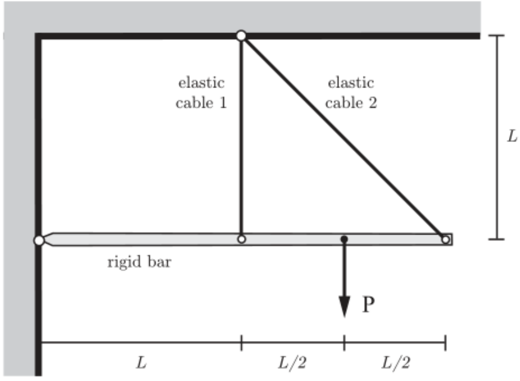
Problem #1 (40%)

A rigid bar is held horizontally by two elastic cables as shown in the figure. The cables have the same Young modulus E and cross section area A. All connections are pinned, and all members can be considered weightless.

A vertical load of value P is applied along the rigid bar as shown. Determine:

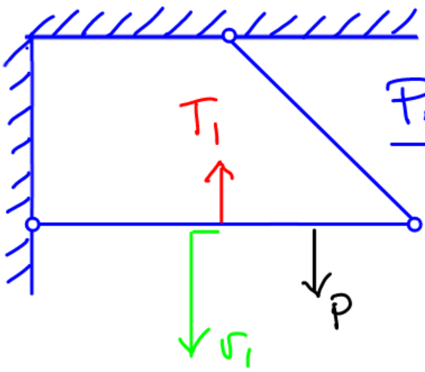
1. The forces in the cables.
2. The displacement of the right tip of the rigid bar.



Statically indeterminate

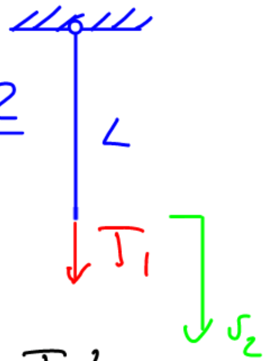
⇒ Force method (degree indet. = 1)

$\boxed{\text{STEP 1}}$ Release the system by e.g. disconnecting cable 1 leaving the force



Problem #1

Problem #2

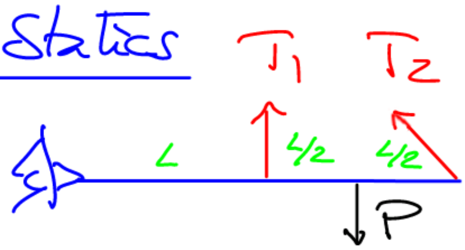


$\downarrow v_2 = T_1 f_1 = T_1 \frac{L}{EA}$

$\boxed{\text{STEP 2}}$ Solve for $v_1\downarrow$ and $v_2\downarrow$ → in terms of P, T_1

Problem #1

Statics



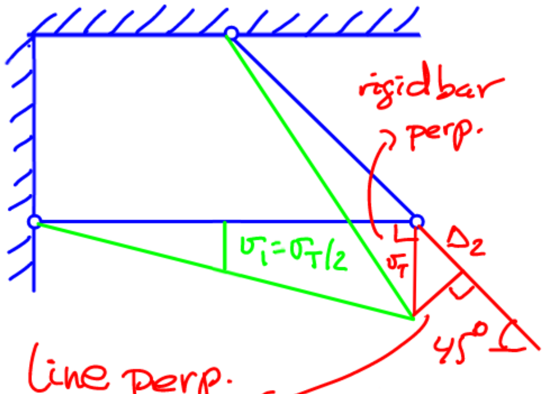
$\Sigma M = 0$

$\Rightarrow P\frac{3L}{2} = T_1 L + T_2 \sin 45 \cdot 2L$

$\Rightarrow T_2 = \frac{\sqrt{2}}{2}\left(\frac{3}{2}P - T_1\right)$

$\hookrightarrow v_1 = \frac{\sqrt{2}L}{2EA}\left(\frac{3}{2}P - T_1\right)$

Kinematics



line perp. to original direction cable 2

$\Rightarrow v_T = 2v_1 = \frac{\Delta_2}{\cos 45}$

$\Rightarrow v_1 = \frac{\Delta_2}{\sqrt{2}} = \frac{1}{\sqrt{2}} T_2 f_2 = T_2 \frac{L}{EA}$

$\hookrightarrow \sqrt{2}L/EA$

Relax

$\boxed{\text{STEP 3}}$ Impose back the compatibility constraint

$$\downarrow v_1 = v_2 \downarrow \;\Rightarrow\; \frac{\sqrt{2}L}{2EA}\left(\frac{3}{2}P - T_1\right) = T_1\frac{L}{EA}$$

$$\Rightarrow \boxed{T_1 = \frac{3P}{2(1+\sqrt{2})}}$$

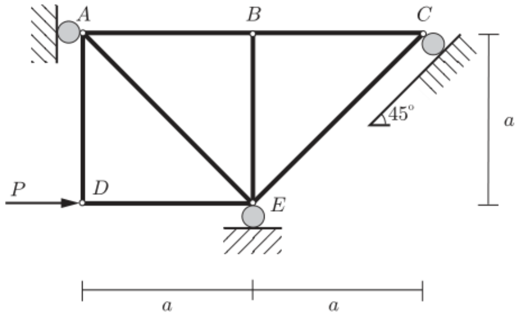
$$T_2 = \frac{\sqrt{2}}{2}\left(\frac{3}{2}P - T_1\right) \Rightarrow \boxed{T_2 = \frac{3P}{2(1+\sqrt{2})}}$$

$\boxed{\text{PART 2}}$ From the kinematics above, the tip of the rigid bar moves vertically down by $\downarrow v_T = 2v_1 \downarrow$

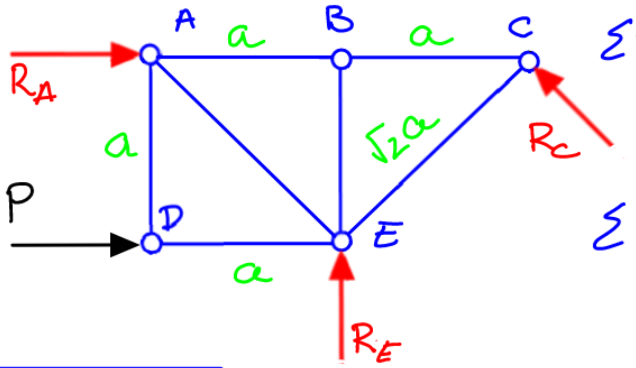
$$\boxed{\downarrow v_T = 2T_1\frac{L}{EA} = \frac{3P}{1+\sqrt{2}}\frac{L}{EA}}$$

Problem #2 (25%)

1. Determine the forces in all the members in the truss of the figure when the horizontal load of value P shown in the figure is applied. Indicate clearly if the member is in tension or compression, and identify all zero-force members, if any.
2. If all the members have the same $0.1 \times 0.1\ m^2$ square cross section, determine the maximum load value P that can be applied with a factor of safety of 1.5 if the material can only take $10MPa$ in tension or compression.



Reactions



$\Sigma M_C = 0 \Rightarrow Pa - R_E a = 0 \Rightarrow \underline{R_E = P}$

$\Sigma F_y = 0 \Rightarrow R_E + R_C \cos 45 = 0$

$\Rightarrow R_C = -\sqrt{2} R_E \Rightarrow \underline{R_E = -\sqrt{2} P}$

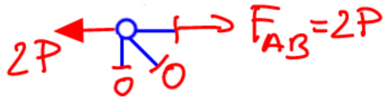
opposite than assumed

$\Sigma F_x = 0 \Rightarrow R_A + P - R_C \sin 45 = 0$

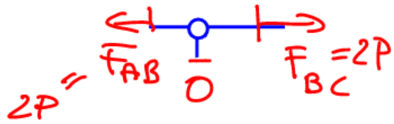
$\Rightarrow \underline{R_A = -2P}$

$\boxed{\text{PART 1}}$ Zero force members $F_{BE} = F_{AD} = F_{AE} = 0$

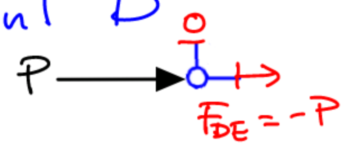
① Joint A



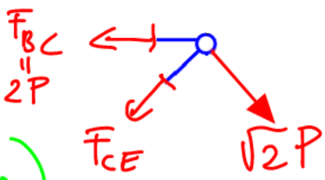
② Joint B



③ Joint D



④ Joint C



$F_{CE} \cos 45 + \sqrt{2} P \cos 45 = 0$

$\Rightarrow F_{CE} = -\sqrt{2} P$

checks

$2P + F_{CE} \cos 45 - \sqrt{2} P \sin 45 = 0$

(Joint E also checks)

$\boxed{\text{PART 2}}$ Maximum force among all members $= 2P$

$$\Rightarrow \frac{2P}{\underset{0.1 \times 0.1 m^2}{A}} \leq \frac{\overset{=10MPa}{\sigma_{ult}}}{FS = 1.5}$$

$$\Rightarrow \boxed{P \leq \frac{10^2}{3} kN = 33.\bar{3} kN = P_{max}}$$

SUMMARY:

$F_{BE} = F_{AD} = F_{AE} = 0$

$F_{AB} = 2P$ (tens.)

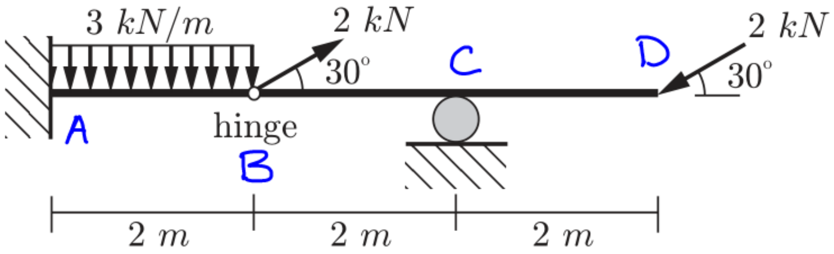
$F_{BC} = 2P$ (tens.)

$F_{CE} = -\sqrt{2}P$ (comp.)

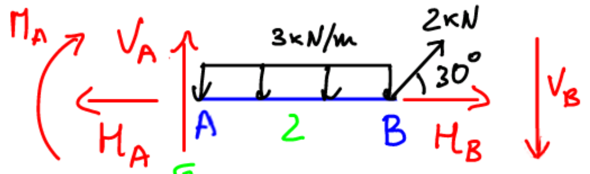
$F_{DE} = -P$ (comp.)

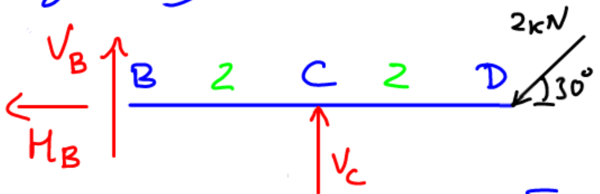
Problem #3 (35%)

Draw the axial force, transversal shear force and bending moment diagrams for the beam shown in the figure. Indicate the characteristic values (min/max values, values at the ends and supports, slopes, linear/parabolic/cubic distributions,...).



Reactions (cut at the hinge B)





$H_A = \overset{=-\sqrt{3}}{H_B} + 2\cos 30 = 0$

$V_A = 3\cdot 2 + \overset{=-1}{V_B} - 2\sin 30 = 4$

$M_A = 2\sin 30\cdot 2 - \underset{-1}{V_B}\cdot 2 - 3\cdot 2\cdot 1 = -2$

$H_B + 2\cos 30 = 0 \Rightarrow H_B = -\sqrt{3}$

$\Sigma M_B = 0 \Rightarrow 2V_C = 4\cdot 2\cdot \sin 30 \Rightarrow V_C = 2$

$\Sigma M_C = 0 \Rightarrow 2V_B = -2\cdot 2\sin 30 \Rightarrow V_B = -1$