Final Exam

This is a **13 question** exam. All questions are equally weighted, and some may have more than one answer.

Once you have finished taking the exam, please send a private message to the instructor on Zoom informing us that you will be signing off and uploading your exam. No need to wait for a reply. You will then have an additional 10 minutes to scan and upload your exams to Gradescope. Exams turned in later than 10 minutes beyond this time will not be graded. If, for whatever reason, you are unable to upload your exam to Gradescope during that time, you may email them to taliasaravi@berkeley.edu.

During this exam, you:

- may use both your notes and the textbook
- may not use the internet or any outside sources, except for Zoom and Gradescope
- may not discuss the exam during the entire exam period and until all exams have been completed

Partial credit will be given for solutions that are able to correctly express the appropriate steps required for solving the problem, even if they did not lead to the correct answer. On the other hand, no credit will be given for answers that are unsupported, even if they are correct.

Academic Integrity:

Berkeley Campus Code of Student Conduct (http://sa.berkeley.edu/student-code-of-conduct):

"The Chancellor may impose discipline for the commission or attempted commission (including aiding or abetting in the commission or attempted commission) of the following types of violations by students, as well as such other violations as may be specified in campus regulations: 102.01 Academic Dishonesty: All forms of academic misconduct including but not limited to cheating, fabrication, plagiarism, or facilitating academic dishonesty."

By signing, you agree that all exam work is your own and that you abided by the above rules. Not abiding by this honor code will lead to penalties. The penalties for academic dishonesty may include failure in the course or potentially expulsion from the university. Please sign below or above Question 1 on your exam.

Signed: _____ Date: _____

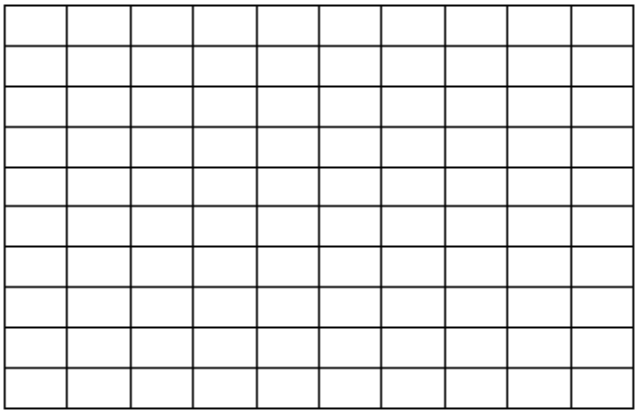
Tables are included in the back for your reference. Good luck!

Problem 1:

The following information regarding the length of time taken to review articles of a particular editor for the year 2002 is given in the table below.

Time (months)	Number of Articles
0 –< 1	45
1 –< 2	17
2 –< 3	18
3 –< 4	19
4 –< 5	12
5 –< 6	14
6 –< 7	13
7 –< 8	22
8 –< 9	11

(i) Construct a relative frequency histogram for the data. Plot your results on the grid below.



(ii) Which interval contains the median review time?

(a) 0 –< 1
(b) 1 –< 2
(c) 2 –< 3
(d) 3 –< 4
(e) 4 –< 5

(ii) Mark the general location of the mean on the histogram. In which direction is the data skewed? Circle: (Right / Left)

Problem 2:

Six hundred stones were examined for cracks, and 18 were found to be cracked. The same 600 stones were examined for discoloration, and 26 were found to be discolored. A total of 562 stones were neither cracked nor discolored. One of the 600 stones is selected at random.

(i) What is the probability that it is cracked, discolored, or both?

(ii) What is the probability that it is cracked but not discolored?

(iii) Given that a stone is discolored, what is the probability that it is also cracked?

Problem 3:

Two solutions, X and Y, are mixed together to form a solution, M, given by $M = X + 1.5Y$. Assume that X has mean 0.125 and standard deviation 0.05, and that Y has mean 0.350 and standard deviation 0.10.

(i) Find μ_M.

(ii) Assuming X and Y to be independent, find σ_M.

Problem 4:

Two machines are operated daily to repair defective toys. Let X be the number of hours that machine 1 is running and let Y be the number of hours that machine 2 is running. Assume that X and Y have a joint PDF given by:

$$f(x,y) = \begin{cases} \frac{3}{2}(x^2+y^2) & 0 < x < 1 \text{ and } 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

(i) Find f_x.

(ii) The conditional probability density function $f_{Y|X}(y \mid 0.5)$ is:

(a) $$f_{Y|X}(y|0.5) = \begin{cases} \frac{3+12y^2}{7} & 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

(b) $$f_{Y|X}(y|0.5) = \begin{cases} \frac{3+12y^3}{8} & 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

(c) $$f_{Y|X}(y|0.5) = \begin{cases} \frac{3+12y^2}{8} & 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

(d) $$f_{Y|X}(y|0.5) = \begin{cases} \frac{3+12y^3}{7} & 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Problem 5:

The number of defective components produced by a certain process in one day has a Poisson distribution with mean 20. Each defective component has probability 0.61 of being repairable.

(i) Find the probability that exactly 15 defective components are produced.

(ii) Given that exactly 15 defective components are produced, find the probability that exactly 10 of them are repairable.

Problem 6:

A battery manufacturer claims that the lifetime of a certain type of battery has a population mean of 40 hours and a standard deviation of 5 hours. Let $\overline{X}$ represent the mean lifetime of the batteries in a simple random sample of size 100.

(i) If the claim is true, what is $P(\overline{X} \leq 36.6)$?

(a) $P(\overline{X} \leq 36.6) < 0.0001$
(b) $0.0001 < P(\overline{X} \leq 36.6) < 0.01$
(c) $0.01 < P(\overline{X} \leq 36.6) < 0.05$
(d) $0.05 < P(\overline{X} \leq 36.6) < 0.10$
(e) $0.10 < P(\overline{X} \leq 36.6) < 0.25$

(ii) Based on the answer to $P(\overline{X} \leq 36.6)$ and assuming the claim to be true, is a sample mean lifetime of 36.6 hours unusually short?

(a) Yes
(b) No

Problem 7:

Two different types of nails were compared in terms of withdrawal strengths (N/mm). For an annularly threaded nail, the ultimate withdrawal strength was modeled as lognormal with $\mu = 3.79$ and $\sigma = 0.219$. For a helically threaded nail, the strength was modeled as lognormal with $\mu = 3.27$ and $\sigma = 0.272$.

An experiment is performed in which withdrawal strengths are measured for several nails of both types. One nail is recorded as having a strength of 20 N/mm, but its type is not given. Which nail type is it most likely to be? Circle all that are true.

(a) We are pretty sure that it is a helically threaded nail
(b) We are pretty sure that it is an annularly threaded nail
(c) Only about 0.01% of annularly threaded nails have strengths as small as 20 N/mm
(d) Only about 0.01% of helically threaded nails have strengths as small as 20 N/mm

Problem 8:

The following MINITAB output presents a confidence interval for a population mean.

One-Sample T: X

Variable	N	Mean	StDev	SE Mean	95% CI
X	10	6.59635	0.11213	0.03546	(6.51613, 6.67656)

Use the output and an appropriate table to compute a 99% confidence interval for the population mean. Round your answer to two decimal places.

Problem 9:

In a certain supermarket, a sample of 60 customers who used a self-service checkout lane averaged 5.1 minutes of checkout time, with a standard deviation of 3.1 minutes. A sample of 72 customers who used a cashier averaged 6.1 minutes with a standard deviation of 2.7 minutes. Can you conclude that the mean checkout time is less for people who use the self-service lane?

(i) Find the p-value.

(ii) Assuming a significance level of $\alpha = 0.05$, we (can / cannot) conclude that the mean checkout time is less for people who use the self-service lane.

Problem 10:

We know the amount of time that a certain rechargeable camera battery lasts is described by the exponential distribution with the rate parameter, lambda, equal to 5 hours.

A fellow CE93 student attempted to create the PDF and CDF for this scenario, and wrote the following lines of code:

```
fig=plt.figure(figsize=(10,5))
axs=[]
x=np.arange(__(a)__,__(b)__,__(c)__)
axs.append(fig.add_subplot(121))
axs[0].plot(__(d)__, expon.pdf(__(e)__,scale=__(f)__))
axs[0].set_title('PDF')
axs[0].set_ylabel('probability density function')
axs[0].set_xlabel('hours the battery lasts')
axs.append(fig.add_subplot(122))
axs[1].plot(__(g)__, expon.cdf(__(h)__,scale=__(i)__))
axs[1].set_title('CDF')
axs[1].set_ylabel('cumulative probability')
axs[1].set_xlabel('hours the battery lasts')
plt.subplots_adjust(wspace=0.5,hspace=0.1)
plt.show()
```

They are having trouble finishing the code, and have asked you for help. Please fill in the missing lines of code by filling in the letters below corresponding to each blank. The blanks for (a)-(c) could have many possible correct answers, so please give one reasonable answer and explain your choices for these lines on the following page.

(a) _____
(b) _____
(c) _____
(d) _____
(e) _____
(f) _____
(g) _____
(h) _____
(i) _____

Problem 10 continued: Explanation for choices (a) - (c):

Problem 11:

Please explain the line below, including the function and the resulting outputs.

```
mu, std = st.norm.fit(sample)
```

Explain:

Problem 12:

What will be the most likely output, given the following Python code?

```
arr = np.array([24,30,10,60,20,40])
arr_1 = (arr > 20)
print (sum(arr_1))
```

A. [True True False True False True]
B. 5
C. 4
D. [1 1 0 0 0 0]
E. None of the above

Explain:

Problem 13:

You are given a dataframe named df. This dataframe has one column named 'birds' which contains the number of bird sightings for each day in a year. Please select the line of code which will return an array of the index/indices for which more than 5 birds were recorded in one day. Please select only one answer.

(a) `np.random.choice(df['birds'])`
(b) `np.where(df['birds']>5)`
(c) `df[('birds')>5]`
(d) `df[0:5]`

Explain why you did not choose 2 of the other options:

Cumulative normal distribution (z table)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
−3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999

Upper percentage points for the Student's *t* distribution

	α								
ν	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	318.309	636.619
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.385	3.646
35	0.255	0.682	1.306	1.690	2.030	2.438	2.724	3.340	3.591
40	0.255	0.681	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.254	0.679	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	0.254	0.677	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576	3.090	3.291