Solutions: Midterm

Recommendation: use Matlab

#2

① $\underline{t} = \underline{\underline{\Sigma}}\,\underline{n} = \begin{bmatrix} -10 & 0 & 50 \\ 0 & 300 & 60 \\ 50 & 60 & 150 \end{bmatrix} \begin{pmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 50/\sqrt{2} \\ 360/\sqrt{2} \\ 210/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 35.4 \\ 255 \\ 149 \end{pmatrix}$ MPa

② $\sigma = \underline{n} \cdot \underline{t} = \begin{pmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} \cdot \begin{pmatrix} 50/\sqrt{2} \\ 360/\sqrt{2} \\ 210/\sqrt{2} \end{pmatrix} = \frac{360+210}{2} = 285$ MPa

③ $\tau = |\underline{t} - \sigma \underline{n}| = 82.9$ MPa

④ $\text{eig}(\underline{\underline{\Sigma}}) = \{-25.2, 143, 322\}$ MPa

⑤ $\tau_{max} = \frac{322 + 25.2}{2} = 173$ MPa

#3

① e

② $50 = \frac{T}{\alpha a b^2} = \frac{T}{0.246 \cdot 10 \cdot 5^2} \Rightarrow T = 3075$ N·mm $= 3.075$ KN·mm

③ $\bar{\theta} = \frac{T}{\beta a b^3 G} = \frac{3.075}{0.229 \cdot 10 \cdot 5^3 \cdot 100} = 107.4 \times 10^{-6}$ rad/mm

$\Delta\theta = \bar{\theta} \cdot L = 107.4 \times 10^{-3}$ rad (with $L = 1000$)

$= 107.4 \times 10^{-3} \cdot \frac{180}{\pi} = 6.2^\circ$

#4 $\epsilon_x = \frac{\partial u}{\partial x} \Rightarrow \Delta = \int_{-a}^{a} \underbrace{\epsilon_x(x, 0)}_{0}\, dx$

$\Rightarrow \Delta = 0$