N.B. Notation and conventions were slightly different in S20

1

NOTE COORDINATE SYSTEM DIFFERS FROM THE ONE WE ARE USING

(a) $\underset{\sim}{M} = E \begin{bmatrix} I_x & -I_{xy} \\ -I_{xy} & I_y \end{bmatrix} \underset{\sim}{k}$

$$\begin{pmatrix} M_x \\ M_y \end{pmatrix} = \begin{pmatrix} -200{,}000 \\ 2.4\times10^6 \end{pmatrix} \quad (N\cdot m)$$

(b) $\cancel{\dfrac{dM_x}{dz}}^{\,0} - V_y = 0$ and $\cancel{\dfrac{dM_y}{dz}}^{\,0} + V_x = 0$

$$\Rightarrow \quad V_x = V_y = 0$$

2

(a) $$2G A_1 \bar{\theta} = q_1 \frac{(2a+b)}{t} + (q_1 - q_2)\, b/(t/4)$$

$$2G A_2 \bar{\theta} = q_2 (5a+b)/t + (q_2 - q_1)\, b/(t/4)$$

$$\left[\begin{matrix} A_1 = ab & A_2 = 2.5ab & G = 100\ kN/mm^2 \\ t = 1mm & a = 300mm & b = 400mm \end{matrix}\right]$$

Solve for q_1, q_2 (I used Mathematica)

$$q_1 = 2.75\times10^7\, \bar{\theta} \qquad q_2 = 2.97\times10^7\, \bar{\theta}$$

$$\Rightarrow (GJ)_{eff} = \frac{2A_1 q_1 + 2A_2 q_2}{\bar{\theta}} = 2.44\times10^7 \ (N\cdot m^2)$$

(b) $$\tau_{max} = \frac{2.97\times10^7}{\underset{\uparrow\ 1mm}{t}}\bar{\theta} = 29.7\underset{\uparrow\ rad/m}{\bar{\theta}} \ \ MPa$$

#3

$$\underbrace{\sigma_{cr} = \frac{E_T \pi^2}{(L/r)^2}}$$

$$\frac{d\sigma}{d\epsilon} \rightarrow E_T = \Delta Y \, e^{-(\epsilon - \epsilon_y)}$$

$$\frac{\sigma - Y_0}{\Delta Y} = 1 - e^{-(\epsilon - \epsilon_y)}$$

$$= 1 - E_T / \Delta Y$$

Find E_T as a function of σ

$$\frac{E_T}{\cancel{\Delta Y}} = +\Delta Y - \frac{\sigma - Y_0}{\cancel{\Delta Y}}$$

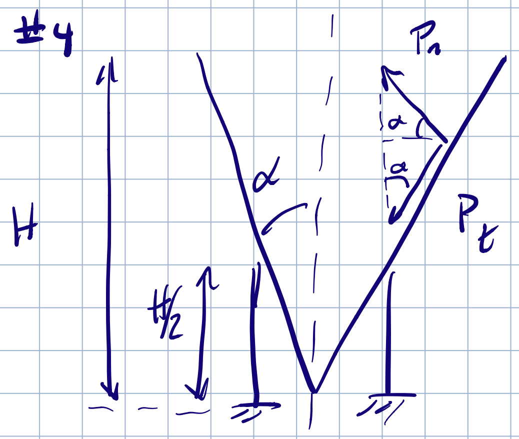
$$= Y_\infty - \sigma$$

$$\sigma = \frac{(Y_\infty - \sigma)\pi^2}{(L/r)^2}$$

$$\left(1 + \frac{\pi^2}{(L/r)^2}\right)\sigma = Y_\infty \pi^2 / (L/r)^2$$

$$\sigma = \frac{Y_\infty \pi^2}{(L/r)^2 \left(1 + \pi^2/(L/r)^2\right)} = \frac{Y_\infty \pi^2}{(L/r)^2 + \pi^2}$$

#4



$$P_v = -P_n \sin\alpha + P_t \cos\alpha$$

$$r_0 = (H - z)\tan\alpha$$

$$r_\theta = r_0 / \cos\alpha$$

$$F_v^{above} = 2\pi \int_0^z P_v \frac{dz}{\cos\alpha} = 2\pi k\left(z + \left(e^{-z/s} - 1\right) s\right)(1 + \tan\alpha)$$

$$F_v^{below} = F_v^{above} - 2\pi \int_0^H P_v \frac{dz}{\cos\alpha}$$

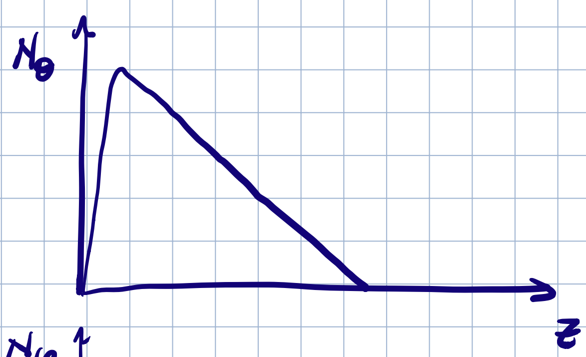
$$= 2\pi k\left((z - H) + \left(e^{-z/s} - e^{-H/s}\right) s\right)(1 + \tan\alpha)$$

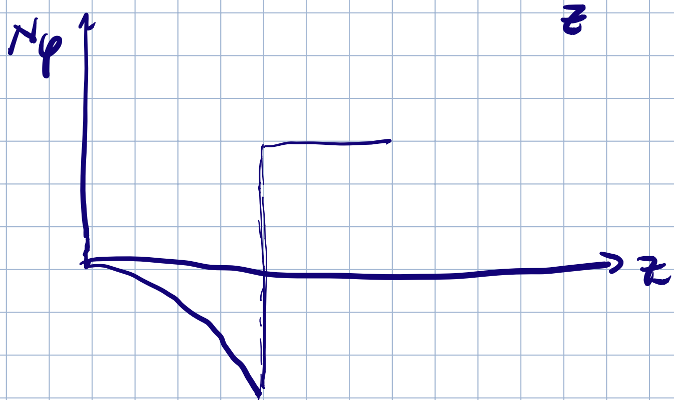
$$N_\theta = -p_n r_\theta = \left(1 - e^{-z/\rho}\right) k (H - z) \sec\alpha \tan\alpha$$

$$N_\varphi = -F_v / (2\pi r_0 \cos\alpha)$$

$$= \begin{cases} -\dfrac{k\left(z + \left(e^{-z/\rho} - 1\right)\rho\right)\csc\alpha\,(1+\tan\alpha)}{H - z} \\[2ex] -\dfrac{k\left((z-H) + \left(e^{-z/\rho} - e^{-H/\rho}\right)\rho\right)\csc\alpha\,(1+\tan\alpha)}{H - z} \end{cases}$$

Rough Sketch





(I performed all the computations with Mathematica)

#5

(b) $a_c = \left(\dfrac{K_{Ic}}{Q\sigma\sqrt{\pi}}\right)^2 = \dfrac{1.45}{P^2}$

$P \quad \dfrac{1 + (b/a)^2}{(b/a)^2 - 1}$

$$N = \frac{1}{C\,(Q\sigma\sqrt{\pi})^m} \; \frac{a^{(1-m/2)}}{1-m/2}\Bigg|_{a_i}^{a_c}$$

where $N = 10^6$ $\quad C = 5.0\times10^{-13}$ $\quad m=4$ $\quad Q = 1.8$

$K_{Ic} = 10$ $\quad a = 0.04$ $\quad b = 0.06$

Mathematica

$$\Rightarrow \quad -1000 + 0.688\,p^2 + 0.00237\,p^4 = 0$$

$$\Rightarrow \quad \underline{\underline{p^{max} = 22.818 \text{ MPa}}}$$

other roots are non-physical

(b)
$$a_c = \frac{1.45}{22.818^2} = \underline{\underline{0.00279 \text{ m}}} \equiv 2.79 \text{ mm}$$

#6

(a)
$$w = \int \frac{1}{r}\int r \int \frac{1}{r}\int \frac{r\,p(r)}{D}\,dr\,dr\,dr\,dr$$

$$+ A + B r^2 + C r^2 \ln r + D \underbrace{\ln r}$$

(b) ∞ disp $\Rightarrow D = 0$

$$= A + Br^2 + Cr^2\ln r + \frac{P_o r^4}{64D} - \frac{P_o r^5}{225DR}$$

(c) ① $w(R) = 0$

② $\frac{\partial w}{\partial r}(R) = 0$

③ $Q_r + \left[\iint_R p(r)\, r\, dr\, d\theta\right] / 2\pi R = 0$

where $Q_r = -D\frac{\partial}{\partial r}\nabla^2 w$

$$= -D\frac{\partial}{\partial r}\left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right]$$

Extra: $A = \dfrac{43 P_0 R^4}{4800 D}$ $\quad B = -\dfrac{29 P_0 R^2}{1440 D}$ $\quad C = 0$

(I solved ①-③ for A, B, C using Mathematica)