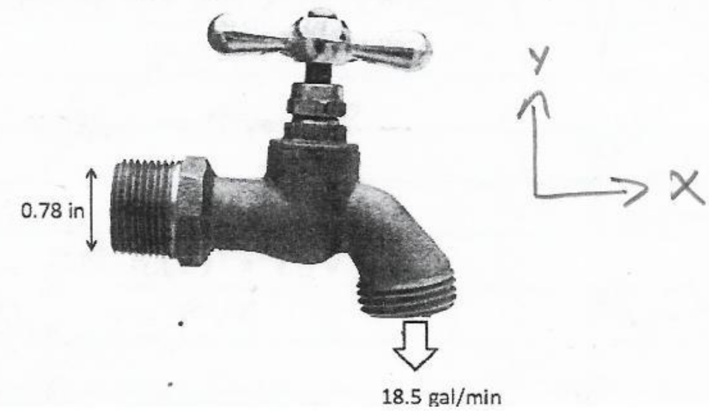
Quiz 3 - October 17, 2022

Please label units in all calculations and write all relevant equations explicitly. Make sure the units make sense!!

1. Lake Merced has a surface area of 2.6 km^2 (650 acres). On a dry year, the average depth of the lake is approximately 3 m. It rained about 0.33 cm/hr the day before. It is also fed by an underground spring that produces 0.13 m^3/s (3 million gallons per day).
   a. How many days of rain will we have to bear for the lake to fill to its average wet year depth of 4.5 m? To get credit for this problem, you MUST write out the relevant equations and substitute each term for the appropriate values.
   b. Name two assumptions you had to make to do this calculation.
   c. On the night of the third week after the day it started raining continuously (a very wet season), a broad-footed mole, the only common species in this area, burrows 3.5 cm diameter horizontal tunnel from outside the lake, only to find himself hit water right at the bottom of the lake. Assuming no obstructions, what kind of flow, in m^3/s, hits the poor guy?

2. Water is flowing from the spigot as depicted below. The flow rate from the spigot is 18.5 gallons per minute, and the inner diameter of the pipe is 0.78 inches (=0.065 ft). The pressure of the water entering is measured to be 13 psi (pounds per square inch) gage. Given the total weight of the spigot and the water in it is 12.8 lbf (pounds in force), what is the reaction force needed to support this system? You can assume the vertical distance between the pipe entrance and the spigot exit is negligible.
NB: Be sure to show the direction(s) of the force!

Density of water is 1.94 $slug/ft^3$.
1 ft^3 = 7.481 gal
1 slug = 1 lbf s^2/ft
(Defined as the mass that is accelerated by 1 ft/s^2 when a net force of one pound is exerted on it)



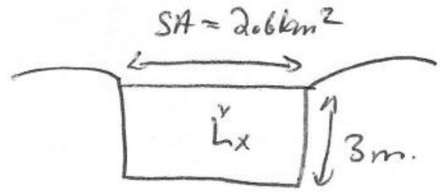
Quiz 3 Cee100

① $SA = 2.6\ km^2 = 2.6\times10^6\ m^2$

Avg Depth = 3m (dry year)

Rain Rate $= 0.33\frac{cm}{hr} = 0.0792\frac{m}{day}$

Underground Spring $= 0.13\ m^3/s$



a) Time needed to fill lake to 4.5m

$$\frac{Dm_{sys}}{Dt} = \frac{dm_{cv}}{dt} + \cancel{m_{out}}^{0} - m_{in} = 0$$

$$m_{cv} = \rho V = \rho A\,h(t) = \rho dV$$

$$m_{in} = -\rho Q_{in} = -\rho VA$$

$$\rho\cdot\frac{dV}{dt} - \rho Q_{in} = 0 \Rightarrow Q_{in} = \frac{dV}{dt}$$

$$\frac{Dm}{Dt} = \frac{d}{dt}\left[\rho A_{surf}\,h(t)\right] - \rho Q_{in} = 0$$

$$A_{surf}\frac{dh}{dt} - Q_{in} = 0$$

$$A_{surf}\int_0^{1.5} dh = Q_{in}\int_0^t dt$$

$$A_{surf}\,h\Big|_0^{1.5} = Q_{in}\,t$$

$$t = \left(\frac{A_{surf}}{Q_{in}}\right)h\Big|_0^{1.5}$$

a) $t = 18$ days

$Q_{in} = Q_{spring} + Q_{rain}$

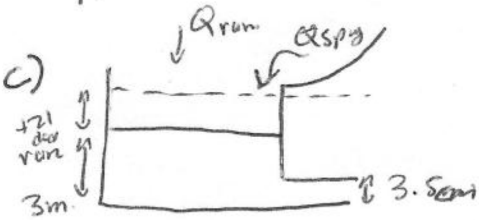
$= \left(11232\frac{m^3}{day}\right) + \left(2.6\times10^6 m^2\right)\left(0.0792\frac{m}{day}\right)$

$Q_{in} = 217152\ \frac{m^3}{day}$

b) • Rain is constant Rate of 0.33cm/hr for all 18 days

• Assume all surface area collecting rain remains constant

c)



$\Delta V = Qt = 21day\left(217152\frac{m^3}{day}\right) = 4560192m^3$

$\Delta V = \Delta h\,(SA)$

$\Delta h = \frac{\Delta V}{SA} = 1.754m$

$$\frac{\cancel{V_1^2}^{0}}{2g} + \frac{\cancel{P_1}^{0}}{\gamma} + z_1 = \frac{V_2^2}{2g} + \frac{\cancel{P_2}^{0}}{\gamma} + \cancel{z_2}^{0}$$

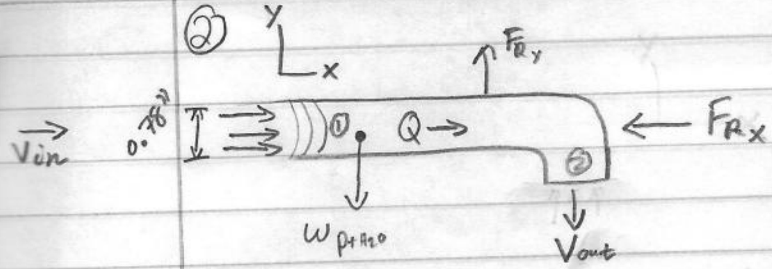
$$V_2 = (2gz_1)^{1/2} = \left(2(9.8)(3+1.754m)\right)^{1/2} = 9.65\ m/s$$

$$Q = VA = \left(9.65\frac{m}{s}\right)\left(\frac{\pi}{4}\left(3.5\times\frac{1}{100cm}\right)^2\right)$$

c) $Q = 9.28\times10^{-3}\ \frac{m^3}{sec}$

CE100 Quiz 3

②



$w_{p+H_2O} = 12.8\ lb$

$Q = 18.5\ gal/min$

$P_{in} = 13\ psi \Rightarrow 13\frac{lb}{in^2} \times \frac{12in}{1ft} \times \frac{12in}{1ft} = 1872\frac{lb}{ft^2}$

$Q_{in} = V_{in}A_{in} \Rightarrow V_{in} = \frac{Q_{in}}{A_{in}} = \frac{18.5\frac{gal}{min} \times \frac{1min}{60sec} \times \frac{1ft^3}{7.481\ gal}}{\frac{\pi}{4}(0.065ft)^2} = 12.4\ ft/sec$

$\Sigma F_x = P_{in}A_{in} - F_{Rx} = \cancel{\frac{dm}{dt}}^0 + \cancel{\Sigma \dot{m}_0 V_0}^0 - \Sigma \dot{m}_1 V_1$

$F_{Rx} = P_{in}A_{in} + \dot{m}_1 V_1$

$= \left(\frac{13lb}{in^2}\right)\left(\frac{\pi}{4}(0.78in)^2\right) + \left(1.94\frac{slug}{ft^3}\right)\left(\frac{18.5gal}{60sec} \times \frac{1ft^3}{7.481gal}\right)\left(12.4\frac{ft}{sec}\right)$

$=$ 11 .+ $0.9915\frac{slug\ ft}{sec^2} = 0.9915\left(\frac{lb\ s^2}{ft}\right)\left(\frac{ft}{s^2}\right)$

$\boxed{F_{Rx} = 7.20 lb}$

$\Sigma F_y = -w_{p+H_2O} + F_{Ry} = \cancel{\frac{dm}{dt}}^0 + \Sigma m_0 V_0 - \cancel{\Sigma m_1 V_1}^0$

$F_{Ry} = w_{p+H_2O} + m_0 V_0$

$= 12.8\ lb + \left(1.95\frac{slug}{ft^3}\right)\left(\frac{18.5gal}{60sec} \times \frac{1ft^3}{7.481gal}\right)\left(-45.65\frac{ft}{s}\right)$

$\boxed{F_{Ry} = 9.13\ lb}$

$\frac{V_0^2}{2g} + \cancel{\frac{P_0}{\gamma}}^0 + \cancel{z_0} = \frac{V_1^2}{2g} + \frac{P_1}{\gamma} + \cancel{z_1}$

$V_0 = \left(V_{in}^2 + \frac{2g P_1}{\rho \cancel{g}}\right)^{1/2}$

$= \left(\left(12.4\frac{ft}{sec}\right)^2 + \frac{2\left(1872\frac{lb}{ft^2}\right)}{1.94\frac{lb\ s^2}{ft^4}}\right)^{1/2}$

$V_0 = 45.65\ ft/s$

$$\boxed{\begin{array}{l} F_{Rx} = 7.20\ lb \leftarrow \\ F_{Ry} = 9.13\ lb \uparrow \end{array}}$$