Answer Key.

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UC Berkeley
Department of Civil Engineering

CE93– QUIZ #1

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October 2, 2019

Name: ______________________________
SID Number: ______________________________

This exam contains 8 pages (including this cover page) and 3 questions. Total of points is 100. You are allowed to use a calculator and a 5"x 8" cheat sheet. Communication with others is not allowed except with the proctors. No leaving early. If you finish early, feel free to meditate, relax, draw something, or try to solve the Bonus question.
Good luck!

Distribution of Marks

Question	Points	Score
1	36	
2	40	
3	24	
Total:	100	

1. Multiple Choice Questions

(a) (12 points) Let A = array([65, 66, 68, 68, 69, 72]) be an array of average annual temperatures in a given city.

Which of the following commands calculates the probability that average annual temperatures are less than 68 in Python?

i. np.sum(A<68)/len(A)
ii. np.sum(A<68)/A
iii. np.sum(A<68)/sum(A)
iv. (A<68)/len(A)

Which of the following plots the Cumulative Frequency Distribution of A in Python?

i. ax.hist(A, cumulative=True, density=True, histtype = 'step')
ii. ax.hist(A, cumulative=True, density=False, histtype = 'step')
iii. ax.hist(A, density=False, histtype = 'step')
iv. ax.cumuldiag(A)

Which of the following calculates the coefficient of variation of A in Python?

i. np.std(A)/np.mean(A)
ii. np.mean(A)/np.std(A)
iii. np.cov(A)
iv. np.coeff(A)

(b) (4 points) A box contains 10 soil samples: 5 of these samples are sand, 3 are clay, and 2 are silt samples. You choose 5 samples at random. What is the probability that 3 samples are sand, 1 sample is clay, and 1 sample is silt?

i. 0.238
ii. 0.722
iii. 0.351
iv. 0.151

$$= \frac{\binom{5}{3}\binom{3}{1}\binom{2}{1}}{\binom{10}{5}}$$

(c) (12 points) An environmental committee has 12 students: 4 from Stanford and 8 from Berkeley. The committee has its meetings around a U-shaped table with 12 seats.

In how many ways can the students be seated around the table?

i. 12!

ii. $\binom{12}{4}$

iii. $\binom{12}{8}$

iv. 12

The committee wishes to elect a President and a Vice President. How many possible outcomes are there for the election?

i. $\frac{2}{12}$

ii. $\binom{12}{2}$

iii. 12

iv. 132 $= 12 \times 11$

Now suppose that the President has to be a Berkeley student, and there is no restriction on the school of the Vice President. How many possible outcomes are there for the election under this assumption?

i. $\binom{12}{2}$

ii. 88 $= 8 \times 11$

iii. 12

iv. 32

(d) (4 points) Two events, A and B, are statistically independent. A has a probability of 0.5, and B has a probability of 0.3. What is the probability of event A happening given that event B has happened?

i. 0.15

ii. 0.6

iii. 1

iv. 0.5 $= P(A)$

(e) (4 points) On her commute to UC Berkeley, Joan chooses between 4 modes of Transportation: Walking, Biking, Bus, or Driving. We know that the probability of Joan choosing to Walk is 0.05, and her probability of driving is 0.4. Which of the below CANNOT be her probability of taking the bus?

i. 0.6 $(\sum P_i \leq 1)$

ii. 0.2

iii. 0.4

iv. 0.45

2. A passenger airplane A is scheduled to take-off from a gate in a given airport at 9 PM. The weather forecasts windy conditions on the night the flight is scheduled, with probabilities 0.2, 0.3, and 0.5 of having heavy, moderate, and light wind, respectively. Based on historical data and the design characteristics of airplane A, we know that the probability of A being delayed is 0.5 if the winds are heavy, 0.2 if the winds are moderate, and 0.05 if the winds are light.

   (a) (10 points) Write the symbolic expression for the event that airplane A is delayed, and calculate its probability.

$$P(A) = P(A|H) \times P(H) + P(A|M) \times P(M) + P(A|L) \times P(L)$$

$$= 0.5 \times 0.2 + 0.2 \times 0.3 + 0.05 \times 0.5$$

$$= 0.185$$

   (b) (10 points) Assume that airplane A was delayed. What is the probability that the wind was not heavy?

$$P(\bar{H}|A) = 1 - P(H|A) = 1 - \left[\frac{P(A|H) \times P(H)}{P(A)}\right]$$

$$= 1 - \frac{0.5 \times 0.2}{0.185}$$

$$\boxed{P(\bar{H}|A) = 0.459}$$

(c) (10 points) Now assume another airplane, airplane B, is scheduled to take-off one hour after airplane A. We also know that if A is delayed, then the probability of B being delayed is 0.8 if the winds are heavy, 0.4 if the winds are moderate, and 0.1 if the winds are light. What is the probability that plane A is delayed and plane B is not delayed?

$$P(A\bar{B}) = P(A) - P(AB)$$

$$\cdot)\ P(AB) = P(AB|H)\times P(H) + P(AB|M)\times P(M) + P(AB|L)\times P(L)$$

$$= P(B|AH)\times P(A|H)\times P(H) + P(B|AM)\times P(A|M)\times P(M) + P(B|AL)\times P(A|L)\times P(L)$$

$$= 0.8\times 0.5\times 0.2 + 0.4\times 0.2\times 0.3 + 0.10\times 0.05\times 0.5$$

$$\boxed{P(A\bar{B}) = 0.0785}$$

$$\cancel{P(A\bar{B}) = P(A) - P(AB) = 0.185 - 0.0785}$$

$$\cancel{\boxed{P(A\bar{B}) = 0.1065}}$$

(d) (10 points) If A was delayed and B was not delayed, what is the probability that the winds were heavy?

$$P(H|A\bar{B}) = \frac{P(A\bar{B}|H)\times P(H)}{P(A\bar{B})} = \frac{P(\bar{B}|AH)\times P(A|H)\times P(H)}{P(A\bar{B})}$$

$$= \frac{[1 - P(B|AH)]\times P(A|H)\times P(H)}{P(A\bar{B})} = \frac{(1-0.8)\times 0.5\times 0.2}{\cancel{0.1065}\ 0.0785}$$

$$\boxed{P(H|A\bar{B}) = \cancel{0.1878}\ 0.25}$$

3. A food-delivery transportation company delivers food from three restaurants (A, B, and C) in a given city. Based on customer complaints, it seems that there are missing items in 3% of deliveries from restaurant A, 5% of deliveries from restaurant B, and 2% of deliveries from restaurant C. Assume that 30% of the delivery orders that the company receives are from restaurant A, 20% are from restaurant B, and 50% are from restaurant C.

(a) (12 points) On a Thursday night, one customer receives an order and calls in to complain about a missing item. What is the probability that the item came from restaurant A?

$$P(A|M) = \frac{P(M|A) \times P(A)}{P(M)}$$

$$= \frac{0.03 \times 0.3}{0.03 \times 0.3 + 0.05 \times 0.2 + 0.02 \times 0.5}$$

$$\boxed{P(A|M) = 0.31}$$

(b) (12 points) Another customer, on another night, orders two food items: one from restaurant A, and one from restaurant B. What is the probability that only one order is missing? Assume restaurants operate independently, i.e. a missing item from one restaurant is independent from whether another restaurant will have a missing item.

$$P(1 \text{ item missing}) = \underbrace{(0.03)}_{P(M_A)}\underbrace{(1-0.05)}_{P(\bar{M}_B)} + \underbrace{(1-0.03)}_{P(\bar{M}_A)}\underbrace{(0.05)}_{P(M_B)}$$

$$\boxed{P(1 \text{ item missing}) = 0.077}$$

(c) (Bonus) Now assume that the delivery company has implemented an inspection service that checks the order for missing items before delivering it to its customers. This service has a success rate of 90%, meaning that it fails to detect a missing item 10% of the time, and that its accuracy is independent of the restaurant from which the food is coming. The system never detects false positives, i.e. it never detects a missing item if the item is not missing. A customer places an order from restaurant B, and it is marked "correct" by the company's new inspection service. What is the probability that the order is in fact missing items?

$$P(M_B \mid C) = \frac{P(M_B \cap C)}{P(C)} = \frac{P(C \mid M_B) \times P(M_B)}{P(C \mid M_B) \times P(M_B) + P(C \mid \bar{M}_B) \times P(\bar{M}_B)}$$

$$= \frac{0.1 \times 0.05}{0.1 \times 0.05 + 1 \times 0.95} = 0.00523$$