Q2 Stress Analysis

20 Points

A set of X-ray measurements have been made to determine the strains at a critical location in a structural component. Another engineer has converted these measurements to the stress values

$$[\tau] = \begin{bmatrix} -10 & 0 & 50 \\ 0 & 300 & 60 \\ 50 & 60 & 150 \end{bmatrix} \text{MPa}\,.$$

1. At this location, what is the traction of the plane with normal vector $\mathbf{n} = \begin{pmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$?

2. At this location, what is the normal stress on this plane?

3. At this location, what is the shear stress on this plane?

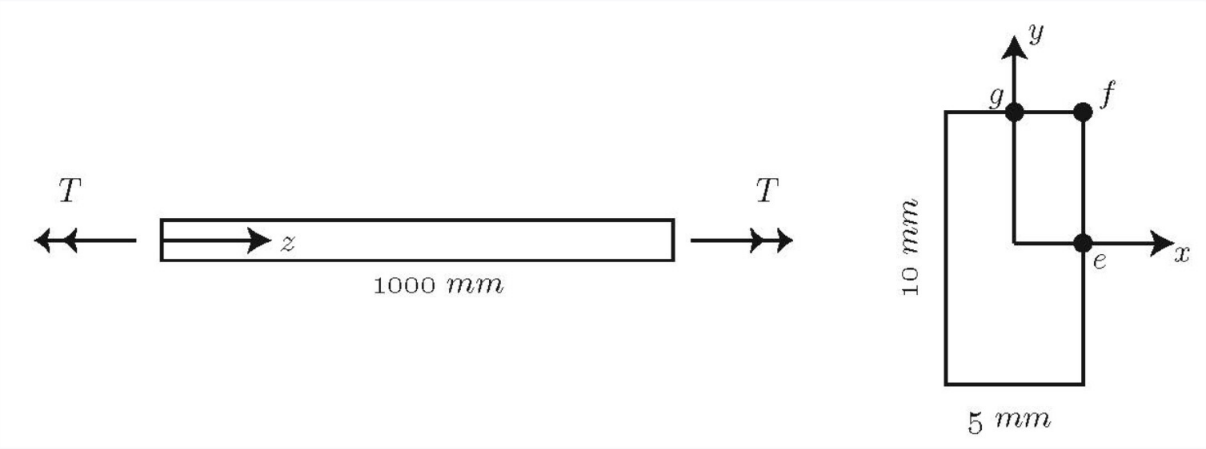
4. At this location, what are the principal stresses?

5. At this location, what is the maximum shear stress?

Q3 Torsion of a Rectangular Bar

12 Points

The torsion bar shown has a 10×5 mm cross-section and is subjected to a torque T.



1. At which point do you expect to find the largest shear stress, e, f, or g?
2. The stress at the point of maximum shear stress has been measured to be 50 MPa. What is the value of the applied torque T ?
3. If the bar has a length $L = 1000$ mm, how many degrees has right end of the bar rotated relative to the left end? Assume the shear modulus of the material is $G = 100$ kN/mm^2

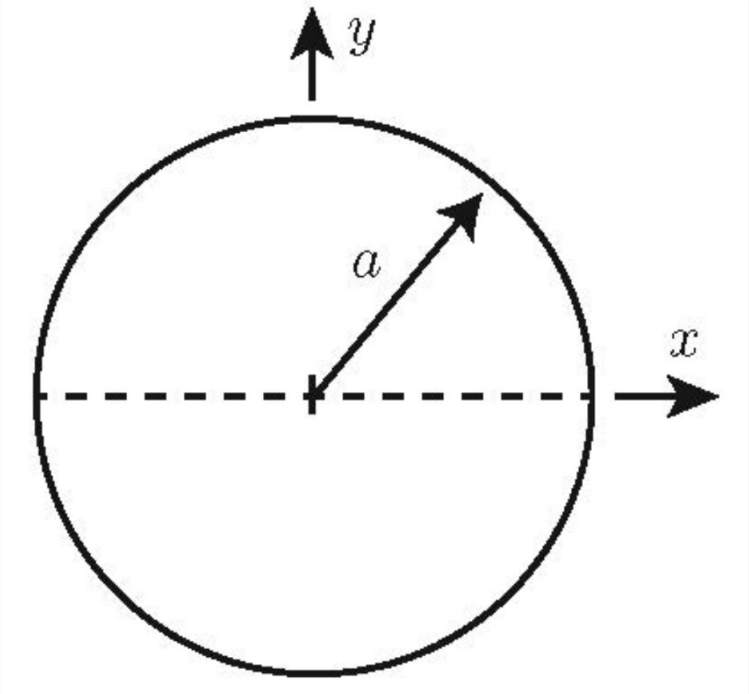
Q4 Strained Disk

4 Points

A thin circular disk of radius a made of a linear elastic isotropic material is subjected to a set of loads. This induces a state of plane-stress in the disk and a strain field of the form:

$$\epsilon_x = Ax^4y^2, \quad \epsilon_y = Ay^4x^2, \quad \epsilon_{xy} = \tfrac{A}{5}xy(x^4+y^4)\,,$$

where A is a given (small) constant.



What is the change in length of the chord shown as a dotted line in the figure?