107.25 | 348.56 | 3398.48 | 10.00 |
| 8 × 8 | 7.5 × 7.5 | 56.25 | 70.31 | 263.67 | 5.33 |
| 8 × 10 | 7.5 × 9.5 | 71.25 | 112.81 | 535.86 | 6.67 |
| 8 × 12 | 7.5 × 11.5 | 86.25 | 165.31 | 950.55 | 8.00 |
| 8 × 14 | 7.5 × 13.5 | 101.25 | 227.81 | 1537.73 | 9.33 |
| 8 × 16 | 7.5 × 15.5 | 116.25 | 300.31 | 2327.42 | 10.67 |
| 8 × 18 | 7.5 × 17.5 | 131.25 | 382.81 | 3349.61 | 12.00 |
| 8 × 20 | 7.5 × 19.5 | 146.25 | 475.31 | 4634.30 | 13.33 |
| 8 × 22 | 7.5 × 21.5 | 161.25 | 577.81 | 6211.48 | 14.67 |
| 8 × 24 | 7.5 × 23.5 | 176.25 | 690.31 | 8111.17 | 16.00 |
| 10 × 10 | 9.5 × 9.5 | 90.25 | 142.90 | 678.76 | 8.33 |
| 10 × 12 | 9.5 × 11.5 | 109.25 | 209.40 | 1204.03 | 10.00 |
| 10 × 14 | 9.5 × 13.5 | 128.25 | 288.56 | 1947.80 | 11.67 |
| 10 × 16 | 9.5 × 15.5 | 147.25 | 380.40 | 2948.07 | 13.33 |
| 10 × 18 | 9.5 × 17.5 | 166.25 | 484.90 | 4242.84 | 15.00 |
| 10 × 20 | 9.5 × 19.5 | 185.25 | 602.06 | 5870.11 | 16.67 |
| 10 × 22 | 9.5 × 21.5 | 204.25 | 731.90 | 7867.88 | 18.33 |
| 12 × 12 | 11.5 × 11.5 | 132.25 | 253.48 | 1457.51 | 12.00 |
| 12 × 14 | 11.5 × 13.5 | 155.25 | 349.31 | 2357.86 | 14.00 |
| 12 × 16 | 11.5 × 15.5 | 178.25 | 460.48 | 3568.71 | 16.00 |
| 12 × 18 | 11.5 × 17.5 | 201.25 | 586.98 | 5136.07 | 18.00 |
| 12 × 20 | 11.5 × 19.5 | 224.25 | 728.81 | 7105.92 | 20.00 |
| 12 × 22 | 11.5 × 21.5 | 247.25 | 885.98 | 9524.28 | 22.00 |
| 12 × 24 | 11.5 × 23.5 | 270.25 | 1058.48 | 12437.13 | 24.00 |

## 13.6. DESIGN OF WOOD BEAMS

Wood beam design is commonly done using the allowable stress method. Design considers the following four criteria:

- Deflections, $\delta$
- Flexural stresses, $f_b$
- Shear stresses, $f_v$
- Bearing stresses, $f_{c\perp}$

**Deflections**

Deflections commonly control the design of wood beams. The limits from Chapter 12 are repeated in Table 13.3 and Table 13.4. In these tables, $l$ = the member span, $L$ refers to the deflection calculated due to live load only, $S$ refers to the deflection calculated due to snow load only, and $KD + L$ refers to the deflection calculated due to combined dead plus live load that occurs after the member is constructed, that is, it is the deflection due to live load plus any long-term deflection under dead loads. For wood, the long-term deflection depends on whether the wood is put into service in unseasoned (wet) condition or seasoned (dry) condition. The deflections are calculated using service loads without load factors.

**Table 13.3 Deflection limits**

| Construction | $L$ | $S$ | $K\,D + L$ |
|---|---|---|---|
| Roof members: | | | |
| Supporting plaster ceiling | $l/360$ | $l/360$ | $l/240$ |
| Supporting non-plaster ceiling | $l/240$ | $l/240$ | $l/180$ |
| Not supporting ceiling | $l/180$ | $l/180$ | $l/120$ |
| Floor members | $l/360$ | – | $l/240$ |
| Exterior walls and interior partitions: | | | |
| With brittle finishes | – | $l/240$ | – |
| With flexible finishes | – | $l/120$ | – |

**Table 13.4 Values of K for creep deflection**

| Wood | |
|---|---|
| Unseasoned | Seasoned |
| 1.0 | 0.5 |

**Flexural stresses, $f_b$**

The allowable stress design method is used to limit the design stresses. The flexural stress acting on the beam is calculated as

$$f_b = \frac{M}{S} \tag{13.1}$$

in which $M$ is moment due to service loads and $S$ is section modulus. The design requirement is that the bending stresses under service loads, $f_b$, do not exceed the adjusted allowable bending stress, that is:

$$f_b \leq C_F \times C_f \times LDF \times F_b \tag{13.2}$$

in which $f_b$ = the calculated normal stress due to bending moment and $F_b$ = the tabulated allowable bending stress.

**Shear stresses, $f_v$**

The shear stress acting on the beam is calculated as

$$f_v = \frac{VQ}{Ib} = 1.5\frac{V}{A} for\ rectangular\ sections \tag{13.3}$$

in which $V$ is moment due to service loads and $A$ is cross-sectional area. The design requirement is that the shear stresses under service loads, $f_v$, do not exceed the adjusted allowable horizontal shear stress, that is:

$$f_v \leq LDF \times F_v \tag{13.4}$$

in which $f_v$ = the calculated normal stress due to shear and $F_v$ = the tabulated allowable horizontal shear stress.

**Bearing at support, $f_{c\perp}$**

The bearing stress acting on the beam at its supports is calculated as

$$f_{c\perp} = \frac{R}{A_b} \tag{13.5}$$

in which $R$ is reaction at the support due to service loads and $A_b$ is bearing area between the end of the beam and its support. The design requirement is that the bearing stresses under service loads, $f_{c\perp}$, do not exceed the adjusted allowable bearing stress perpendicular to the grain of the wood, that is:

$$f_{c\perp} \leq LDF \times F_{c\perp} \tag{13.6}$$

in which $f_{c\perp}$= the calculated normal stress due to bearing and $F_{c\perp}$ = the tabulated allowable bearing stress perpendicular to the grain of the wood.

**Additional considerations:**

- Need to ensure lateral stability by bracing beams at their ends and possibly along their spans.
- Cross-grain bending is not allowed as the strength is very weak. See Figure 13.11.
- Never notch a beam, as this can lead to splitting and reduced strength. See Figure 13.12.
- In CE 120, we only cover bearing perpendicular to the grain, not at an angle to the grain. See Figure 13.13.

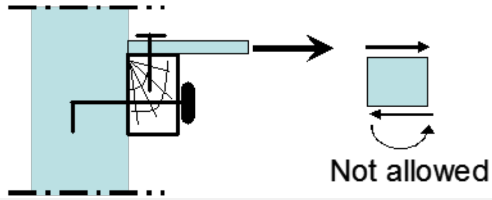



Figure 13.11 Cross-grain bending is not allowed.

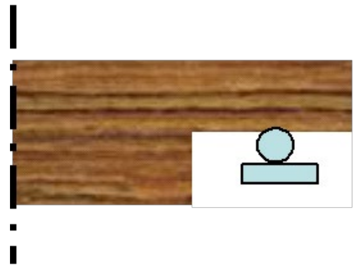
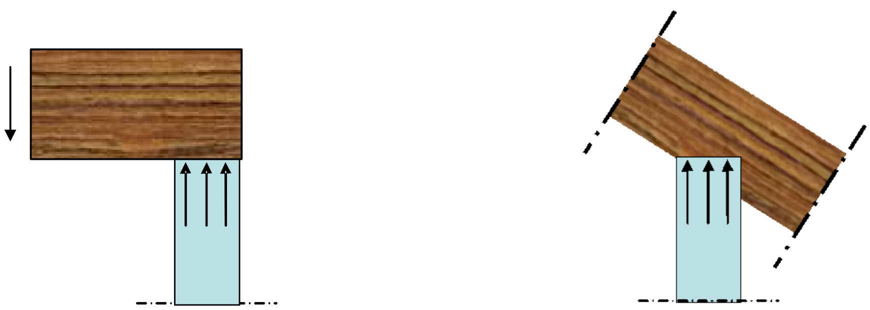


Figure 13.12 Avoid notching the end of beams.

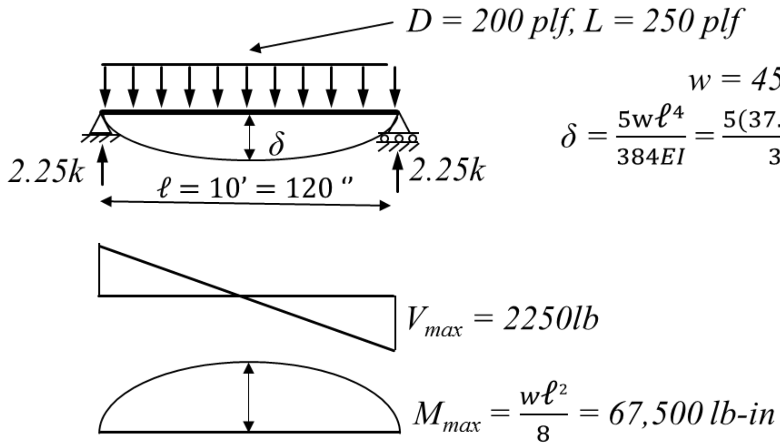


(a) Bearing perpendicular to grain. (b) Bearing at angle to grain. (Not covered.)

Figure 13.13 End bearing between beams and supports.

Example 1: Wood beam design. See below.

Example – Beam Design                Douglas Fir North, Select Structural




*w = 450 plf = 37.5 lb/in.*

$$\delta = \frac{5w\ell^4}{384EI} = \frac{5(37.5)(120)^4}{384EI} = 1.01 * 10^8/\text{EI (in)}$$

Deflection:

$E = 1{,}800{,}000$ psi

$\delta_{\text{max, L}} \leq \frac{\text{span}}{360} = \frac{120}{360} = 0.33\ in.$

$\left(\frac{250}{450}\right) * 1.01 * 10^8 * \frac{1}{1{,}800{,}000} * \frac{1}{I} \leq 0.33\ in \rightarrow I \geq 95\ in^4$     (4x8 OK)

$\delta_{\text{max, L+0.5D}} \leq \frac{\text{span}}{240} = \frac{120}{240} = 0.5\ in.$

$\left(\frac{350}{450}\right) 1.01 * 10^8 * \frac{1}{1{,}800{,}000} * \frac{1}{I} \leq 0.5\ in \rightarrow I \geq 87\ in^4$     (4x8 OK)

Moment:

$f_b = \frac{M}{S} = \frac{67500}{S} \leq 1800\text{ psi} = F_b \rightarrow S \geq 37.5\ in^3$  (4x10 OK)

Shear:

$f_v = \frac{3V}{2A} = \left(\frac{3}{2}\right)\frac{2250}{A} \leq 95\text{ psi} = F_v \rightarrow A \geq 35.5\ in^3$  (4x12 OK)

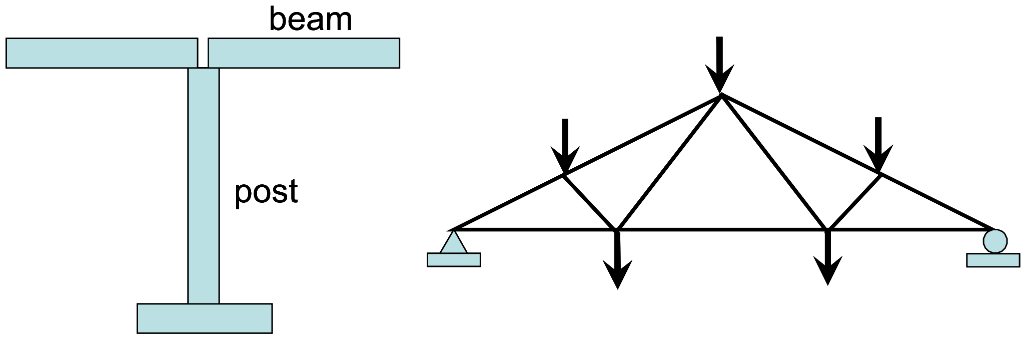
Bearing

$f_{c\perp} = \frac{R}{A_b} = \frac{2250}{Ab} \leq 625 \rightarrow A_b \geq 3.6\ in^3 = l_b * b \ \rightarrow l_b \geq 1.03\ in$

Final solution: Choose 4X12 with bearing not less than 1.5 in.

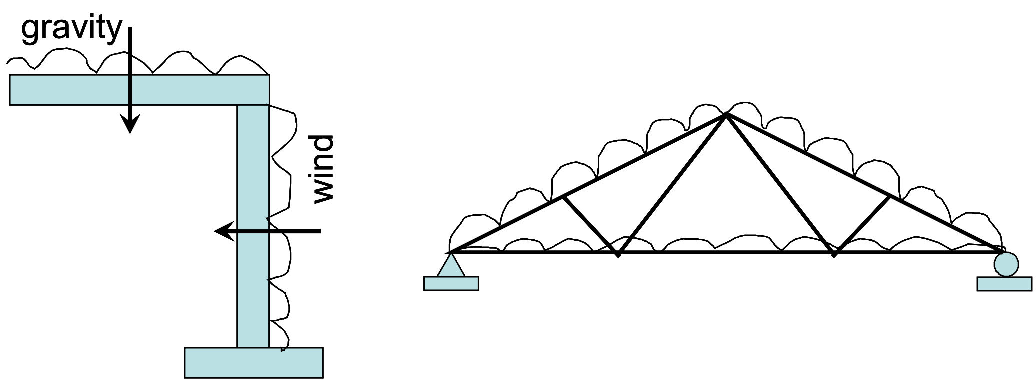
## 13.7. WOOD COMPRESSION MEMBERS

In CE120, we consider only pure compression members.




Pure compression [post on the left or top chord on right]

We do not consider members with axial compression and bending, as below.

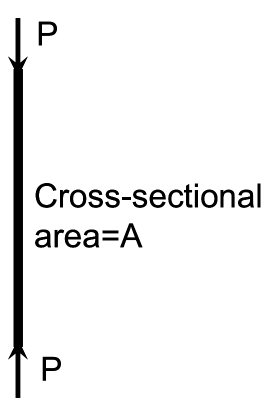



Bending + Compression (not considered)

Design of compression members must consider:

- Slenderness effects on member axial strength, and
- Bearing on the supporting elements.

**Slenderness effects**



The axial stress acting on the column is calculated as

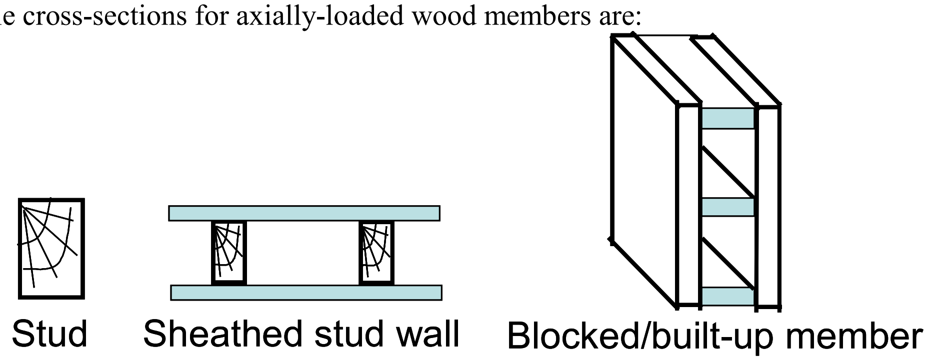
$$f_c = \frac{P}{A} \tag{13.7}$$

in which $P$ is axial force due to service loads and $A$ is cross-sectional area of the compression member. The design requirement is that the axial stress under service loads, $f_c$, shall not exceed the adjusted allowable axial stress parallel to the grain of the wood, that is:

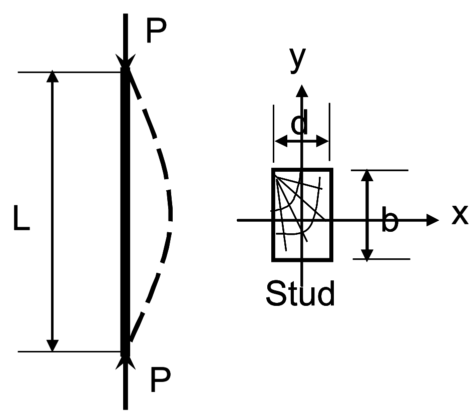
$$f_c \leq LDF \times F_c \times C_p = F_c^* \times C_p \tag{13.8}$$

in which $f_c$= the calculated axial stress due, $F_c$ = the tabulated allowable bearing stress parallel to the grain of the wood, and $C_p$ = a slenderness adjustment factor.

Possible cross-sections for axially-loaded wood members are:




The slenderness effect, represented by $C_p$, is determined as follows. Buckling will occur about the y-axis with weaker moment of inertia.




Define general slenderness ratio = $\frac{kL}{r}$ where $k$ is the effective length factor.

$$r = \sqrt{\frac{I_y}{A}} = \text{radius of gyration} = \sqrt{\frac{\left(bd^3\right)/12}{bd}} = d\sqrt{1/12}$$

In wood design, we will replace $\frac{kL}{r}$ by $\frac{kL}{d}\sqrt{12}$. Therefore, the critical load is given by

$$P_{cr} = \frac{\pi^2 EI}{(kL)^2} = \frac{\pi^2 EA}{(kL/r)^2} = \frac{\pi^2}{12}\frac{EA}{(kL/d)^2}$$

Buckling is assumed to occur at deformations exceeding the linear elastic limit, and this therefore requires a modified modulus of elasticity written as $E_m = \frac{E}{(1.5)/(0.85)} = \frac{E}{1.76}$ in which E is the tabulated modulus. Thus, the buckling stress for pure buckling of a very slender member becomes

$$f_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E_m}{\left(\frac{kl}{r}\right)^2} = 0.82\frac{E_m}{\left(\frac{kl}{d}\right)^2} \equiv F_{cE} \tag{13.9}$$

in which $F_{cE}$ = the Euler buckling stress for pure buckling.

For less slender members, crushing of the wood fibers may limit the axial stress, preventing achieving the pure Euler buckling state. Factor $C_p$ represents this effect. $C_p$ is expressed by:

$$C_p = \frac{1 + F_{cE}/F_c^*}{2c} - \sqrt{\left(\frac{1 + F_{cE}/F_c^*}{2c}\right)^2 - \frac{F_{cE}/F_c^*}{c}} \tag{13.10}$$

in which $c$ = buckling and crushing interaction factor = 0.8 for sawn lumber, $F_{cE}$ is defined by Eq. (13.9), and $F_c^* = LDF \times F_c$. Equation (13.10) is plotted in comparison with the Euler buckling equation in Figure 13.14. Note that it is not permitted to design members having $kl/d$ exceeding 50.

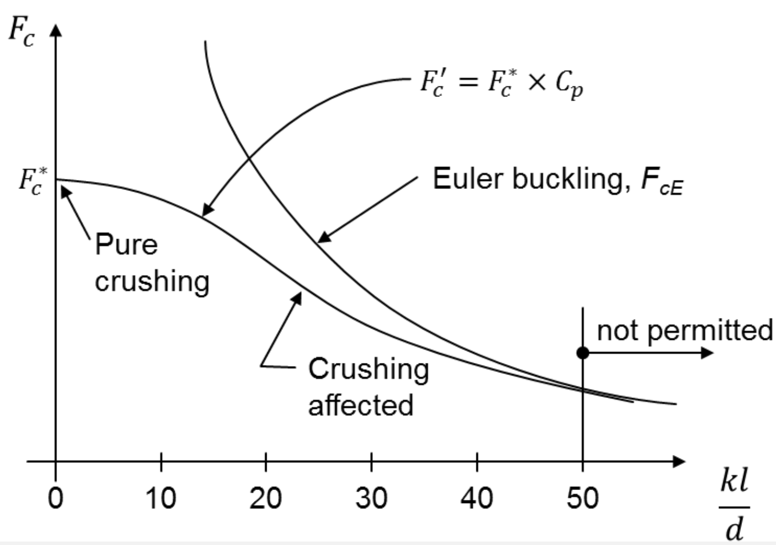



Figure 13.14 Allowable stress as effected by slenderness and crushing factor $C_p$.

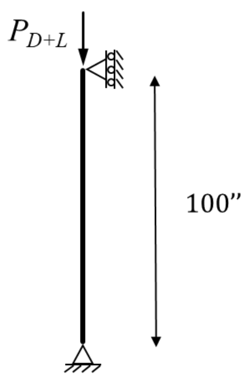
**Bearing on the supporting elements**

Bearing pressure on the supporting members must not exceed allowable bearing stress. Normally, this is the allowable compressive stress acting perpendicular to the grain of the supporting member, that is $F_{c\perp}$. Some building codes permit this stress to be increased because of local effects, but this is not considered in CE 120. Therefore, the design requirement is given by

$$f_{c\perp} \leq LDF \times F_{c\perp} \tag{13.11}$$

in which $f_{c\perp}$= the calculated normal stress due to bearing and $F_{c\perp}$ = the tabulated allowable bearing stress perpendicular to the grain of the wood.

Example 2: The compression member below is a 2x4 made of Douglass Fir and is stud grade. Assume bearing is not a concern.




*a) Is the use of this member permitted if unbraced?*
*b) If yes, what is the maximum allowable load? If not, brace the 2” direction and repeat*
*c) Find the maximum allowable compressive force, $F_c$’.*

$F_c = 675$ psi, $E = 1.5 * 10^6$ psi
LDF = 1.0 for D+L load

$kl/d = \frac{1*100}{1.5"} = 66.7 > 50 \rightarrow$ Not permitted $\rightarrow$
*Provide bracing over height for weak-axis bending*

$kl/d = \frac{1*100}{3.5"} = 28.6 < 50 \rightarrow$ *OK for strong axis*

***Find $F_c$’***

$$E_m = E\Big/\left(\frac{1.5}{0.85}\right) = \frac{0.85*(1.5*10^6)}{1.5} = 8.5*10^5 \ psi$$

$$F_{cE} = \frac{0.82*Em}{(\frac{kl}{d})^2} = \frac{0.82*(8.5*10^5)}{28.6^2} = 852 \ psi$$

$$F_c^* = F_c * LDF = (675)*(1.0) = 675 \ psi, \ c = 0.8$$

$$\rightarrow \boxed{C_p = \frac{1+F_{cE}/F_c^*}{2c} - \sqrt{\left(\frac{1+F_{cE}/F_c^*}{2c}\right)^2 - \frac{F_{cE}/F_c^*}{c}}} = 0.764$$

$$\rightarrow F_c' = F_c * C_p * LDF = (675)(0.764)(1.0) = 516 \ psi$$

$$\rightarrow P_{max} = F_c' * A = 516*1.5*3.5 = \mathbf{2710 \ lb}$$

# 14. Design of Steel Structures

## 14.1. INTRODUCTION

This chapter introduces steel as a structural material and describes design of simple elements of steel construction. Steel design can be carried out using either allowable stress design or Load and Resistance Factor Design (LRFD). This chapter presents only LRFD, as it is more prevalent in professional practice. The design provisions are covered in detail in *Steel Construction Manual (AISC)*. At UC Berkeley, CE 122 provides more in-depth treatment of the design of steel structures.

## 14.2. STEEL CHARACTERISTICS

Structural steel is a steel construction material that is available in standard shapes and sizes, and in a range of material properties. Figure 14-1 illustrates some of the available shapes and their designations. For example, the wide-flange shape, which can be very efficient for beams and columns, is designated W in U.S. practice. Each of these shapes is produced in a variety of sizes and weights. Properties for W sections are presented later in this chapter.

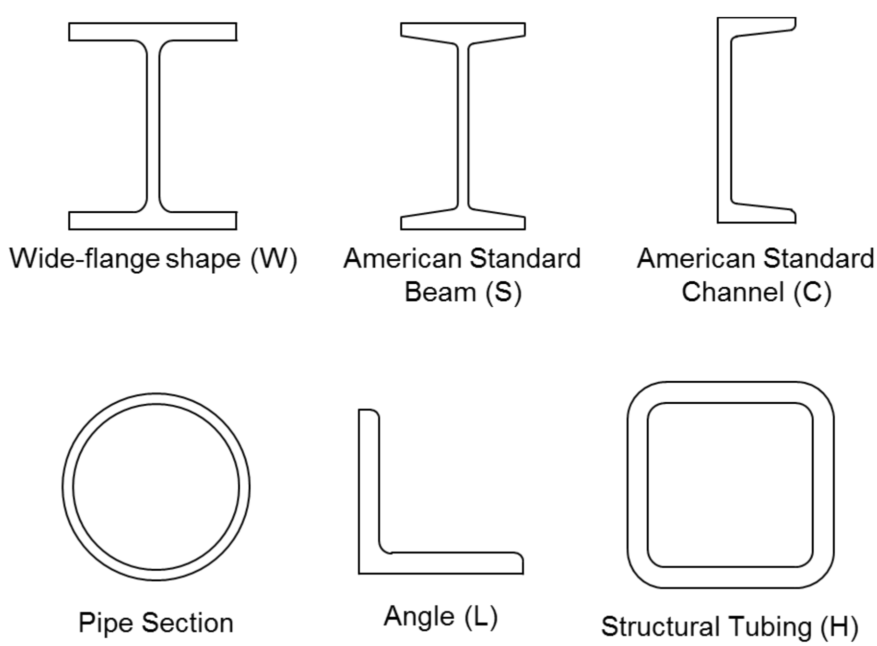



Figure 14-1 **Sample sections for structural steel members.**

The designer can also specify that rolled plates and standard shapes be welded together to produce a variety of built-up members (Figure 14-2).

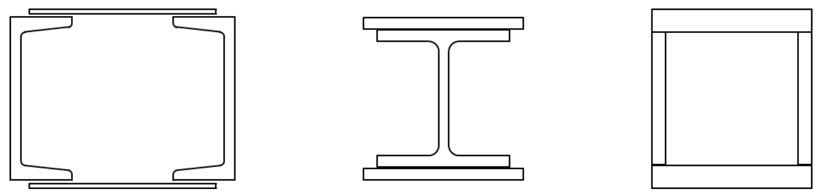


**Figure 14-2** Sample built-up members.

## 14.3. MECHANICAL PROPERTIES OF STRUCTURAL STEEL

Modern structural steels are referred to by their ASTM designations. The standard designation is A#, where # is a number referring to a specific ASTM standard. Examples include ASTM A36 and A53. ASTM A36 is a standard steel alloy that is a common structural steel in the United States. It is readily welded by all welding processes. ASTM A 53 is a carbon steel alloy used in structural pipe and tubing.

**Figure 14-3** shows stress-strain relations for structural steels. Lower-strength steels usually show a distinct yield plateau, followed by strain-hardening. Higher-strength steels may or may not show a distinct yield plateau. The *elastic modulus* of structural steel is relatively independent of the strength, and can be taken as $E_s$ = 30,000 ksi. *Yield strength* of structural steel varies with the ASTM designation. ASTM A36 steel has nominal yield strength of 36 ksi. For design purposes, we will assume the steel has an elasto-plastic stress-strain relation, that is, it is linearly elastic until the yield stress, and then strains without hardening beyond the yield point. The assumed behavior for A36 steel is shown by the broken line in **Figure 14-3**

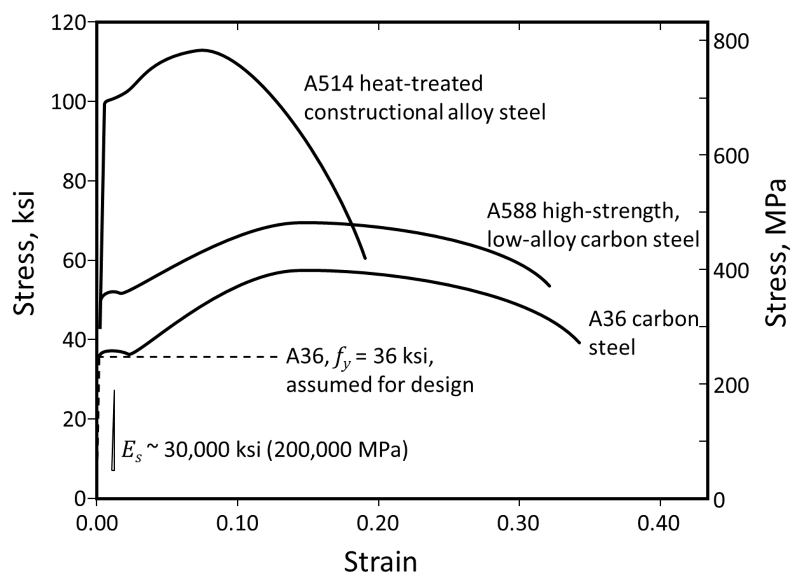


**Figure 14-3** Steel stress-strain relations.

*Unit weight* of structural steel is 490 pcf.

*Fire resistance* is a measure of the ability of a building element to resist a fire. Structural steel material properties are sensitive to high temperatures that can occur during fires in buildings or on bridges. Therefore, structural steels require fire protection measures to avoid fire-induced failures. Fire protection can be achieved by encasing structural steel in reinforced concrete or by applying spray-on fire-resistive coatings (**Figure 14-4**).

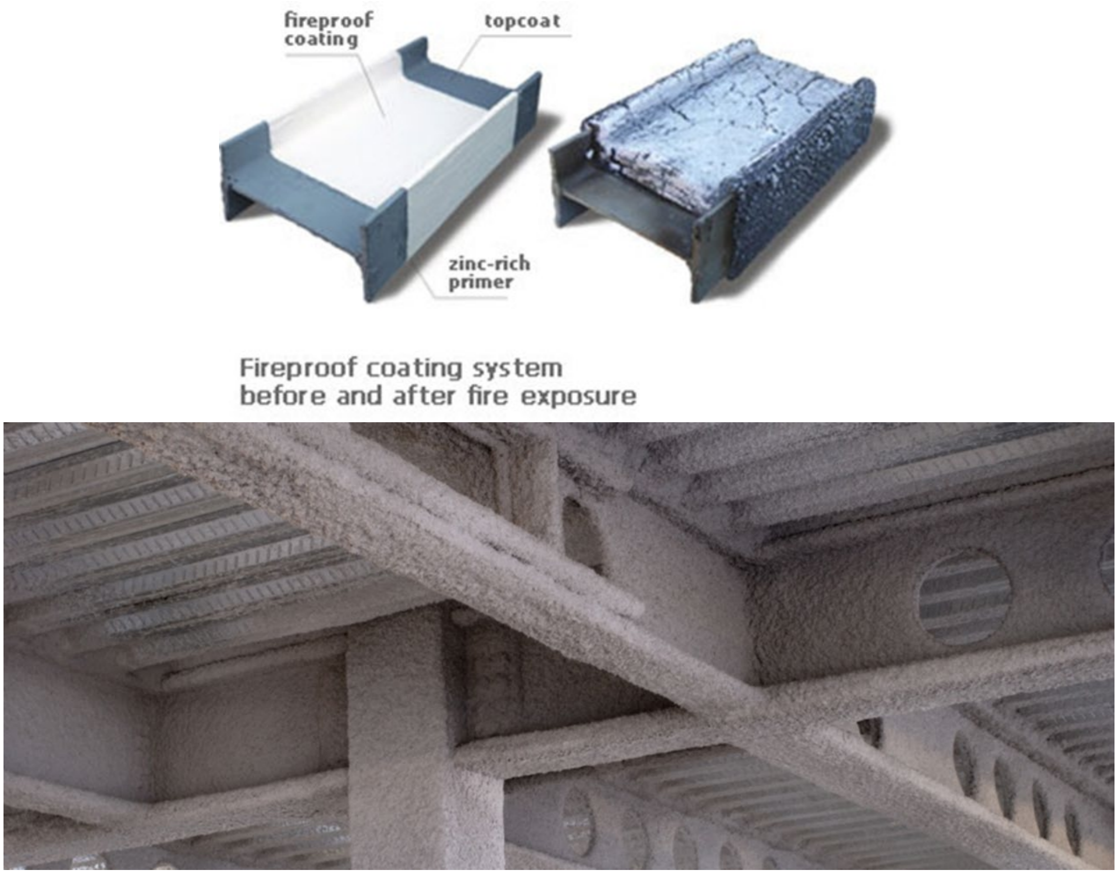



**Figure 14-4** Fire protection of structural steel members. Top: Intumescent paint before and after fire exposure (http://vmp-holding.com/). Bottom: Cementitious fire spray (http://www.sharpfibre.com/).

## 14.4. SECTION PROPERTIES OF WIDE FLANGE MEMBERS

The standard shapes shown in Figure 14-1 are available in a wide variety of standard sizes. Section designations along with the dimensional and structural properties can be downloaded from http://www.aisc.org/content.aspx?id=2868. Section properties for W sections are tabulated at the end of this chapter. For example, a W21X44 has approximate depth of 21 inches and weight of 44 plf. Actual properties, as tabulated are weight $W = 44.0$ plf, area $A = 13.0$ in.$^2$, depth $d = 20.7$ in., etc.

## 14.5. DESIGN STRENGTHS FOR STRUCTURAL STEEL BEAMS

Design of structural steel members usually is done in accordance with the Load and Resistance Factor (LRFD) method, which was introduced in Chapter 12. To use the LRFD method, we need to define nominal and design strengths for structural steel members. For our purposes, we are interested in the moment and shear strengths.

**Design moment strength, $\phi M_n$**

Consider the symmetric cross section of a wide flange member (**Figure 14-5**a). The member is assumed to be made using steel that has an elasto-plastic stress strain relation, with yield stress $f_y$. Under the action of applied moments, the member develops longitudinal strains that vary linearly over the member depth as shown in **Figure 14-5**b. Yielding of the extreme fiber (**Figure 14-5**c)occurs when the strain at the extreme fiber reaches $\varepsilon_y$. The moment at the onset of yielding is designated as the yield moment $M_y$. From principles of linear-elastic structural mechanics, up to the onset of yielding and for axial force $P = 0$, we can relate the moment and extreme fiber stress by

$$f = \frac{Mc}{I} \tag{14.1}$$

in which $c$ = distance from neutral axis to extreme point on section, $f$ = stress at extreme point on section, $M$ = applied moment, and $I$ = moment of inertia of the cross section about the neutral axis. Setting $f = f_y$ and solving for moment $M$, we obtain

$$M_y = \frac{f_y I}{c} = f_y S \tag{14.2}$$

in which $M_y$ = yield moment. In the right side of Eq. (14.2), we have substituted $S = I/c$, in which $S$ is referred to as the *section modulus*.

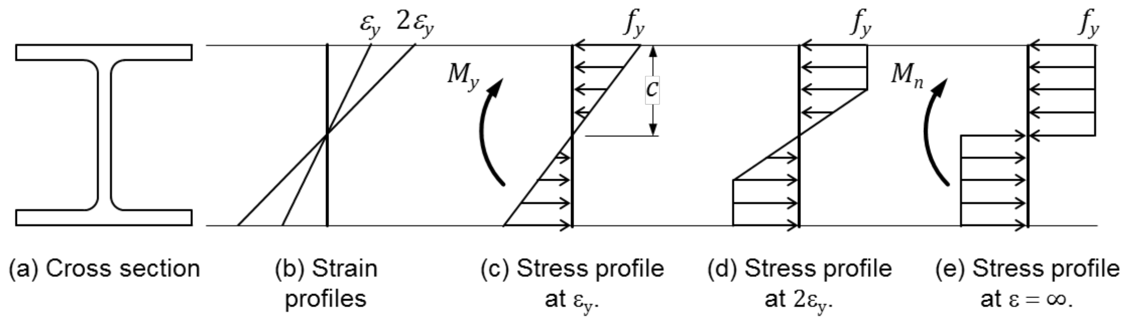



**Figure 14-5** Moment strength of structural steel cross section.

If we continue to bend the section until the beam reaches extreme fiber strain $2\varepsilon_y$, the stresses will take on the profile shown in **Figure 14-5**d. Note that the beam has reached the yield strain at half the depth $c$ measured from the neutral axis toward the extreme fiber, which explains the yield stress occurring over the outer quarter of the beam depth at both top and bottom of the beam cross section.

If we imagined bending the beam to infinite curvature, such that the beam achieved infinite strain over the full depth, then the beam would be fully yielded over its full depth, as shown in **Figure 14-5**e. This condition defines the nominal moment strength $M_n$, also known as the *plastic moment*. We write the nominal moment strength as

$$M_n = f_y Z \tag{14.3}$$

in which $Z$ is defined as the *plastic modulus*.

In the LRFD method, the *design moment strength* is defined as $\phi M_n$, where $M_n$ is defined by Eq. (14.3) and $\phi = 0.9$.

**Example 1**: What is the yield moment for an A36 W21X44 section bent about the strong axis? The section carries zero axial force.

**Solution**: See Figure 14-6. The strong axis is the horizontal axis for which the flanges will resist maximum tension and compression. From the section table at the end of the chapter, this is axis X. The section modulus for bending about the strong axis is $S_x = 81.6$ in.$^3$ From Eq. (14.2), the yield moment under zero axial force is $M_y = \frac{f_y I}{c} = f_y S = (36\ ksi)(81.6\ in.^3) = 2940\ k-in.$

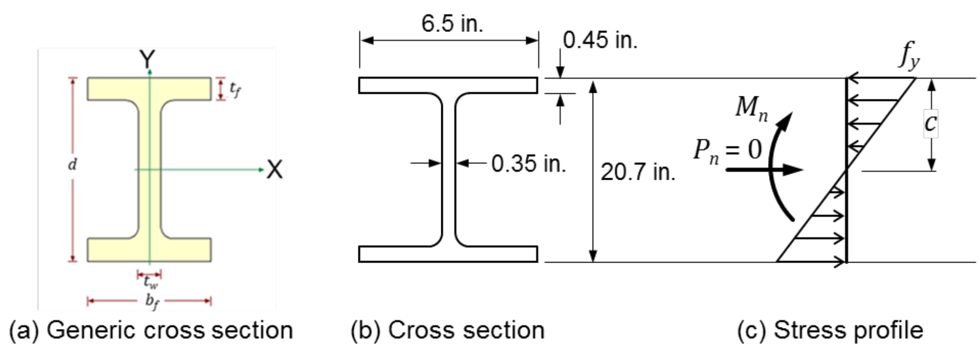



Figure 14-6 Cross-sectional dimensions of W21X44 (not drawn to scale) and stress conditions for yield moment $M_y$.

**Example 2**: Use first principles to calculate the plastic modulus $Z$ for an A36 W21X44 section bent about the strong axis.

**Solution**: The cross-sectional dimensions read from the section table (end of this chapter) are shown in Figure 14-7a. The stress profile associated with development of the plastic moment $M_n$ is shown in Figure 14-7b. Axial force $P_n = 0$.

Because of symmetry of the cross section and of the stresses about the X axis, we know that the neutral axis depth $c$ must be half the section depth. However, for the more general case, we set depth $c$ as a variable and use the requirements of equilibrium of axial forces to solve for $c$. Assuming that the web extends through full depth and that the flange therefore has width equal to flange width minus web width, the stress resultants are as follows:

$$C_f = t_w(b_f - t_w)f_y$$
$$C_w = t_w c f_y$$
$$T_f = t_w(b_f - t_w)f_y$$
$$T_w = t_w(d - c)f_y$$

Summing axial forces on the free-body diagram of Figure 14-7c:

$$\overrightarrow{\Sigma F_x} = P_n + T_f + T_w - C_f - C_w = 0 \qquad (14.4)$$

Substituting the expressions for the stress resultants in Eq. (14.5) and solving for $c$ we obtain $c = d/2$. The stress resultants can now be solved as

$C_w = T_w = (t_w)(d/2)(f_y) = (0.35)(20.7/2)(36) = 130$ kips.
$C_f = T_f = (t_f)(b_f - b_w)(f_y) = (0.45)(6.5 – 0.35)(36) = 99.6$ kips.

The centroidal locations of each of these resultants is shown in Figure 14-7c.

Summing moments about the neutral axis in Figure 14-7c, clockwise positive, we can write

$$\Sigma M = M_n - 10.13T_f - 5.18T_w - 10.13C_f - 5.18C_w = 0 \qquad (14.5)$$

Substituting the resultants into this expression, and solving for $M_n$, results in $M_n = 3370$ k-in. Using Eq. (14.3), we solve for plastic modulus $Z = M_n/f_y = 3370$ k-in./36 ksi = 93.6 in.[3] Note that the value given in the section table (at the end of this chapter) is Z = 95.4 in.[3] The small discrepancy is due to approximations in defining the shape in this example problem.

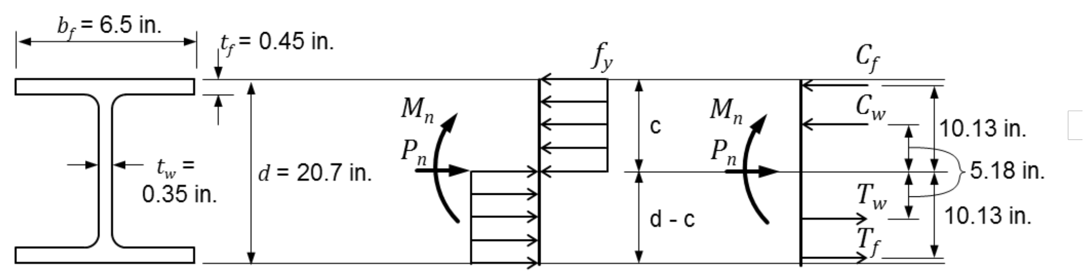



Figure 14-7 Cross-sectional dimensions of W21X44 (not drawn to scale) and stress conditions for nominal (plastic) moment $M_n$.

## Design shear strength, $\phi V_n$

Consider the member shown in Figure 14-8. The nominal shear strength is defined by the shear yield stress $f_{yv}$ and the cross-sectional area of the web $t_w d$. According to the von Mises yield criterion, the shear yield strength is $f_{yv} = \frac{f_y}{\sqrt{3}} = 0.577 f_y$. In design of steel members, this is approximated by $f_{yv} = 0.6 f_y$. Thus, the nominal shear strength is defined as

$$V_n = 0.6 f_y t_w d \qquad (14.6)$$

In the LRFD method, the *design shear strength* is defined as $\phi V_n$, where $V_n$ is defined by Eq. (14.6) and $\phi = 0.9$.

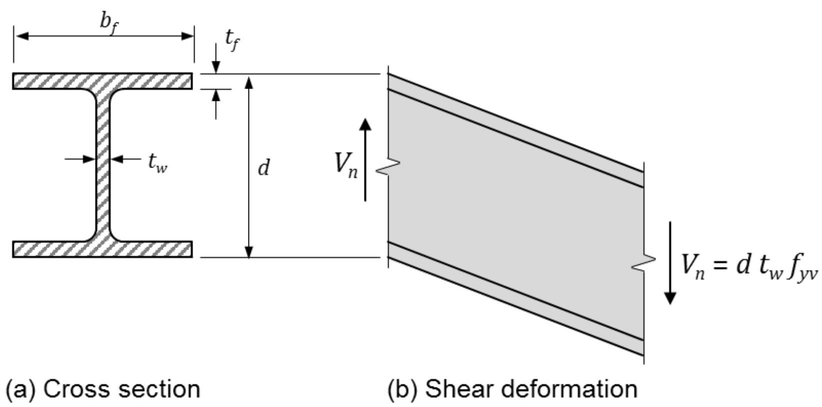



Figure 14-8 Cross section and shear deformed shape of W section.

## 14.6. DESIGN EXAMPLES

The examples of this section illustrate how to organize calculations and check required strength and serviceability for beams. The design methods and deflection limits are described in Chapter 12.

**Example 3**: A cantilever beam supports uniformly distributed service dead load of 3.5 klf and concentrated service live load of 8 kips (Figure 14-9). The member supports a floor. Select a minimum weight A36 W21 section that satisfies design requirements considering bending moment, shear, and deflections.

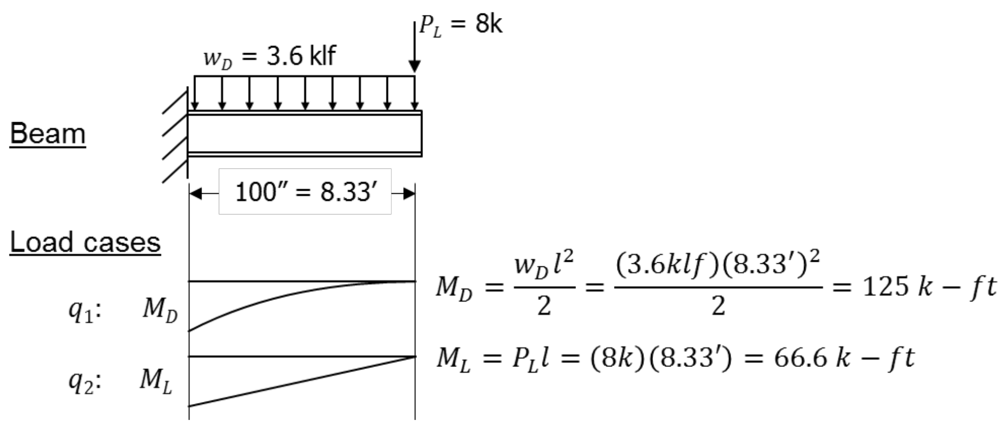



Figure 14-9 Example 3: Cantilever beam design.

**Solution**: The beam and loading are shown in Figure 14-9. The self-weight of the W section is conservatively estimate at 100 plf. Therefore, the dead load is shown as $w_D$ = 3.5 klf + 0.1 klf = 3.6 klf. Two load cases are of interest, $q_1$ for dead load and $q_2$ for live load. We will assume that the design is controlled by maximum moments, and then check shears and deflections.

Using the LRFD method, the design load combinations are LC1 and LC2, as follows:

LC1: $M_u = 1.4M_D$ = 1.4 x 125 k-ft = 175 k-ft.
LC2: $M_u = 1.2M_D + 1.6M_L$ = 1.2 x 125 k-ft + 1.6 x 66.6 k-ft = 257 k-ft = 3080 k-in. ←controls

The strength design requirement is expressed by $M_u \le \phi M_n = \phi \times Z \times f_y$. Solving for Z we find $Z \ge M_u/\phi f_y = 3080\ k - in./(0.9 \times 36\ ksi) = 95.1\ in.^3$ The lightest W21 that satisfies this requirement is a W21X44, which has $Z_x = 95.4$ in.$^3$ Note that this section weighs 44 plf, which is within the assumed conservatively assumed value of 100 plf.

We next check shear. The controlling load case is $V_u = 1.2V_D + 1.6V_L = 1.2 \text{ x } 30\text{k} + 1.6 \text{ x } 8\text{k} = 48.8$ kips. The strength design requirement is expressed by $V_u \le \phi V_n = \phi \times t_w d \times f_{yv}$. The selected W21X44 has $t_w = 0.35$ in. and $d = 20.7$ in. Using $\phi = 0.9$, we find $\phi V_n = 141\ kips$, which exceeds $V_u$, therefore, this section meets requirements for shear.

Lastly, we check deflections. The selected W21X 44 has $I_x = 843$ in.$^4$. The steel modulus is $E =$ 30,000 ksi. From Chapter 12, the allowable deflection under live load is $l/360 = 100" / 360 = 0.28\ in.$ The deflection under service live load is $\delta_L = \frac{P_L l^3}{3EI} = [(8\ kips)(100")^3]/[(3)(30{,}000\ ksi)(843\ in.^3)] = 0.11\ in.$ The calculated deflection is less than allowable, so the design satisfies the deflection requirement.

**Example 4**: A propped cantilever beam supports uniformly distributed service dead load of 1 klf (including self-weight) and uniformly distributed service live load of 2 klf. Find the design moments $M_u$ at points b and c and select a minimum weight A36 W18 section considering moment strength only.

**Solution**: The beam and loading are shown in Figure 14-10.

First, define the different load cases involving dead and live loads. Note that live load can be placed either in span ac, span cd, or span ad. We can use influence lines to identify that loadings in spans ac and cd are critical. Calculate moment diagrams for each load case.

Second, combine the different load cases using the load combinations required for LRFD. Three different load combinations are required, case 1 involving only dead load, and cases 2 and 3 involving dead load plus live load in two different patterns. Using the principle of superposition, the moment values for the different load combinations are simply linear combinations of the various load cases. For example, in load case 2, the load combination is $1.2M_D + 1.6M_L$, which for point b results in $M_u =$ 1.2 x 34 k-ft + 1.6 x 100 k-ft = 201 k-ft.

The largest moment along the span is 201 k-ft = 2410 k-in.[1] Thus, the required nominal (plastic) moment strength is $M_n = M_u/\phi$ = 2410 k-in. / 0.9 = 2680 k-in. Then, according to Eq. (14.3), the required plastic modulus is $Z = M_n/f_y$ = 2680/36 = 74.4 in.$^3$ Select a W18X40, which provides Z = 78.4 in.$^3$

The final design would also need to consider shear and deflections. Those aspects are not considered in this example.

[1] Note that the maximum moment in span abc does not occur exactly at mid-span point b. However, we want a practical method of combining moments for multiple load cases, which requires us to select a single point for superposition of results. Furthermore, the value at b is very nearly the maximum value for the loading considered, such that it is acceptable to design the beam based on the moment at b rather than exploring to find the exact maximum moment. For some other loading cases, especially those involving heavy concentrated loads, this approximation might not be sufficient.

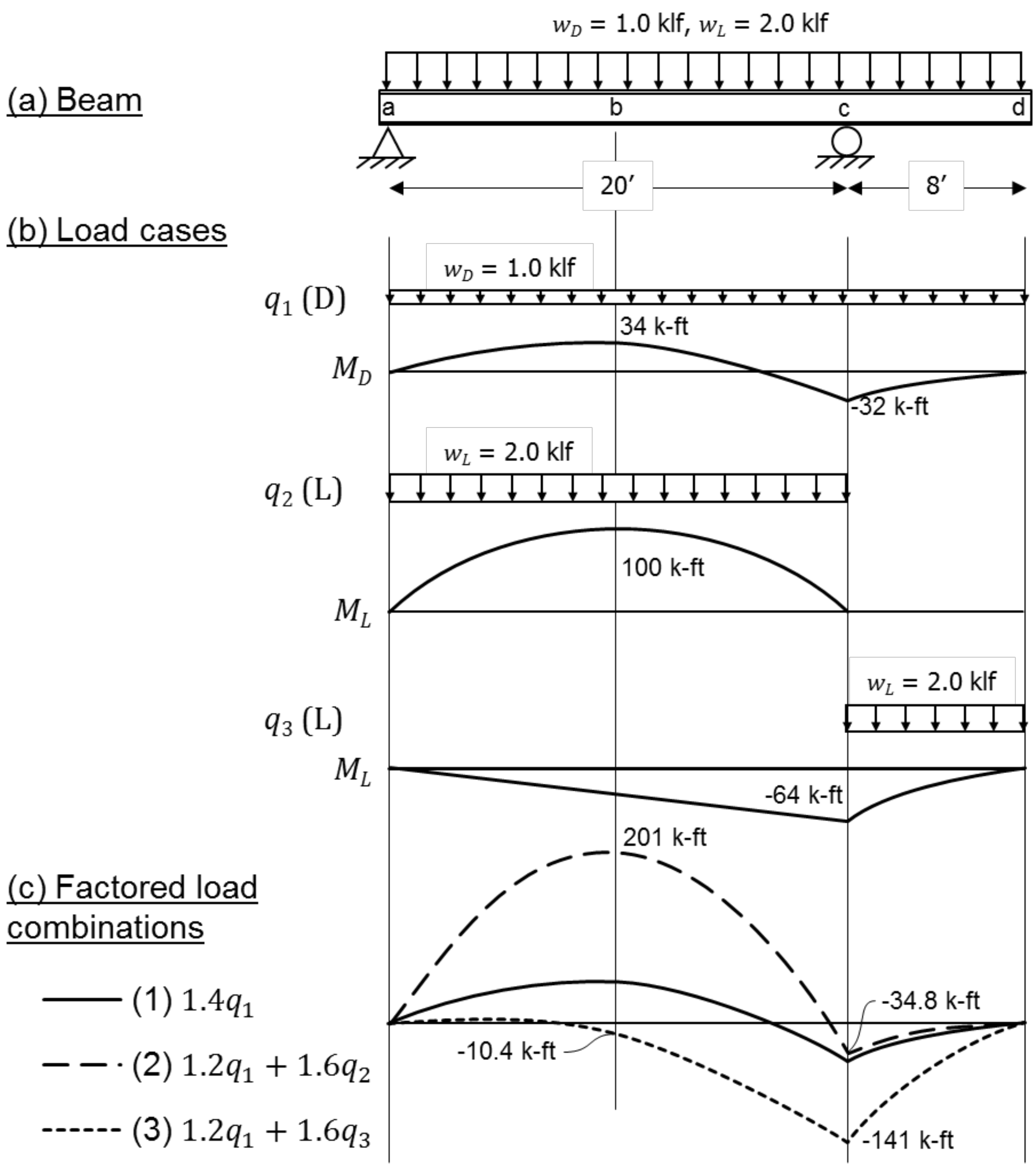
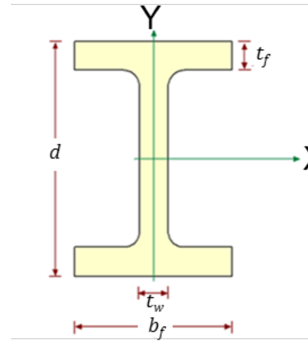



Figure 14-10 Example 4: Propped cantilever beam design.

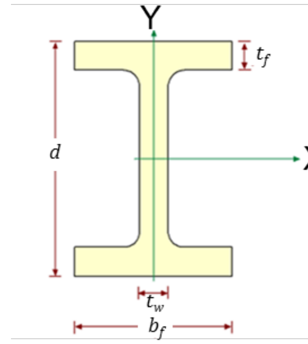
**W Shapes – Properties for designing**




W = weight in plf
A = area, in.$^2$
I in moment of inertia, in.$^4$
Z = plastic modulus, in.$^3$
S = section modulus, in.$^3$
r = radius of gyration, in.
J = torsional constant, in.$^3$
All other dimensions in inches.

| AISC_Manual_Label | $W$ | $A$ | $d$ | $b_f$ | $t_w$ | $t_f$ | $I_x$ | $Z_x$ | $S_x$ | $r_x$ | $I_y$ | $Z_y$ | $S_y$ | $r_y$ | $J$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| W33X152 | 152 | 44.9 | 33.5 | 11.6 | 0.635 | 1.06 | 8160 | 559 | 487 | 13.5 | 273 | 73.9 | 47.2 | 2.47 | 12.4 |
| W33X141 | 141 | 41.5 | 33.3 | 11.5 | 0.605 | 0.960 | 7450 | 514 | 448 | 13.4 | 246 | 66.9 | 42.7 | 2.43 | 9.70 |
| W33X130 | 130 | 38.3 | 33.1 | 11.5 | 0.580 | 0.855 | 6710 | 467 | 406 | 13.2 | 218 | 59.5 | 37.9 | 2.39 | 7.37 |
| W33X118 | 118 | 34.7 | 32.9 | 11.5 | 0.550 | 0.740 | 5900 | 415 | 359 | 13.0 | 187 | 51.3 | 32.6 | 2.32 | 5.30 |
| W30X148 | 148 | 43.6 | 30.7 | 10.5 | 0.650 | 1.18 | 6680 | 500 | 436 | 12.4 | 227 | 68.0 | 43.3 | 2.28 | 14.5 |
| W30X132 | 132 | 38.8 | 30.3 | 10.5 | 0.615 | 1.00 | 5770 | 437 | 380 | 12.2 | 196 | 58.4 | 37.2 | 2.25 | 9.72 |
| W30X124 | 124 | 36.5 | 30.2 | 10.5 | 0.585 | 0.930 | 5360 | 408 | 355 | 12.1 | 181 | 54.0 | 34.4 | 2.23 | 7.99 |
| W30X116 | 116 | 34.2 | 30.0 | 10.5 | 0.565 | 0.850 | 4930 | 378 | 329 | 12.0 | 164 | 49.2 | 31.3 | 2.19 | 6.43 |
| W30X108 | 108 | 31.7 | 29.8 | 10.5 | 0.545 | 0.760 | 4470 | 346 | 299 | 11.9 | 146 | 43.9 | 27.9 | 2.15 | 4.99 |
| W30X99 | 99.0 | 29.0 | 29.7 | 10.5 | 0.520 | 0.670 | 3990 | 312 | 269 | 11.7 | 128 | 38.6 | 24.5 | 2.10 | 3.77 |
| W30X90 | 90.0 | 26.3 | 29.5 | 10.4 | 0.470 | 0.610 | 3610 | 283 | 245 | 11.7 | 115 | 34.7 | 22.1 | 2.09 | 2.84 |
| W27X178 | 178 | 52.5 | 27.8 | 14.1 | 0.725 | 1.19 | 7020 | 570 | 505 | 11.6 | 555 | 122 | 78.8 | 3.25 | 20.1 |
| W27X161 | 161 | 47.6 | 27.6 | 14.0 | 0.660 | 1.08 | 6310 | 515 | 458 | 11.5 | 497 | 109 | 70.9 | 3.23 | 15.1 |
| W27X146 | 146 | 43.2 | 27.4 | 14.0 | 0.605 | 0.975 | 5660 | 464 | 414 | 11.5 | 443 | 97.7 | 63.5 | 3.20 | 11.3 |
| W27X129 | 129 | 37.8 | 27.6 | 10.0 | 0.610 | 1.10 | 4760 | 395 | 345 | 11.2 | 184 | 57.6 | 36.8 | 2.21 | 11.1 |
| W27X114 | 114 | 33.6 | 27.3 | 10.1 | 0.570 | 0.930 | 4080 | 343 | 299 | 11.0 | 159 | 49.3 | 31.5 | 2.18 | 7.33 |
| W27X102 | 102 | 30.0 | 27.1 | 10.0 | 0.515 | 0.830 | 3620 | 305 | 267 | 11.0 | 139 | 43.4 | 27.8 | 2.15 | 5.28 |
| W27X94 | 94.0 | 27.6 | 26.9 | 10.0 | 0.490 | 0.745 | 3270 | 278 | 243 | 10.9 | 124 | 38.8 | 24.8 | 2.12 | 4.03 |
| W27X84 | 84.0 | 24.7 | 26.7 | 10.0 | 0.460 | 0.640 | 2850 | 244 | 213 | 10.7 | 106 | 33.2 | 21.2 | 2.07 | 2.81 |
| W24X162 | 162 | 47.8 | 25.0 | 13.0 | 0.705 | 1.22 | 5170 | 468 | 414 | 10.4 | 443 | 105 | 68.4 | 3.05 | 18.5 |
| W24X146 | 146 | 43.0 | 24.7 | 12.9 | 0.650 | 1.09 | 4580 | 418 | 371 | 10.3 | 391 | 93.2 | 60.5 | 3.01 | 13.4 |
| W24X131 | 131 | 38.6 | 24.5 | 12.9 | 0.605 | 0.960 | 4020 | 370 | 329 | 10.2 | 340 | 81.5 | 53.0 | 2.97 | 9.50 |
| W24X117 | 117 | 34.4 | 24.3 | 12.8 | 0.550 | 0.850 | 3540 | 327 | 291 | 10.1 | 297 | 71.4 | 46.5 | 2.94 | 6.72 |
| W24X104 | 104 | 30.7 | 24.1 | 12.8 | 0.500 | 0.750 | 3100 | 289 | 258 | 10.1 | 259 | 62.4 | 40.7 | 2.91 | 4.72 |
| W24X103 | 103 | 30.3 | 24.5 | 9.00 | 0.550 | 0.980 | 3000 | 280 | 245 | 10.0 | 119 | 41.5 | 26.5 | 1.99 | 7.07 |
| W24X94 | 94.0 | 27.7 | 24.3 | 9.07 | 0.515 | 0.875 | 2700 | 254 | 222 | 9.87 | 109 | 37.5 | 24.0 | 1.98 | 5.26 |
| W24X84 | 84.0 | 24.7 | 24.1 | 9.02 | 0.470 | 0.770 | 2370 | 224 | 196 | 9.79 | 94.4 | 32.6 | 20.9 | 1.95 | 3.70 |
| W24X76 | 76.0 | 22.4 | 23.9 | 8.99 | 0.440 | 0.680 | 2100 | 200 | 176 | 9.69 | 82.5 | 28.6 | 18.4 | 1.92 | 2.68 |
| W24X68 | 68.0 | 20.1 | 23.7 | 8.97 | 0.415 | 0.585 | 1830 | 177 | 154 | 9.55 | 70.4 | 24.5 | 15.7 | 1.87 | 1.87 |
| W24X62 | 62.0 | 18.2 | 23.7 | 7.04 | 0.430 | 0.590 | 1550 | 153 | 131 | 9.23 | 34.5 | 15.7 | 9.80 | 1.38 | 1.71 |
| W24X55 | 55.0 | 16.2 | 23.6 | 7.01 | 0.395 | 0.505 | 1350 | 134 | 114 | 9.11 | 29.1 | 13.3 | 8.30 | 1.34 | 1.18 |
| W21X147 | 147 | 43.2 | 22.1 | 12.5 | 0.720 | 1.15 | 3630 | 373 | 329 | 9.17 | 376 | 92.6 | 60.1 | 2.95 | 15.4 |
| W21X132 | 132 | 38.8 | 21.8 | 12.4 | 0.650 | 1.04 | 3220 | 333 | 295 | 9.12 | 333 | 82.3 | 53.5 | 2.93 | 11.3 |
| W21X122 | 122 | 35.9 | 21.7 | 12.4 | 0.600 | 0.960 | 2960 | 307 | 273 | 9.09 | 305 | 75.6 | 49.2 | 2.92 | 8.98 |
| W21X111 | 111 | 32.6 | 21.5 | 12.3 | 0.550 | 0.875 | 2670 | 279 | 249 | 9.05 | 274 | 68.2 | 44.5 | 2.90 | 6.83 |
| W21X101 | 101 | 29.8 | 21.4 | 12.3 | 0.500 | 0.800 | 2420 | 253 | 227 | 9.02 | 248 | 61.7 | 40.3 | 2.89 | 5.21 |
| W21X93 | 93.0 | 27.3 | 21.6 | 8.42 | 0.580 | 0.930 | 2070 | 221 | 192 | 8.70 | 92.9 | 34.7 | 22.1 | 1.84 | 6.03 |

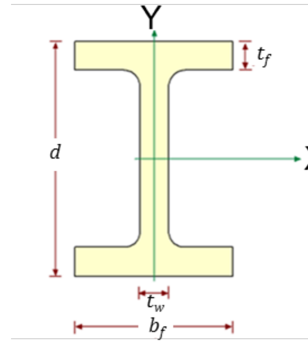
**W Shapes – Properties for designing**




W = weight in plf
A = area, in.$^2$
I in moment of inertia, in.$^4$
Z = plastic modulus, in.$^3$
S = section modulus, in.$^3$
r = radius of gyration, in.
J = torsional constant, in.$^3$
All other dimensions in inches.

| AISC_Manual_Label | *W* | *A* | *d* | $b_f$ | $t_w$ | $t_f$ | $I_x$ | $Z_x$ | $S_x$ | $r_x$ | $I_y$ | $Z_y$ | $S_y$ | $r_y$ | *J* |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| W21X83 | 83.0 | 24.4 | 21.4 | 8.36 | 0.515 | 0.835 | 1830 | 196 | 171 | 8.67 | 81.4 | 30.5 | 19.5 | 1.83 | 4.34 |
| W21X73 | 73.0 | 21.5 | 21.2 | 8.30 | 0.455 | 0.740 | 1600 | 172 | 151 | 8.64 | 70.6 | 26.6 | 17.0 | 1.81 | 3.02 |
| W21X68 | 68.0 | 20.0 | 21.1 | 8.27 | 0.430 | 0.685 | 1480 | 160 | 140 | 8.60 | 64.7 | 24.4 | 15.7 | 1.80 | 2.45 |
| W21X62 | 62.0 | 18.3 | 21.0 | 8.24 | 0.400 | 0.615 | 1330 | 144 | 127 | 8.54 | 57.5 | 21.7 | 14.0 | 1.77 | 1.83 |
| W21X55 | 55.0 | 16.2 | 20.8 | 8.22 | 0.375 | 0.522 | 1140 | 126 | 110 | 8.40 | 48.4 | 18.4 | 11.8 | 1.73 | 1.24 |
| W21X48 | 48.0 | 14.1 | 20.6 | 8.14 | 0.350 | 0.430 | 959 | 107 | 93.0 | 8.24 | 38.7 | 14.9 | 9.52 | 1.66 | 0.803 |
| W21X57 | 57.0 | 16.7 | 21.1 | 6.56 | 0.405 | 0.650 | 1170 | 129 | 111 | 8.36 | 30.6 | 14.8 | 9.35 | 1.35 | 1.77 |
| W21X50 | 50.0 | 14.7 | 20.8 | 6.53 | 0.380 | 0.535 | 984 | 110 | 94.5 | 8.18 | 24.9 | 12.2 | 7.64 | 1.30 | 1.14 |
| W21X44 | 44.0 | 13.0 | 20.7 | 6.50 | 0.350 | 0.450 | 843 | 95.4 | 81.6 | 8.06 | 20.7 | 10.2 | 6.37 | 1.26 | 0.770 |
| | | | | | | | | | | | | | | | |
| W18X119 | 119 | 35.1 | 19.0 | 11.3 | 0.655 | 1.06 | 2190 | 262 | 231 | 7.90 | 253 | 69.1 | 44.9 | 2.69 | 10.6 |
| W18X106 | 106 | 31.1 | 18.7 | 11.2 | 0.590 | 0.940 | 1910 | 230 | 204 | 7.84 | 220 | 60.5 | 39.4 | 2.66 | 7.48 |
| W18X97 | 97.0 | 28.5 | 18.6 | 11.1 | 0.535 | 0.870 | 1750 | 211 | 188 | 7.82 | 201 | 55.3 | 36.1 | 2.65 | 5.86 |
| W18X86 | 86.0 | 25.3 | 18.4 | 11.1 | 0.480 | 0.770 | 1530 | 186 | 166 | 7.77 | 175 | 48.4 | 31.6 | 2.63 | 4.10 |
| W18X76 | 76.0 | 22.3 | 18.2 | 11.0 | 0.425 | 0.680 | 1330 | 163 | 146 | 7.73 | 152 | 42.2 | 27.6 | 2.61 | 2.83 |
| W18X71 | 71.0 | 20.9 | 18.5 | 7.64 | 0.495 | 0.810 | 1170 | 146 | 127 | 7.50 | 60.3 | 24.7 | 15.8 | 1.70 | 3.49 |
| W18X65 | 65.0 | 19.1 | 18.4 | 7.59 | 0.450 | 0.750 | 1070 | 133 | 117 | 7.49 | 54.8 | 22.5 | 14.4 | 1.69 | 2.73 |
| W18X60 | 60.0 | 17.6 | 18.2 | 7.56 | 0.415 | 0.695 | 984 | 123 | 108 | 7.47 | 50.1 | 20.6 | 13.3 | 1.68 | 2.17 |
| W18X55 | 55.0 | 16.2 | 18.1 | 7.53 | 0.390 | 0.630 | 890 | 112 | 98.3 | 7.41 | 44.9 | 18.5 | 11.9 | 1.67 | 1.66 |
| W18X50 | 50.0 | 14.7 | 18.0 | 7.50 | 0.355 | 0.570 | 800 | 101 | 88.9 | 7.38 | 40.1 | 16.6 | 10.7 | 1.65 | 1.24 |
| W18X46 | 46.0 | 13.5 | 18.1 | 6.06 | 0.360 | 0.605 | 712 | 90.7 | 78.8 | 7.25 | 22.5 | 11.7 | 7.43 | 1.29 | 1.22 |
| W18X40 | 40.0 | 11.8 | 17.9 | 6.02 | 0.315 | 0.525 | 612 | 78.4 | 68.4 | 7.21 | 19.1 | 10.0 | 6.35 | 1.27 | 0.810 |
| W18X35 | 35.0 | 10.3 | 17.7 | 6.00 | 0.300 | 0.425 | 510 | 66.5 | 57.6 | 7.04 | 15.3 | 8.06 | 5.12 | 1.22 | 0.506 |
| | | | | | | | | | | | | | | | |
| W16X100 | 100 | 29.4 | 17.0 | 10.4 | 0.585 | 0.985 | 1490 | 198 | 175 | 7.10 | 186 | 54.9 | 35.7 | 2.51 | 7.73 |
| W16X89 | 89.0 | 26.2 | 16.8 | 10.4 | 0.525 | 0.875 | 1300 | 175 | 155 | 7.05 | 163 | 48.1 | 31.4 | 2.49 | 5.45 |
| W16X77 | 77.0 | 22.6 | 16.5 | 10.3 | 0.455 | 0.760 | 1110 | 150 | 134 | 7.00 | 138 | 41.1 | 26.9 | 2.47 | 3.57 |
| W16X67 | 67.0 | 19.6 | 16.3 | 10.2 | 0.395 | 0.665 | 954 | 130 | 117 | 6.96 | 119 | 35.5 | 23.2 | 2.46 | 2.39 |
| W16X57 | 57.0 | 16.8 | 16.4 | 7.12 | 0.430 | 0.715 | 758 | 105 | 92.2 | 6.72 | 43.1 | 18.9 | 12.1 | 1.60 | 2.22 |
| W16X50 | 50.0 | 14.7 | 16.3 | 7.07 | 0.380 | 0.630 | 659 | 92.0 | 81.0 | 6.68 | 37.2 | 16.3 | 10.5 | 1.59 | 1.52 |
| W16X45 | 45.0 | 13.3 | 16.1 | 7.04 | 0.345 | 0.565 | 586 | 82.3 | 72.7 | 6.65 | 32.8 | 14.5 | 9.34 | 1.57 | 1.11 |
| W16X40 | 40.0 | 11.8 | 16.0 | 7.00 | 0.305 | 0.505 | 518 | 73.0 | 64.7 | 6.63 | 28.9 | 12.7 | 8.25 | 1.57 | 0.794 |
| W16X36 | 36.0 | 10.6 | 15.9 | 6.99 | 0.295 | 0.430 | 448 | 64.0 | 56.5 | 6.51 | 24.5 | 10.8 | 7.00 | 1.52 | 0.545 |
| W16X31 | 31.0 | 9.13 | 15.9 | 5.53 | 0.275 | 0.440 | 375 | 54.0 | 47.2 | 6.41 | 12.4 | 7.03 | 4.49 | 1.17 | 0.461 |
| W16X26 | 26.0 | 7.68 | 15.7 | 5.50 | 0.250 | 0.345 | 301 | 44.2 | 38.4 | 6.26 | 9.59 | 5.48 | 3.49 | 1.12 | 0.262 |
| | | | | | | | | | | | | | | | |
| W14X132 | 132 | 38.8 | 14.7 | 14.7 | 0.645 | 1.03 | 1530 | 234 | 209 | 6.28 | 548 | 113 | 74.5 | 3.76 | 12.3 |
| W14X120 | 120 | 35.3 | 14.5 | 14.7 | 0.590 | 0.940 | 1380 | 212 | 190 | 6.24 | 495 | 102 | 67.5 | 3.74 | 9.37 |
| W14X109 | 109 | 32.0 | 14.3 | 14.6 | 0.525 | 0.860 | 1240 | 192 | 173 | 6.22 | 447 | 92.7 | 61.2 | 3.73 | 7.12 |
| W14X99 | 99.0 | 29.1 | 14.2 | 14.6 | 0.485 | 0.780 | 1110 | 173 | 157 | 6.17 | 402 | 83.6 | 55.2 | 3.71 | 5.37 |
| W14X90 | 90.0 | 26.5 | 14.0 | 14.5 | 0.440 | 0.710 | 999 | 157 | 143 | 6.14 | 362 | 75.6 | 49.9 | 3.70 | 4.06 |

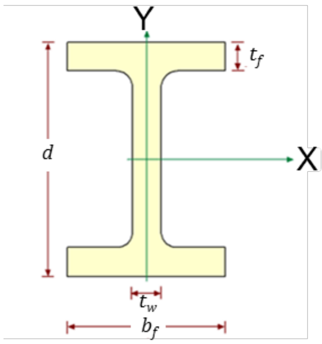
**W Shapes – Properties for designing**




W = weight in plf
A = area, in.$^2$
I in moment of inertia, in.$^4$
Z = plastic modulus, in.$^3$
S = section modulus, in.$^3$
r = radius of gyration, in.
J = torsional constant, in.$^3$
All other dimensions in inches.

| AISC_Manual_Label | W | A | d | $b_f$ | $t_w$ | $t_f$ | $I_x$ | $Z_x$ | $S_x$ | $r_x$ | $I_y$ | $Z_y$ | $S_y$ | $r_y$ | J |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| W14X82 | 82.0 | 24.0 | 14.3 | 10.1 | 0.510 | 0.855 | 881 | 139 | 123 | 6.05 | 148 | 44.8 | 29.3 | 2.48 | 5.07 |
| W14X74 | 74.0 | 21.8 | 14.2 | 10.1 | 0.450 | 0.785 | 795 | 126 | 112 | 6.04 | 134 | 40.5 | 26.6 | 2.48 | 3.87 |
| W14X68 | 68.0 | 20.0 | 14.0 | 10.0 | 0.415 | 0.720 | 722 | 115 | 103 | 6.01 | 121 | 36.9 | 24.2 | 2.46 | 3.01 |
| W14X61 | 61.0 | 17.9 | 13.9 | 10.0 | 0.375 | 0.645 | 640 | 102 | 92.1 | 5.98 | 107 | 32.8 | 21.5 | 2.45 | 2.19 |
| W14X53 | 53.0 | 15.6 | 13.9 | 8.06 | 0.370 | 0.660 | 541 | 87.1 | 77.8 | 5.89 | 57.7 | 22.0 | 14.3 | 1.92 | 1.94 |
| W14X48 | 48.0 | 14.1 | 13.8 | 8.03 | 0.340 | 0.595 | 484 | 78.4 | 70.2 | 5.85 | 51.4 | 19.6 | 12.8 | 1.91 | 1.45 |
| W14X43 | 43.0 | 12.6 | 13.7 | 8.00 | 0.305 | 0.530 | 428 | 69.6 | 62.6 | 5.82 | 45.2 | 17.3 | 11.3 | 1.89 | 1.05 |
| W14X38 | 38.0 | 11.2 | 14.1 | 6.77 | 0.310 | 0.515 | 385 | 61.5 | 54.6 | 5.87 | 26.7 | 12.1 | 7.88 | 1.55 | 0.798 |
| W14X34 | 34.0 | 10.0 | 14.0 | 6.75 | 0.285 | 0.455 | 340 | 54.6 | 48.6 | 5.83 | 23.3 | 10.6 | 6.91 | 1.53 | 0.569 |
| W14X30 | 30.0 | 8.85 | 13.8 | 6.73 | 0.270 | 0.385 | 291 | 47.3 | 42.0 | 5.73 | 19.6 | 8.99 | 5.82 | 1.49 | 0.380 |
| W14X26 | 26.0 | 7.69 | 13.9 | 5.03 | 0.255 | 0.420 | 245 | 40.2 | 35.3 | 5.65 | 8.91 | 5.54 | 3.55 | 1.08 | 0.358 |
| W14X22 | 22.0 | 6.49 | 13.7 | 5.00 | 0.230 | 0.335 | 199 | 33.2 | 29.0 | 5.54 | 7.00 | 4.39 | 2.80 | 1.04 | 0.208 |
| W12X336 | 336 | 98.9 | 16.8 | 13.4 | 1.78 | 2.96 | 4060 | 603 | 483 | 6.41 | 1190 | 274 | 177 | 3.47 | 243 |
| W12X305 | 305 | 89.5 | 16.3 | 13.2 | 1.63 | 2.71 | 3550 | 537 | 435 | 6.29 | 1050 | 244 | 159 | 3.42 | 185 |
| W12X279 | 279 | 81.9 | 15.9 | 13.1 | 1.53 | 2.47 | 3110 | 481 | 393 | 6.16 | 937 | 220 | 143 | 3.38 | 143 |
| W12X252 | 252 | 74.1 | 15.4 | 13.0 | 1.40 | 2.25 | 2720 | 428 | 353 | 6.06 | 828 | 196 | 127 | 3.34 | 108 |
| W12X230 | 230 | 67.7 | 15.1 | 12.9 | 1.29 | 2.07 | 2420 | 386 | 321 | 5.97 | 742 | 177 | 115 | 3.31 | 83.8 |
| W12X210 | 210 | 61.8 | 14.7 | 12.8 | 1.18 | 1.90 | 2140 | 348 | 292 | 5.89 | 664 | 159 | 104 | 3.28 | 64.7 |
| W12X190 | 190 | 56.0 | 14.4 | 12.7 | 1.06 | 1.74 | 1890 | 311 | 263 | 5.82 | 589 | 143 | 93.0 | 3.25 | 48.8 |
| W12X170 | 170 | 50.0 | 14.0 | 12.6 | 0.960 | 1.56 | 1650 | 275 | 235 | 5.74 | 517 | 126 | 82.3 | 3.22 | 35.6 |
| W12X152 | 152 | 44.7 | 13.7 | 12.5 | 0.870 | 1.40 | 1430 | 243 | 209 | 5.66 | 454 | 111 | 72.8 | 3.19 | 25.8 |
| W12X136 | 136 | 39.9 | 13.4 | 12.4 | 0.790 | 1.25 | 1240 | 214 | 186 | 5.58 | 398 | 98.0 | 64.2 | 3.16 | 18.5 |
| W12X120 | 120 | 35.2 | 13.1 | 12.3 | 0.710 | 1.11 | 1070 | 186 | 163 | 5.51 | 345 | 85.4 | 56.0 | 3.13 | 12.9 |
| W12X106 | 106 | 31.2 | 12.9 | 12.2 | 0.610 | 0.990 | 933 | 164 | 145 | 5.47 | 301 | 75.1 | 49.3 | 3.11 | 9.13 |
| W12X96 | 96.0 | 28.2 | 12.7 | 12.2 | 0.550 | 0.900 | 833 | 147 | 131 | 5.44 | 270 | 67.5 | 44.4 | 3.09 | 6.85 |
| W12X87 | 87.0 | 25.6 | 12.5 | 12.1 | 0.515 | 0.810 | 740 | 132 | 118 | 5.38 | 241 | 60.4 | 39.7 | 3.07 | 5.10 |
| W12X79 | 79.0 | 23.2 | 12.4 | 12.1 | 0.470 | 0.735 | 662 | 119 | 107 | 5.34 | 216 | 54.3 | 35.8 | 3.05 | 3.84 |
| W12X72 | 72.0 | 21.1 | 12.3 | 12.0 | 0.430 | 0.670 | 597 | 108 | 97.4 | 5.31 | 195 | 49.2 | 32.4 | 3.04 | 2.93 |
| W12X65 | 65.0 | 19.1 | 12.1 | 12.0 | 0.390 | 0.605 | 533 | 96.8 | 87.9 | 5.28 | 174 | 44.1 | 29.1 | 3.02 | 2.18 |
| W12X58 | 58.0 | 17.0 | 12.2 | 10.0 | 0.360 | 0.640 | 475 | 86.4 | 78.0 | 5.28 | 107 | 32.5 | 21.4 | 2.51 | 2.10 |
| W12X53 | 53.0 | 15.6 | 12.1 | 10.0 | 0.345 | 0.575 | 425 | 77.9 | 70.6 | 5.23 | 95.8 | 29.1 | 19.2 | 2.48 | 1.58 |
| W12X50 | 50.0 | 14.6 | 12.2 | 8.08 | 0.370 | 0.640 | 391 | 71.9 | 64.2 | 5.18 | 56.3 | 21.3 | 13.9 | 1.96 | 1.71 |
| W12X45 | 45.0 | 13.1 | 12.1 | 8.05 | 0.335 | 0.575 | 348 | 64.2 | 57.7 | 5.15 | 50.0 | 19.0 | 12.4 | 1.95 | 1.26 |
| W12X40 | 40.0 | 11.7 | 11.9 | 8.01 | 0.295 | 0.515 | 307 | 57.0 | 51.5 | 5.13 | 44.1 | 16.8 | 11.0 | 1.94 | 0.906 |
| W12X35 | 35.0 | 10.3 | 12.5 | 6.56 | 0.300 | 0.520 | 285 | 51.2 | 45.6 | 5.25 | 24.5 | 11.5 | 7.47 | 1.54 | 0.741 |
| W12X30 | 30.0 | 8.79 | 12.3 | 6.52 | 0.260 | 0.440 | 238 | 43.1 | 38.6 | 5.21 | 20.3 | 9.56 | 6.24 | 1.52 | 0.457 |
| W12X26 | 26.0 | 7.65 | 12.2 | 6.49 | 0.230 | 0.380 | 204 | 37.2 | 33.4 | 5.17 | 17.3 | 8.17 | 5.34 | 1.51 | 0.300 |
| W12X22 | 22.0 | 6.48 | 12.3 | 4.03 | 0.260 | 0.425 | 156 | 29.3 | 25.4 | 4.91 | 4.66 | 3.66 | 2.31 | 0.848 | 0.293 |
| W12X19 | 19.0 | 5.57 | 12.2 | 4.01 | 0.235 | 0.350 | 130 | 24.7 | 21.3 | 4.82 | 3.76 | 2.98 | 1.88 | 0.822 | 0.180 |
| W12X16 | 16.0 | 4.71 | 12.0 | 3.99 | 0.220 | 0.265 | 103 | 20.1 | 17.1 | 4.67 | 2.82 | 2.26 | 1.41 | 0.773 | 0.103 |

**W Shapes – Properties for designing**




W = weight in plf
A = area, in.$^2$
I in moment of inertia, in.$^4$
Z = plastic modulus, in.$^3$
S = section modulus, in.$^3$
r = radius of gyration, in.
J = torsional constant, in.$^3$
All other dimensions in inches.

| AISC_Manual_Label | *W* | *A* | *d* | $b_f$ | $t_w$ | $t_f$ | $I_x$ | $Z_x$ | $S_x$ | $r_x$ | $I_y$ | $Z_y$ | $S_y$ | $r_y$ | *J* |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| W12X14 | 14.0 | 4.16 | 11.9 | 3.97 | 0.200 | 0.225 | 88.6 | 17.4 | 14.9 | 4.62 | 2.36 | 1.90 | 1.19 | 0.753 | 0.0704 |
| | | | | | | | | | | | | | | | |
| W10X45 | 45.0 | 13.3 | 10.1 | 8.02 | 0.350 | 0.620 | 248 | 54.9 | 49.1 | 4.32 | 53.4 | 20.3 | 13.3 | 2.01 | 1.51 |
| W10X39 | 39.0 | 11.5 | 9.92 | 7.99 | 0.315 | 0.530 | 209 | 46.8 | 42.1 | 4.27 | 45.0 | 17.2 | 11.3 | 1.98 | 0.976 |
| W10X33 | 33.0 | 9.71 | 9.73 | 7.96 | 0.290 | 0.435 | 171 | 38.8 | 35.0 | 4.19 | 36.6 | 14.0 | 9.20 | 1.94 | 0.583 |
| W10X30 | 30.0 | 8.84 | 10.5 | 5.81 | 0.300 | 0.510 | 170 | 36.6 | 32.4 | 4.38 | 16.7 | 8.84 | 5.75 | 1.37 | 0.622 |
| W10X26 | 26.0 | 7.61 | 10.3 | 5.77 | 0.260 | 0.440 | 144 | 31.3 | 27.9 | 4.35 | 14.1 | 7.50 | 4.89 | 1.36 | 0.402 |
| W10X22 | 22.0 | 6.49 | 10.2 | 5.75 | 0.240 | 0.360 | 118 | 26.0 | 23.2 | 4.27 | 11.4 | 6.10 | 3.97 | 1.33 | 0.239 |

# 15. Design of Reinforced Concrete Structures

## 15.1. INTRODUCTION

This chapter introduces reinforced concrete as a structural material and describes design of simple elements of reinforced concrete construction. Reinforced concrete design is almost always carried out using the Load and Resistance Factor Design (LRFD) method. However, in the terminology of the American Concrete Institute Building Code, it is known as the *strength design method*. The design provisions are covered in detail in the *Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary*. At UC Berkeley, CE 123 provides more in-depth treatment of the design of reinforced concrete structures.

## 15.2. REINFORCED CONCRETE CONSTRUCTION

Reinforced concrete refers to construction made of concrete that is reinforced with (usually) steel reinforcement. Concrete is very efficient in resisting compressive stress but less effective in resisting tensile stress. By appropriate proportioning and placement of steel reinforcement within concrete, the reinforcement can provide the required tensile resistance. Steel can also resist compression and can control cracking in concrete construction. Concrete cover over the reinforcement provides protection against fire and corrosive agents.

Reinforced concrete is widely used in building and infrastructure projects. Figure 15-1 shows some examples.

(a) Mid-rise concrete building, UC Berkeley campus. (Image courtesy of Studios Architecture)

(b) High-rise moment frame-wall building in Pacific Northwest of United States. (Photograph courtesy of Cary Kopczynski)


(c) Hoover Dam and Hoover Dam Bypass (http://www.atkinsglobal.com/en-gb/projects/hoover-dam-bypass)

Figure 15-1 Reinforced concrete construction examples.

Reinforced concrete can be either *cast-in-place* or *precast*. In cast-in-place construction, the reinforcement is supported in formwork and concrete is cast to form the members in their final position (Figure 15-2). In precast construction, individual members are cast in a precast yard, shipped to the site, and connected together to complete the structure (Figure 15-3).

Figure 15-2 Casting concrete in a reinforced concrete building.

Figure 15-3 Construction of the San Francisco Oakland Bay Bridge Skyway. Note the segmental precast construction, in which precast segments are hoisted into position, then prestressed and grouted in place. (http://www.hatchertechnical.com/)

Whether cast-in-place or precast, the members can be either prestressed or non-prestressed. In *non-prestressed construction*, the reinforcing steel is held in place in the forms with the concrete cast around the reinforcement, without any prestressing of the reinforcement. In this case, the reinforcement experiences stress only under application of external load (or volume change due to effects of shrinkage and temperature change). In *prestressed concrete*, the reinforcement is stressed before the application of external load. This can be done in one of two ways. In *pre-tensioned concrete*, the reinforcement is prestressed in tension (usually in precasting beds at a precasting plant), the concrete is cast, and then the reinforcement is released. Because the reinforcement is bonded to the concrete, it transfers compressive force to the concrete, thereby prestressing it. In *post-tensioned concrete*, the prestressing steel is placed in greased conduits, the concrete is cast, and after the concrete has hardened the prestressing steel is stressed in tension and held in tension through anchorages within the concrete.

Figure 15-4 illustrates some of the differences in behavior of non-prestressed and prestressed concrete. If the non-prestressed beam is cast without camber, then under load the resulting moments will induce curvature, leading to deflections and possibly concrete cracking. In contrast, the prestressed beam develops an initial upward deflection due to the eccentrically placed prestressing steel. External load will produce curvature in the opposite direction. In the ideal design, the prestressing is designed to exactly counter the deflection under load, such that there is no deflection in the final design. In this condition, there also is no tensile strain or cracking in the concrete. Of course, the non-prestressed beam can be built with an initial camber to achieve the same result of zero deflection under design load, but tensile strain and possible cracking are still likely. Apart from this simple introduction, prestressed concrete is beyond the scope of CE 120 and this Reader.

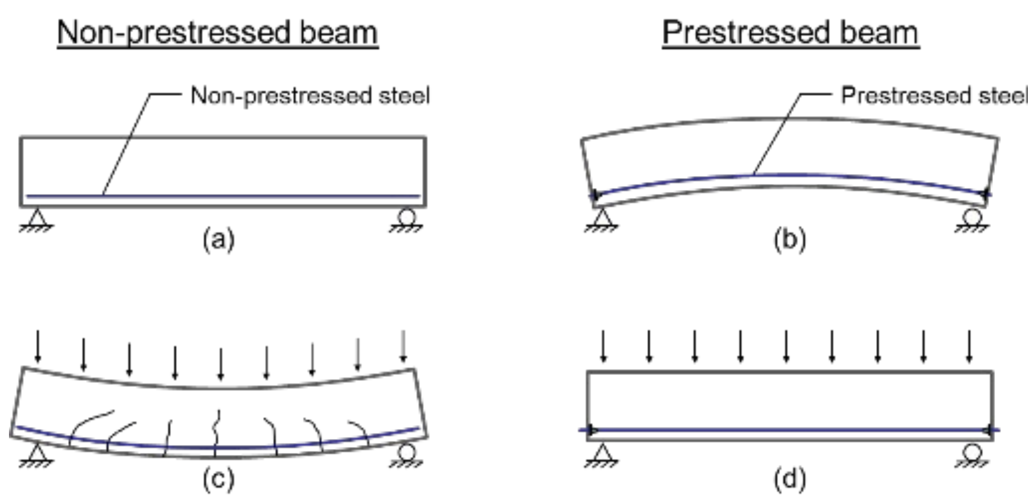



Figure 15-4 Non-prestressed and prestressed beams.

## 15.3. MECHANICAL PROPERTIES OF CONCRETE AND REINFORCEMENT

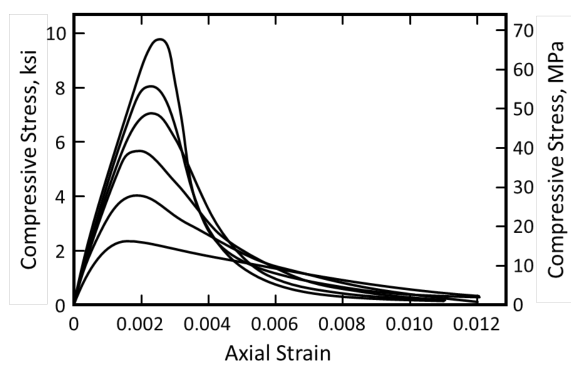
### 15.3.1. Concrete

Concrete is a composite material consisting mainly of aggregates held together by a binding agent. Aggregate usually includes fine aggregates (sand) and course aggregates (gravel or crushed stone). The binding agent is usually portland cement. Through addition of controlled amounts of mixing water, portland cement gains an adhesive characteristic. In addition to aggregates, cement, and water, modern concretes usually have admixtures, which are added before or during mixing. Some admixtures can improve workability, thereby reducing required water and improving potential strength. Others can modify setting and hardening characteristics of the plastic concrete and can improve thermal and freeze-thaw cracking resistance.

Concrete compressive strength is the property most commonly specified by structural engineers. The specified strength is generally given the designation $f_c'$. Unless another age is indicated in the design drawings or specifications, the specified strength generally refers to the 28-day compressive strength, which can be checked by conducting a standard uniaxial compression test on concrete cylinders.

Figure 15-5 shows a standard cylinder test and measured stress-strain relations for normalweight aggregate concrete samples of various strengths. In some markets such as the San Francisco Bay Area, the maximum compressive strength that can be obtained using local aggregates is around 10,000 psi (70 MPa). In other markets (for example, Seattle and Chicago) superior aggregate qualities enable compressive strengths of around 20 ksi (140 MPa).

(a)




(b)

Figure 15-5 (a) Uniaxial compression test on 6 x 12 in. (150 x 300 mm) cylinder at UC Berkeley laboratories (photo courtesy of L. Stepanov). (b) Stress-strain relations of normalweight concretes under uniaxial compressive loading (after Wischers, 1979, as reported by ACI 363R-92, 1992)

Although the stress-strain relation for concrete is nonlinear even at low stress levels, we commonly assume it to be linear for stresses up to around $0.5f_c'$. The commonly accepted definition of concrete modulus is a chord modulus from the stress-strain point at 50 microstrain to the stress-strain point at $0.4f_c'$ (ASTM C469). ACI 318 uses Eq. (15-1) as an empirical estimate of the concrete modulus.

$$E_c = 33 w_c^{1.5} \sqrt{f_c'} \text{, psi}$$
$$E_c = 0.043 w_c^{1.5} \sqrt{f_c'} \text{, MPa} \tag{15-1}$$

in which $w_c$ = density of concrete in lb/ft$^3$ (kg/m$^3$). For normalweight aggregate, this can be simplified to

$$E_c = 57{,}000 \sqrt{f_c'} \text{, psi}$$
$$E_c = 4700 \sqrt{f_c'} \text{, MPa} \tag{15-2}$$

Note that these empirical equations define modulus (units of force per area) in terms of $\sqrt{f_c'}$, which has units of square root of force per area). Therefore, the constant must also have units of square root of force per area. Consequently, these equations only work using either psi or MPa units for $f_c'$, with the result that $E_c$ has corresponding units of psi or MPa.

Concrete density for concrete with normal-weight aggregate is around 145 pcf. The density of reinforced concrete is somewhat higher. Standard practice is to take the *density of reinforced concrete at 150 pcf*. Light-weight concretes are achieved by using light-weight aggregates, allowing densities for structural lightweight concrete around 120 pcf.

### 15.3.2. Steel

Steels used in reinforced concrete include bars and wires for nonprestressed applications, and strands, wires, and bars for prestressed applications. In this reader, we consider only bars for nonprestressed applications.

Standard, nonprestressed bars are produced in standard sizes. Table 15-1 lists the standard sizes used in the United States. The bars can be identified in one of two ways, either using U.S. customary inch-lb units or metric units. In the U.S. customary unit system, the bar size number is approximately equal to the nominal diameter in eighths of an inch. The same bar can also be identified by its nominal diameter in mm. Thus, a No. 3 bar (3/8-inch diameter) in the U.S. customary system is the same as a No. 10 bar (10-mm diameter) in the metric system.

Table 15-1 ASTM standard reinforcement bar sizes (ASTM A615)

| Bar size, no. U.S. (metric) | Nominal diameter, in. (mm) | Nominal area, in.$^2$ (mm$^2$) |
|---|---|---|
| 3 (10) | 0.375 (9.5) | 0.11 (71) |
| 4 (13) | 0.500 (12.7) | 0.20 (129) |
| 5 (16) | 0.625 (15.9) | 0.31 (199) |
| 6 (19) | 0.750 (19.1) | 0.44 (284) |
| 7 (22) | 0.875 (22.2) | 0.60 (387) |
| 8 (25) | 1.000 (25.4) | 0.79 (510) |
| 9 (29) | 1.128 (28.7) | 1.00 (645) |
| 10 (32) | 1.270 (32.3) | 1.27 (819) |
| 11 (36) | 1.410 (35.8) | 1.56 (1006) |
| 14 (43) | 1.693 (43.0) | 2.25 (1452) |
| 18 (57) | 2.257 (57.3) | 4.00 (2581) |

ASTM standards control the deformation patterns on deformed bars to ensure appropriate bond characteristics while avoiding sharp-cornered deformations that reduce fatigue life (Figure

15-6). In addition, standard markings identify the bar characteristics. Various types of reinforcement can provide enhanced corrosion resistance in highly corrosive environments. Epoxy-coated reinforcement is one option (Figure 15-6 b and c). In the United States, green epoxy coating indicates the epoxy was applied before fabricating (cutting, bending) reinforcement. This practice ensures a uniform coating with minimal cost, but fabrication after coating can cause damage to the epoxy coating. Purple or grey epoxy coating indicates the epoxy was applied after the bars were fabricated – these bars should not be bent after coating. Other alternatives are zinc coated reinforcement, stainless steel reinforcement, and galvanized reinforcement.

(a)
(b)
(c)

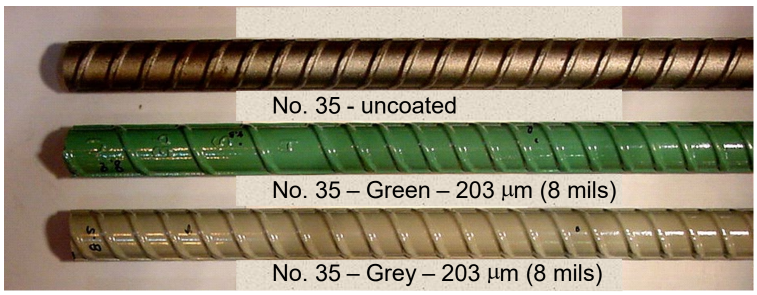



Figure 15-6 Photograph of No 11 (No. 35 metric) reinforcing bars (a) uncoated (A615), (b) green epoxy coating (ASTM A775), and (c) grey (alternately purple) epoxy coating (ASTM A934). Bar deformation patterns other than those shown are also used.

*Reinforcement grade* refers to the nominal yield strength of the reinforcement in ksi (MPa). In the United States, deformed reinforcing bars are available in Grades 40 (280), 50 (350), 60 (420), 75 (520), 80 (550), 100 (690), and 120 (830), where the number refers to the nominal yield strength in ksi (MPa). In general, only Grade 60 (420) bars are widely available. Most producers can provide 60-ft (18-m) lengths of stock without special order.

Figure 15-7 plots characteristic stress-strain relations for different types of deformed reinforcement. The initial modulus is approximately $E_s$ = 29,000 ksi (200,000 MPa). Actual yield strength tends to be higher than the specified strength, followed by strain-hardening. For design purposes, the steel is assumed to be linear to the nominal yield point, followed by plastic yielding without strain-hardening. The design relation for Grade 60 reinforcement is shown as the broken line in the figure.

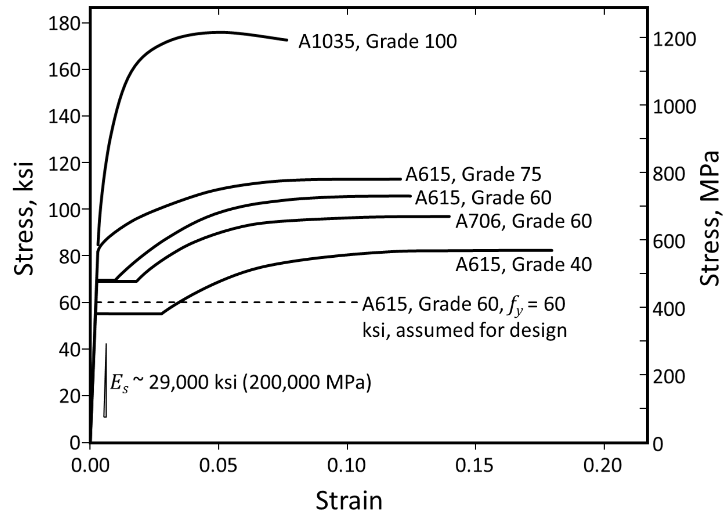



Figure 15-7 Characteristic engineering stress versus engineering strain relations for A615, A706, and A1035 deformed bars in tension.

## 15.4. TYPICAL REINFORCED CONCRETE BEAMS AND COLUMNS

Reinforced concrete beams are built in a variety of cross-sectional shapes, including rectangular, flanged (where a beam is monolithic with the floor slab that it supports), I-shaped (common in precast bridges), and box-shaped (common in bridge construction). Beams are commonly provided with both longitudinal and transverse reinforcement, as shown in Figure 15-8.

To understand the requirements for reinforcement, considering the simply-supported beam shown in Figure 15-8. The shear and moment diagrams corresponding to the uniform load are shown. Bending moment results in flexural tension along the bottom face of the beam. Under service loads, this flexural tension is likely to be sufficient to induce cracks. The cracks initiate at the maximum moment section (midspan) and propagate perpendicular to the flexural tension stress, that is, they propagate vertically into the beam. Longitudinal reinforcement, shown blue, is provided to resist flexural tension across these cracks.

As load increases, flexural tension cracking spreads away from the beam midspan. These additional cracks initiate as flexurally driven cracks, oriented vertically. As they propagate upward into the beam, however, the presence of shear stress causes the direction of principal tensile stress to rotate, as suggested by the square element drawn near point c. Consequently, the orientation of the cracks changes as they propagate upward into the beam. These cracks are sometimes referred to as *diagonal tension cracks*, or simply as *shear cracks*. *Transverse reinforcement*, shown green, is provided to cross these cracks, thereby preventing them from opening excessively and helping to resist shear forces acting on the beam. The individual pieces of transverse reinforcement in a beam are commonly referred to as *stirrups*.

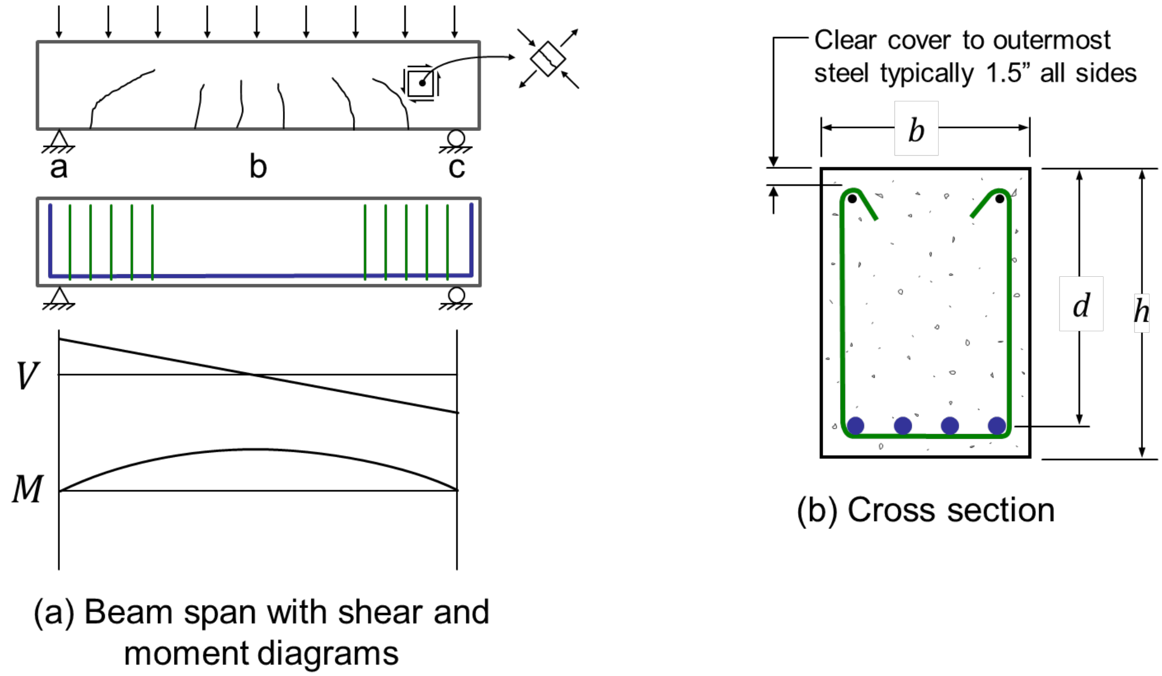



Figure 15-8 Longitudinal and transverse reinforcement in beams.

In a building or bridge frame, the beams and columns can be cast monolithically to form a moment-resisting frame. Figure 15-9 illustrates a moment-resisting frame. Note that effective moment transfer at the connections requires that the beam and column longitudinal reinforcement be extended into and be anchored within the beam-column joints. These subjects are discussed further in CE 123.

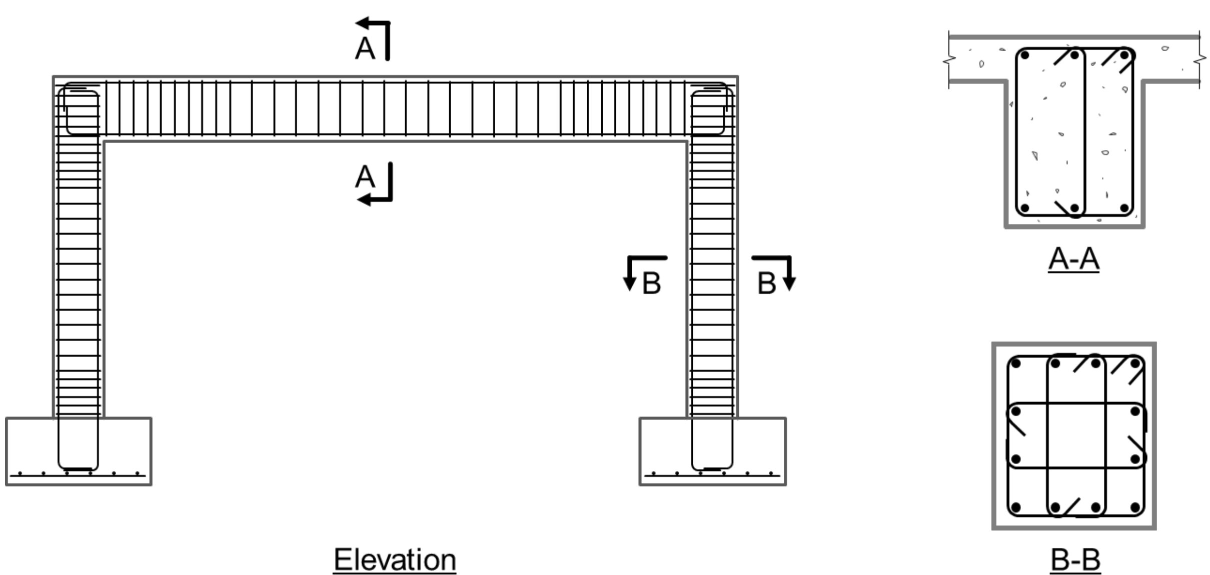



Figure 15-9 One-bay, one-story, moment-resisting frame comprising beam, columns, and beam-column joints.

## 15.5. DESIGN STRENGTHS FOR REINFORCED CONCRETE BEAMS

Design of reinforced concrete members usually is done in accordance with the Load and Resistance Factor (LRFD) method, which was introduced in Chapter 12. ACI 318 refers to this method as the strength design method. To use the strength design method, we need to define nominal and design strengths for reinforced concrete members. For our purposes, we are interested in the moment and shear strengths of beams.

**Design moment strength, $\phi M_n$**

Consider the beam cross section shown in Figure 15-10. The member is assumed to have one layer of longitudinal reinforcement, with yield stress $f_y$, positioned with its centroid a distance $d$ from the extreme compression fiber. Under the action of applied moments, the member develops longitudinal strains that vary linearly over the member depth as shown in Figure 15-10b. For practical longitudinal steel ratios ($0.005 \leq \rho \leq 0.02$, where $\rho = A_s/bd$), the beam longitudinal reinforcement will yield well before the compression zone crushes. Beam strength is reached when the concrete crushes at a concrete compressive strain around $\varepsilon_{cu} = 0.003$. Consequently, the stresses acting on the beam cross section at failure are as shown in Figure 15-10c. Note that the vertical line in Figure 15-10c represents a thin slice of the beam cross section. It is a *free-body diagram* showing the nominal moment acting on one side of the slice and the internal stresses acting on the other side.

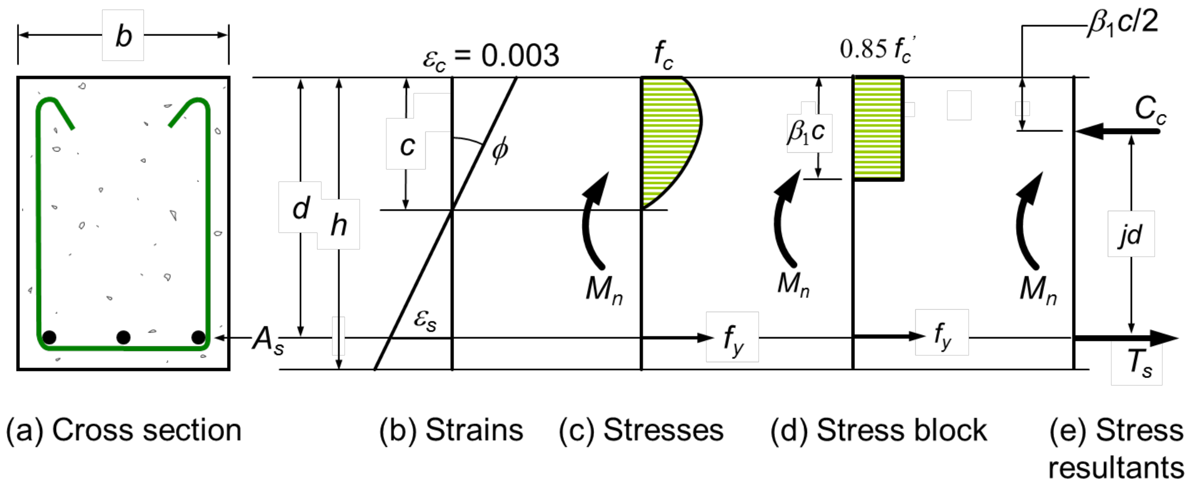



Figure 15-10 Moment strength of reinforced concrete beam.

We can solve for $M_n$ in Figure 15-10c by summing moments about any convenient point. To simplify the challenge of integrating over the concrete compressive stress block of Figure 15-10c, we replace it by an equivalent rectangular compressive stress block, as shown in Figure 15-10d. The stress intensity $0.85f_c'$ is suitable for all concrete compressive strengths. The depth of the stress block is $\beta_1 c$, where $\beta_1$ varies from 0.85 to 0.65 as a function of the concrete compressive strength. For the purpose of calculating the nominal moment strength, we do not need to define the precise value of $\beta_1$, as this term cancels in the moment equations. Given this simplified stress block, the stress resultants of Figure 15-10e can be determined as

$$T_s = A_s f_y \tag{15-3}$$

in which $A_s$ = area of longitudinal tension reinforcement and $f_y$ = steel yield stress. For the commonly used Grade 60 reinforcement, $f_y$ = 60 ksi. Additionally,

$$C_c = 0.85 f_c' b \beta_1 c \tag{15-4}$$

Summing axial forces on the free-body diagram of Figure 15-10e, we find that $T_s = C_c$, and substituting in Eq. (15-3) and (15-4), we find that the neutral axis depth $c$ is

$$c = \frac{A_s f_y}{0.85 f_c' b \beta_1} \tag{15-5}$$

Summing moments about the centroid of the rectangular stress block, we find

$$M_n = T_s \times jd \tag{15-6}$$

in which $j$ is a multiplier on the effective depth $d$, and $jd$ can be solved as

$$jd = d - \frac{\beta_1 c}{2} = d - \frac{1}{2}\frac{A_s}{b}\frac{f_y}{0.85 f_c'} = d\left(1 - 0.59 \frac{A_s}{bd}\frac{f_y}{f_c'}\right) = d\left(1 - 0.59 \rho \frac{f_y}{f_c'}\right) \tag{15-7}$$

in which $\rho = A_s/bd$. For practical designs in which $0.005 \le \rho \le 0.02$, and for $f_y$ = 60 ksi and $f_c'$ = 5 ksi, the value of the term in parentheses is $0.86 \le j \le 0.96$. A good rule of thumb is to use $jd = 0.9d$ as an approximation. Thus, in CE 120, the nominal moment strength can be written as

$$M_n = A_s f_y \times 0.9d \tag{15-8}$$

In the strength design method, as in the LRFD method, the design strength is given by $\phi M_n$, in which $\phi$ = strength reduction factor. *For moment strength design, $\phi$ = 0.9.*

**Example 1**: What is the design moment strength $\phi M_n$ for the section shown in Figure 15-11? Concrete has $f'_c$ = 4 ksi and steel is Grade 60.

**Solution**: See Figure 15-11. For No. 8 longitudinal bars, according to Table 15-1, the diameter is 1 inch and the cross-sectional area is 0.79 in.$^2$ (Note that we could calculate these quantities knowing the nominal diameter is 8/8 inches.) Therefore, the total steel area is $A_s$ = 3 x 0.79 = 2.37 in.$^2$ To find the effective depth $d$, assume typical concrete cover of 1.5 inches over the stirrups, add 0.5 in. for the stirrup diameter and 0.5 in. for half the longitudinal bar diameter, leading to $d$ = 24" – 1.5" - 0.5" - 0.5" = 21.5 in. The nominal moment strength is $M_n = A_s$ x $f_y$ x 0.9$d$ = (2.37 in.$^2$)(60 ksi)(0.9 x 21.5 in.) = 2750 k-in. The design moment strength is $\phi M_n$ = 0.9 x 2750 k-in. = 2480 k-in.

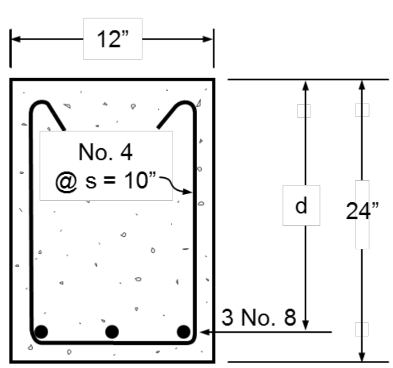



Figure 15-11 Cross-sectional dimensions of beam for Examples 1 and 2.

**Design shear strength, $\phi V_n$**

As discussed in Section 15.4, shear tends to produce cracks that are inclined relative to the beam longitudinal axis. Transverse reinforcement is required to control crack widths, thereby enabling the interlocked segments of concrete on either side of a crack to continue to transfer some shear, and also providing additional shear strength directly through the reinforcement. The nominal shear strength is defined as follows:

$$V_n = V_c + V_s \tag{15-9}$$

in which $V_c$ = shear strength attributable to the concrete and $V_s$ = shear strength attributable to the transverse reinforcement.

The shear strength attributable to the concrete is based test observations, and is expressed as

$$\begin{aligned} V_c &= 2\sqrt{f_c'}bd, \text{ psi} \\ V_c &= 0.17\sqrt{f_c'}bd, \text{ MPa} \end{aligned} \tag{15-10}$$

in which $V_c$ = concrete contribution to shear strength (lb or Newtons), $f_c'$ = concrete compressive strength in psi or MPa, $b$ = width of the web of the beam in inches or mm, and $d$ = effective depth of the beam in inches or mm.

The shear strength attributable to transverse reinforcement is obtained by cutting a 45-degree crack through the beam and summing the vertical force resisted by the stirrups. We can write that the number $n$ of stirrups crossed by a crack is to $d$ as one stirrup is to the spacing $s$, that is, $n = d/s$. Therefore, the total force resisted by stirrups along a 45-degree crack is

$$V_s = nA_v f_{yt} = \frac{A_v f_{yt} d}{s} \tag{15-11}$$

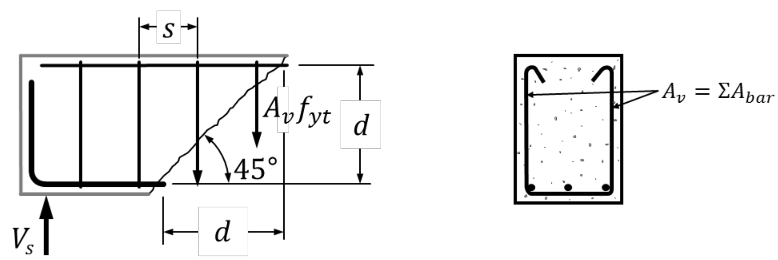



Figure 15-12 Contribution $V_s$ of transverse reinforcement to shear strength.

In the strength design method, as in the LRFD method, the design strength is given by $\phi V_n$, in which $\phi$ = strength reduction factor. *For shear strength design of concrete beams, $\phi$ = 0.75.*

**Example 2**: What is the design shear strength $\phi V_n$ for the section shown in Figure 15-11? Concrete has $f'_c$ = 4 ksi and steel is Grade 60.

**Solution**: From Eq. (15-10), $V_c = 2\sqrt{f_c'}bd = 2\sqrt{4000} \times 12 \times 21.5 = 32{,}600\ lb$. From Eq. (15-11), $V_s = \frac{A_v f_{yt} d}{s} = \frac{(2\times0.20)(60{,}000)(21.5)}{10} = 51{,}600\ lb$. From Eq. (15-9), $V_n = V_c + V_s = 32{,}600\ lb + 51{,}600\ lb = 84{,}200\ lb = 84.2\ kips$. The design shear strength is $\phi V_n$ = 0.75 x 84.2 kips = 63.2 kips.

## 15.6. ADDITIONAL DESIGN CONSIDERATIONS

Design of reinforced concrete beams should consider additional aspects, some of which are building code requirements and others of which are practical construction considerations. A few of these are summarized below:

**Cover and spacing of reinforcement**

Concrete cover over reinforcement is important to protect reinforcement from corrosion and fire, and to ensure the reinforcement is adequately bonded to the surrounding concrete. Different conditions require different cover. For most practical designs, clear cover over reinforcement should not be less than 1.5 inches. For footings exposed to earth, clear cover should not be less than 3 inches.

Placement of concrete requires that reinforcement be spaced to facilitate flow of concrete around the reinforcement. As a minimum, bar spacing should not be less than the larger of 1 inch and the bar diameter.

**Beam dimensions**

Beams can have any cross sections required for serviceability and strength. Beams deeper than 36 inches require additional reinforcement along the side faces to control cracking. The ratio of beam width *b* to beam depth $h$ is not constrained by codes. However, efficient designs commonly fall in the range $1 \leq \frac{h}{b} \leq 2$.

**Longitudinal reinforcement ratios**

To control performance within acceptable objectives, building codes place limits on the minimum and maximum amounts of longitudinal reinforcement. As a practical matter, the longitudinal reinforcement ratio $\rho = A_s/bd$ should fall in the range $0.005 \leq \rho \leq 0.02$. These limits will satisfy building code requirements.

**Moment design constraints**

Sometimes the size of the beam is restricted by architectural requirements. In this case, moment design requires only the selection of the reinforcement to meet the strength requirements.

Other times, the size of the beam is not constrained. In this case, it is best to select a design with modest longitudinal reinforcement ratio, as this will ease construction and avoid deflection problems. Combining Eq. (15-6) and (15-7),

$$M_n = T_s \times jd = A_s f_y d\left(1 - 0.59\rho\frac{f_y}{f_c'}\right) = \frac{A_s}{bd} f_y bd^2\left(1 - 0.59\rho\frac{f_y}{f_c'}\right)$$
$$= \rho f_y bd^2\left(1 - 0.59\rho\frac{f_y}{f_c'}\right) \sim 0.9\rho f_y bd^2 \tag{15-12}$$

If we assume a ratio for $b/d$, for example $b/d = ½$, then this expression becomes

$$M_n \sim \frac{0.9\rho f_y d^3}{2} \tag{15-13}$$

Using Eq. (15-13), with an assumed steel ratio 0.01 (which is known to produce reasonably efficient designs that perform well), we can quickly solve for required depth $d$ and the estimate the beam depth $h \sim d + 2.5$ inches. This and other approaches will be practiced in CE 123.

**Transverse reinforcement spacing and limits**

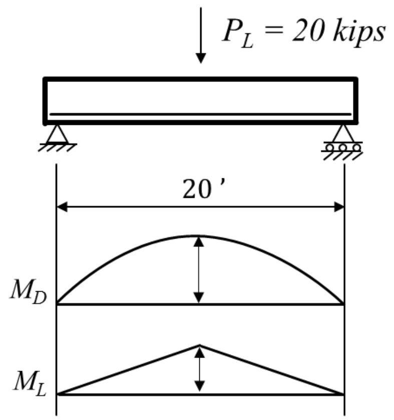
In California and other seismically active regions, building codes require that transverse reinforcement be provided even if the concrete cross section provides adequate strength for design loads without transverse steel. Therefore, always provide stirrups in beams.

Stirrups need to intersect all possible shear (diagonal) cracks. Therefore, building codes require that maximum spacing of stirrups not exceed $d/2$. Some additional requirements may apply.

In addition, the spacing of stirrups should not exceed the spacing required based on consideration of the required strength. That is, the following design requirement must always be satisfied: $\phi V_n \geq V_u, where\ V_u = V_c + V_s$.

## 15.7. DESIGN EXAMPLES

**Example 3**: Select dimensions and longitudinal reinforcement required for a beam to resist the loads shown. Consider moment only. Concrete has $f'_c$ = 4 ksi and steel is Grade 60.




Design for $\rho = 0.01$

*Estimate beam size:*
*~ span/12* → 20''

Use 10'' ∗ 20''

$w = \frac{10*20}{144} * 150$ pcf = 208 plf

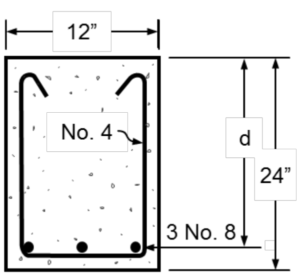
This is relatively minor load. Upsize to 400 plf to be on the safe side.

$M_D = \frac{wL^2}{8} = \frac{0.4*20^2}{8} = 20$ k-ft

$M_L = \frac{Pl}{4} = \frac{20k*20'}{4} = 100$ k-ft

$M_u = \max(1.4\,M_D,\ 1.2\,M_D + 1.6\,M_L)$
= max ( 28 k-ft , 184 k-ft )
So $M_u$ = 184 k-ft = 2210 k-in

$M_u = 2210 \text{ k-in} \le \varphi M_n = (0.9)(A_s f_y)(0.9d) = 0.9\rho f_y 0.9\frac{d^3}{2}$




$\rightarrow d = \sqrt[3]{\frac{2210*2}{0.9^2*0.01*60}} = 21$ in

$b = 1/2 * d = 10.5$ in

$A_s = \rho bd = (0.01)(10.5)(21) = 2.20$ in$^2$

Use 3 No8 bars $\rightarrow A_s = 2.37$ in$^2$

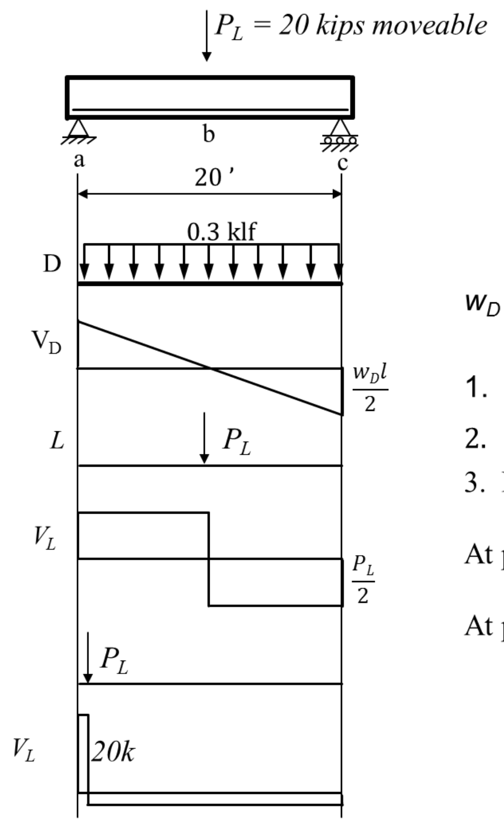
$b$ = 12''

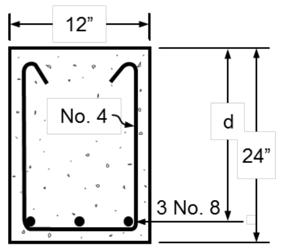
$h = d + 2.5'' = 23.5'' \rightarrow h = 24''$

Same beam as before

Check: $M_n = (A_s f_y)(0.9d) = (2.37)(60)(0.9)(21.5) = 2750$ k-in
$\varphi M_n = 0.9 * 2750 = 2480 \text{ k-in} > M_u \rightarrow$ OK

**Example 4**: Given the properties and loads given below, select spacing of transverse reinforcement.




$f'_c = 4$ ksi
$f_y = 60$ ksi
$b = 12''$
$d \sim 24''\text{-}2.5'' = 21.5''$

$$w_D = \frac{24*12}{144} * 150 \text{ pcf} = 300 \text{ plf}$$

1. $\frac{w_D l}{2} = \frac{0.3*20}{2} = 3$ kips
2. $\frac{P_L}{2} = 10$ kips
3. $P_L = 20$ kips

At point a : $V_u = 1.2\, V_D + 1.6\, V_L$
$= (1.2)(3) + (1.6)(20) = 35.6$ kips
At point b : $V_u = 0 + 1.6 * 10 = 16$ kips

**Check point a**

$V_u \leq \varphi V_n = \varphi(V_c + V_s)$
$\rightarrow \frac{V_u}{\varphi} = \frac{35.6}{0.75} = 47.5 \text{ kips} \leq V_c + V_s$
$V_c = 2\sqrt{f'_c}bd = 2\sqrt{4000}\,(12)(21.5) = 32{,}500 \text{ lb} = 32.6$ kips
$\varphi V_c = 0.75 * 32.6 = 24.5$ kips

$\rightarrow V_s \geq 47.5 - 32.6 = 14.9$ kips
$\rightarrow V_s \geq \frac{A_v f_y d}{s} = \frac{(0.2*2)(60)(21.5)}{s} \geq 14.9$ kips
$\rightarrow s \leq 34.7$ in

but $s_{max} = d/2 = 21.5/2 \rightarrow$ use No, 4 ⊔ @ 10"

**Check point b**

$V_u = 16$ kips
$\rightarrow \frac{V_u}{\varphi} = \frac{16}{0.75} = 21.3 \leq V_c + V_s$
$V_c = 32.6$ kips ; $\varphi V_c = 0.75 * 32.6 = 24.5$ kips
$\rightarrow V_s$ negative

but $s_{max} = d/2 = 21.5/2 \rightarrow$ use No. 4 ⊔ @ 10"