UC Berkeley

Department of Civil and Environmental Engineering

CE93 Fall 2022
Midterm 2

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Name: ____

SID Number: ____

- This exam contains 11 printed pages (including the cover page). Write your name on all pages of the question paper and your cheat sheet.
- You are allowed to use a calculator and a 2-sided cheat sheet of size 8.5 x 11 inches. Submit your cheat sheet with the exam booklet to score 10 points on the exam.
- Please show all steps you took to reach the answer. Points will be liberally awarded for each correct step towards the solution. If you need more space than provided, please use the back side of the sheets, and clearly write the question number corresponding to your work on the back pages, if any.
- Please wait till the end of the exam to leave.

Question	**Points**	**Score**
1	24	
2	26	
3	18	
4	12	
Cheat sheet	10	
Total	**90**	

Question 1: (24 points)

The truck traffic on a certain highway can be described as a Poisson process with a mean arrival rate of 1 truck per minute.

(a) What is the probability that there will be at least two trucks passing a weigh station on this highway in a 5-min period? (**8 points**)

$X \sim \text{Poisson}(1)$

(a) $X_5 \sim \text{Poisson}(5)$

$$P(X_5 \geq 2) = 1 - P(X_5 < 2)$$
$$= 1 - (P(X_5 = 0) + P(X_5 = 1))$$
$$= 1 - \left(\frac{e^{-5} 5^0}{0!} + \frac{e^{-5} 5^1}{1!}\right)$$
$$= 1 - e^{-5}(1 + 5)$$
$$= 1 - 6e^{-5}$$
$$\simeq 0.9596$$

(b) The weight of each truck is random, and the probability that a truck is overloaded is 10%. What is the probability that at most one of the next five trucks stopping at the weigh station will be overloaded? (**8 points**)

$X \sim \text{Bin}(5, 0.1)$

$$P(X \leq 1) = P(X = 0) + P(X = 1)$$
$$= \binom{5}{0} 0.1^0 (0.9)^5 + \binom{5}{1} 0.1^1 (0.9)^4$$
$$= \left(\frac{5!}{5!\,0!}\right) \times 1 \times 0.9^5 + \left(\frac{5!}{4!\,1!}\right) \times 0.1 \times 0.9^4$$
$$\simeq 0.5905 + 0.3281$$
$$= 0.9186$$

(c) Suppose no trucks passed the weigh station in 5 minutes. What is the probability that no truck will pass the weigh station in the next 2 minutes? (**8 points**)

$X \sim \text{Poisson}(1) \rightarrow T \sim \text{Exp}(1)$

$P(T > 5+2 \mid T > 5) = \dfrac{P(T > 5+2)}{P(T > 5)} = \dfrac{e^{-(5+2)}}{e^{-5}} = e^{-2}$

$= P(T > 2)$ (memoryless property)

$\therefore P(T > 5+2 \mid T > 5) = P(T > 2) \simeq \underline{\underline{0.1353}}$

Question 2: (26 points)

The load on a column of a high-rise reinforced concrete building may be composed of the dead load (D), the live load (L), and the earthquake-induced load (E).

Suppose the properties of these load components are as follows:

μ_D = 2,000 tons, σ_D = 210 tons

μ_L = 1,500 tons, σ_L = 350 tons

μ_E = 2,500 tons, σ_E = 450 tons

The total load carried by the column would be T = D + L + E.

(a) If the D, L, and E are statistically independent, what is the standard deviation of the total load? (**6 points**)

$\text{Var}(T) = \text{Var}(D+L+E) = \text{Var}(D) + \text{Var}(L) + \text{Var}(E)$

$= 210^2 + 350^2 + 450^2$

$= 369100$

$\therefore \sigma_T = \sqrt{\text{Var}(T)} \simeq \underline{\underline{607.536}}$ tons

(b) The load carrying capacity (C) of the column has the following properties:
μ_C = 10,000 tons, σ_C = 1,500 tons
The column will be overloaded if the total load is greater than its load carrying capacity. If all the variables D, L, E, and C have a Normal distribution, what is the probability that the column will be overloaded? (**12 points**)

$C \sim N(10000, 1500^2) \Rightarrow C \sim N(10000, 2250000)$

$T \sim N(2000+1500+2500, 210^2+350^2+450^2)$

$\Rightarrow T \sim N(6000, 369100)$

Overloaded if $T > C \Rightarrow$ if $T - C > 0$.

Let $X = T - C$.

Since T and C have a Normal distribution, X has a Normal distribution too.

$X \sim N(6000-10000, 369100+2250000)$

$\Rightarrow \mu_X = -4000, \ \sigma_X = 1618.36$

$P(X>0) = 1 - P(X \le 0) = 1 - P(Z \le z_0)$

$z_0 = \frac{0 - \mu_X}{\sigma_X} = \frac{0-(-4000)}{1618.36} \simeq 2.47$

$P(Z \le z_0) = 0.9932 \Rightarrow P(\text{overloading}) = 1 - 0.9932 = \underline{0.0068}$

(c) There are 50 columns, each with μ_C = 10,000 tons, σ_C = 1,500 tons in a structure. What is the distribution (and its parameters) of the mean load carrying capacity of the columns in the structure? What is the coefficient of variation of the mean load carrying capacity? (**8 points**)

$n = 50, \ \mu_C = 10000$ tons, $\sigma_C = 1500$ tons

$\Rightarrow \bar{X} \sim N\left(10000, \frac{1500^2}{50}\right) \rightarrow$ from CLT for sample mean

$\Rightarrow \mu_{\bar{X}} = 10000$ tons

$\sigma_{\bar{X}} = \sqrt{45000}$ tons $\simeq 212.13$

$\text{c.o.v.} = \frac{\sigma_{\bar{X}}}{|\mu_{\bar{X}}|} \simeq \underline{0.0212}$

(d) If the dead load (D) and the earthquake load (E) are correlated with a correlation coefficient ($\rho_{D,E}$) of 0.6 and the live load (L) is uncorrelated with the dead load and the earthquake load, what is the coefficient of variation of the total load? (10 points – ***BONUS QUESTION****)*

$$C.O.V. = \frac{\sigma_T}{|\mu_T|}$$

Since $T = D + L + E$ and $\rho_{D,E} = 0.6$, $\rho_{D,L} = 0$, $\rho_{E,L} = 0$.

$$\sigma_T^2 = \sigma_D^2 + \sigma_L^2 + \sigma_E^2 + 2\,Cov(D, E)$$

$$\Rightarrow \sigma_T^2 = 210^2 + 350^2 + 450^2 + 2 \times 0.6 \times 210 \times 450$$

$$\Rightarrow \sigma_T^2 = 482500$$

Also, $\mu_T = E(T) = E(D + L + E) = 2000 + 1500 + 2500 = 6000$

$$\Rightarrow C.O.V. = \frac{\sqrt{482500}}{6000} \simeq 0.1104$$

Question 3: (18 points)

The occurrences of floods in the town may be modeled as a Poisson process. The following table shows the record of floods during a 10-year period for a town.

Year	Number of floods	Year	Number of floods
1994	1	1999	0
1995	0	2000	2
1996	1	2001	0
1997	1	2002	0
1998	0	2003	1

(a) Using the count data from the table, estimate the rate parameter of the Poisson process. (**6 points**)

$x = 6$

$t = 10$

$\Rightarrow \hat{\lambda} = \frac{x}{t} = \underline{\underline{0.6}}$

(b) What is the uncertainty (standard deviation) of the estimate? (**6 points**)

$\sigma_{\hat{\lambda}} = \sqrt{\frac{\hat{\lambda}}{t}} = \sqrt{\frac{0.6}{10}} \simeq \underline{\underline{0.2449}}$

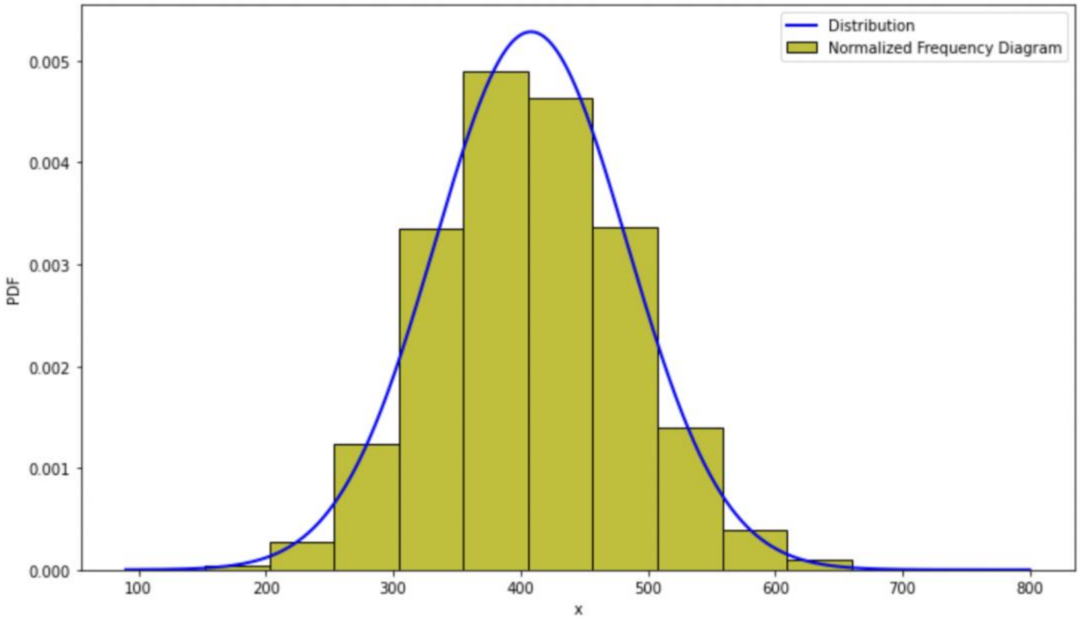
(c) Assuming the rate parameter of the Poisson process remains the same, how many years of data must be used to reduce the uncertainty in the estimate to 0.1? (**6 points**)

If $\sigma_{\hat{\lambda}} = 0.1$, $\Rightarrow 0.1 = \sqrt{\frac{\hat{\lambda}}{t}} = \sqrt{\frac{0.6}{t}}$

$\Rightarrow t = \frac{0.6}{(0.1)^2} = \underline{\underline{60 \text{ years}}}$

Question 4: (12 points)

(a) The figure shows the normalized frequency diagram from a sample of data. We also fit a distribution to the data and plot the PDF of the fitted distribution on top of the normalized frequency diagram. Which of the following lines of code would produce the plot of the probability density function in the figure below?




```
# A Exponential Distribution
plt.plot(x,pdf_exponential,color='blue',linestyle='-',lw=2,label='Exponential')
```

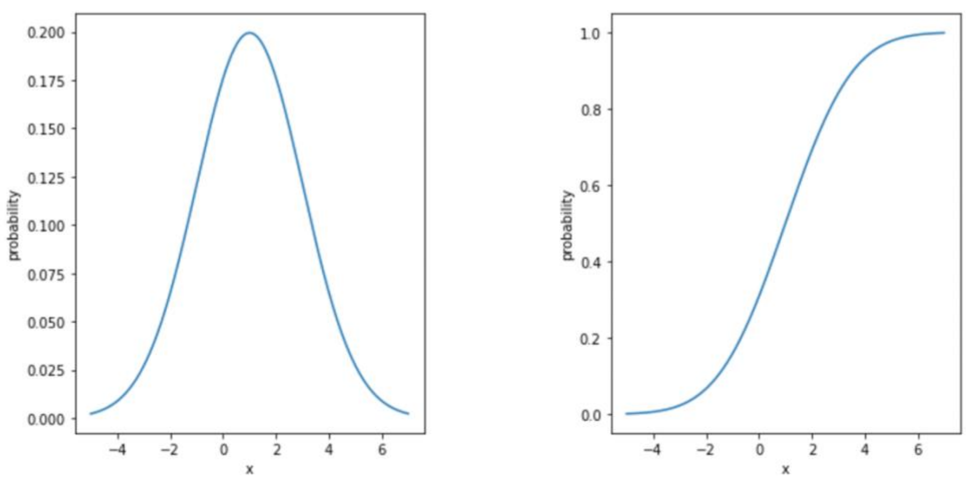
```
# B Normal Distribution
plt.plot(x,pdf_normal, color='blue', lw=2, label='Normal')
```

```
# C Uniform Distribution
plt.plot(x,pdf_uniform, color='blue',linestyle=':', lw=3, label='Uniform')
```

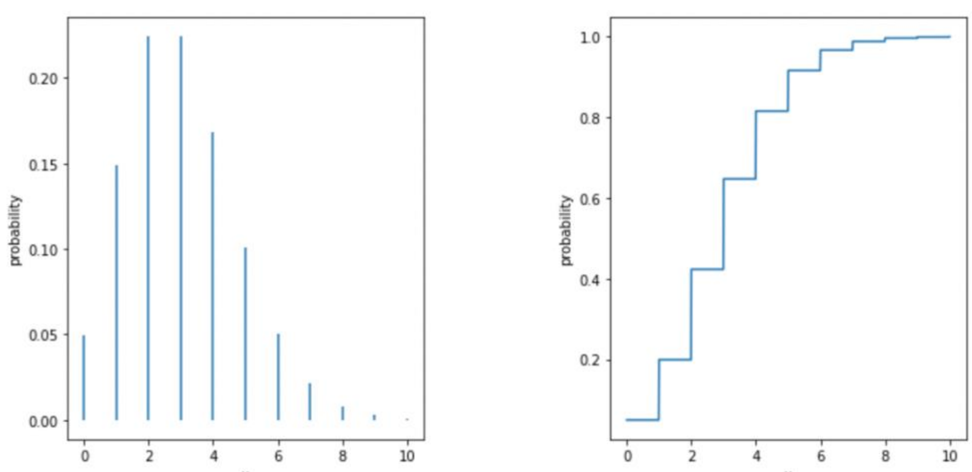
```
# D Poisson Distribution
plt.plot(x,pdf_poisson, color='blue',linestyle='--', lw=3, label='Poisson')
```

(b) Which of the following graphs would this code snippet produce?

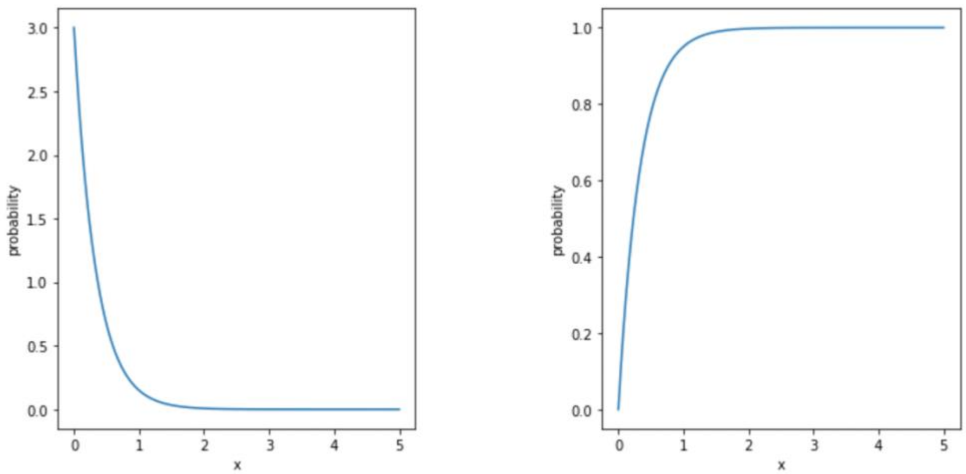
```
p = 0.3
x = np.arange(0,20+0.01,0.01)

fig = plt.figure(figsize = (10,5))
axs = []
axs.append(fig.add_subplot(121))
axs[0].vlines(x,ymin = 0,ymax = geom.pmf(x,p))
axs[0].set_xlabel("x")
axs[0].set_ylabel("probability")

axs.append(fig.add_subplot(122))
axs[1].plot(x,geom.cdf(x,p))
axs[1].set_xlabel("x")
axs[1].set_ylabel("probability")

plt.tight_layout(w_pad=10)
plt.show()
```



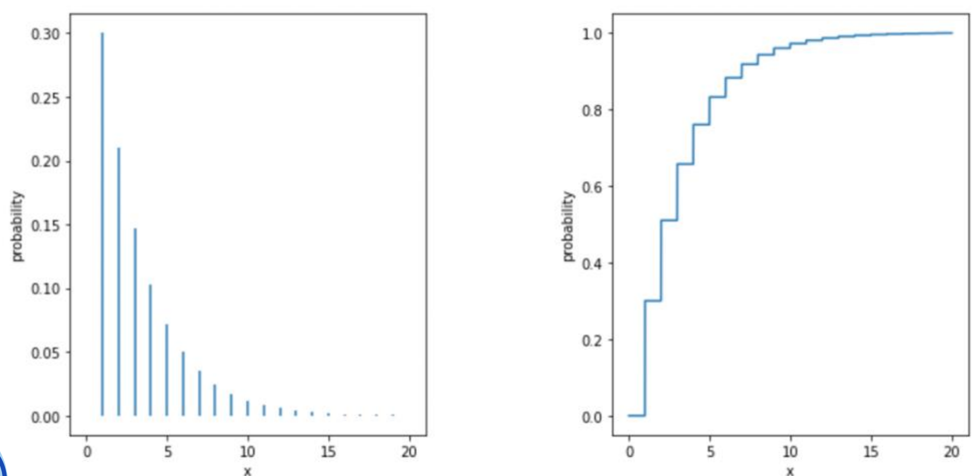
a.



b.


3.0
2.5
2.0
1.5
1.0
0.5
0.0
probability
0 1 2 3 4 5
x
1.0
0.8
0.6
0.4
0.2
0.0
probability
0 1 2 3 4 5
x

c.

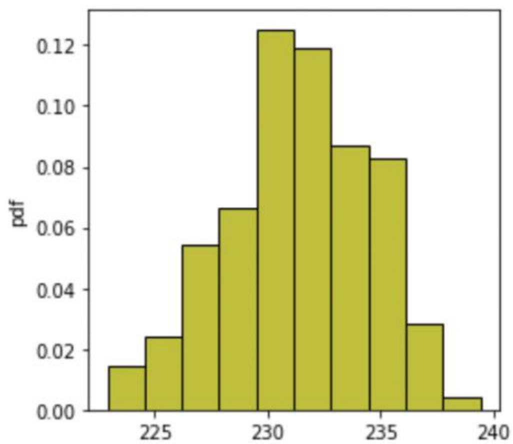

0.30
0.25
0.20
0.15
0.10
0.05
0.00
probability
0 5 10 15 20
x
1.0
0.8
0.6
0.4
0.2
0.0
probability
0 5 10 15 20
x

d.

(c) This code snippet uses some concepts from Lab 7. Variable 'UV' is an array containing the UV irradiance data measured in a particular month, and 'month' is an array with the same index as 'UV' containing the corresponding month (1 is January, 2 is February, etc.).

Comment line by line, what is the code trying to do?

```
mjj = UV[(month==5) | (month==6) | (month==7)]

mean_list_mjj = []

for i in range(300):
    mean_list_mjj.append(np.mean(mjj.sample(n=50, replace=True)))

plt.figure(figsize=(4,4))
plt.hist(mean_list_mjj, ec='black', density=True, color='y')
plt.ylabel('pdf')
plt.show()
```



Line 1: Get UV measurements of month May, June, July and store in mjj variable;
Line 2: create empty arrays to store means;
Line 3/4: loop 300 times for sampling 50 times from mjj array and get the mean of each, and store in the empty array;
Line 5: initialize figure size 4,4;
Line 6 plot the mean list as histogram;
Line 7 give label;
Line 8 show graph

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998