CE 11 Engineered Systems & Sustainability: Final Exam			Total Score ↓
Date: 12.15.2021,	Total questions: **14**	Total points: **110**	
Name:		Time: 2 hr 40 min	

Question:	1	2	3	4	5	6	7	8
Points:	10	5	3	10	10	3	10	5
Score:								

Question:	9	10	11	12	13	14		Total
Points:	10	10	10	10	10	4		110
Score:								

Instructions:

1. **Show your work. When needed, use words to explain your reasoning.**
2. **Use a reasonable number of sig figs in your answers (2-3 will suffice).**
3. **The exam is open-book, open-notes. Important constants and unit conversions are provided on the last page of the exam.**
4. **No use of the internet or communication with other people is allowed while taking the exam.**
5. **Exam will end at 10:50pm to allow for time to upload exams to gradescope. Any exams not uploaded by 11:00pm will receive a zero.**

10 1. **Gaseous Fuel Storage:** A natural gas-fired power plant consistently generates 50 MW of electricity. The power plant has an efficiency of 50% (calculated using lower heating value of the fuel). You need to size a gas storage tank at a pressure of 60 atm capable of storing a 10-hour supply of gas for your power plant. Assume the temperature is 20°C. How big should it be? Report your answer in m^3. Helpful data:
Lower Heating Value of Natural Gas = 47.1 MJ/kg
Approximate Molecular Weight of Natural Gas = 19 g/mol

Solution:

We need to use the equation PV=ηRT to solve this problem. We are solving for V. We have P (60 atm), T (20 + 273.15 = 293.15 K). R is a constant. So we just need to calculate the moles of gas to store (η). A natural gas-fired power plant generating 50 MW of power for 10 hours will generate:

$$50 \text{ MW} \times 10 \text{ h} = 500 \text{ MWh}$$

Our power plant is 50% efficient, so we need to double that to get the energy content of the gas we need to store:

$$\frac{500 \text{ MWh}}{0.50} = 1000 \text{ MWh}$$

Now we should convert MWh of gas to MJ. This will make it easier to use the lower heating value to calculate the mass, and ultimately, moles of gas:

$$1000 \text{ MWh} \times 3600 \text{ MJ/MWh} = 3.6 \times 10^6 \text{ MJ}$$

We use the MJ of gas to calculate mass of gas:

$$\frac{3.6 \times 10^6 \text{ MJ gas}}{47.1 \text{ MJ/kg}} = 7.64 \times 10^4 \text{ kg gas}$$

$$\frac{7.64 \times 10^7 \text{ g gas}}{19 \text{ g/mol}} = 4.02 \times 10^6 \text{ mol gas}$$

Now we can plug this into PV=ηRT:

$$(60 \text{ atm})V = (4.02 \times 10^6 \text{ mol})(0.0821 \frac{\text{L-atm}}{\text{mol-K}})(293.15 \text{ K})$$

Don't forget to divide your answer by 1000 to convert from L to m^3:

$$V = 1610 \text{ m}^3$$

$\boxed{5}$ 2. **Time Value of Money:** You are designing a structure that may be subject to sea level rise in the coming decades. If you have an annual interest rate of 6%, how much should you be willing to pay today in mitigation measures to avoid $1 million of flooding damage-related costs 20 years from now? You may exclude any other effects like inflation.

Solution: We know that the savings at $t = 20$ years will be $1 million and $i = 0.06$ so we can plug that into the basic question for accounting for the time value of money:

$$X(1 + 0.06)^{20} = \$1 \times 10^6$$

Then solve for X:

$$X = \$312,000$$

$\boxed{3}$ 3. **Functional Units for Life-Cycle Assessment:** Which functional unit is likely to be most appropriate for comparing the GHG emissions associated with traveling by BART (our Bay Area regional rail transit system) or by car? Select only one.

a. Vehicle-kilometer traveled
b. MJ of fuel
c. Gallons of fuel
d. Vehicle-MJ
e. Passenger-kilometer traveled
f. Tonne-kilometer traveled

Solution: The correct answer is e (Passenger-kilometer traveled). This is because the goal of BART and a car are to move people from one place to another. It is worth noting that even this is not a perfect comparison. BART and cars might take a different route to get from a given point to another (perhaps BART takes a more direct route and the roads are more circuitous).

$\boxed{10}$ 4. **Groundwater Pumping:** You are a regulator trying to determine how much groundwater a local farmer is pumping. The hydraulic conductivity in the subsurface is 30 m/day. We don't know the water level at the well or the pumping well's radius, but we *do* have measurements at 2 observation wells. We also know the aquifer is 35 m thick. Well 1 is located 30 m away from the center point of the pumping well and is drawn down by 2 m. Well 2 is located 50 m away and is drawn down by 0.2 m. How much water is the farmer's well extracting? Report your answer in cubic meters per day (round to the nearest cubic meter).

Solution:

First we should take what we know about the draw-down at the two wells and convert those into head (height of the water table) at each well:

$$h_1 = 35 \text{ m} - 2 \text{ m} = 33 \text{ m}$$

$$h_2 = 35 \text{ m} - 0.2 \text{ m} = 34.8 \text{ m}$$

We also know:

$$r_1 = 30 \text{ m and } r_2 = 50 \text{ m}$$

$$K = 30 \text{ m/d}$$

We have the following equation that we used in our last homework:

$$K = \frac{Q \ln \frac{r_1}{r_w}}{\pi(h_1^2 - h_w^2)}$$

The trick here is the figure out that we we can sub in the radius (distance) for the second well instead of needing the radius of the pumping well itself.

$$K = \frac{Q \ln \frac{r_1}{r_2}}{\pi(h_1^2 - h_2^2)}$$

Rearrange to solve for Q:

$$Q = \frac{K\pi(h_1^2 - h_2^2)}{\ln \frac{r_1}{r_2}}$$

$$Q = \frac{(30 \text{ m/d})\pi((33 \text{ m})^2 - (34.8 \text{ m})^2)}{\ln(\frac{30 \text{ m}}{50 \text{ m}})}$$

$$Q = 22,516 \text{ m}^3\text{/day}$$

$\boxed{10}$ 5. **Planetary Energy Balance:** If the current average temperature on Earth is 15 °C, how much warmer would it need to get to achieve an *increase* in black body radiation equivalent to 10^{15} Watts? Report your answer as ΔT (in °C).

> **Solution:**
>
> This is just the reverse of a problem from Midterm 1 (with the numbers changed a bit). First we need to calculate the the outgoing radiation in watts per square meter using the surface area of the Earth:
>
> $$\Delta E = 1.0 \times 10^{15} \text{ W}$$
> $$\Delta L = \frac{1.0 \times 10^{15} \text{ W}}{4\pi(6.3781 \times 10^{6} \text{ m})^2}$$
> $$\Delta L = 1.956 \text{ W/m}^2$$
>
> Now let's figure out the new temperature:
>
> $$\Delta L = L_f - L_0$$
> $$1.956 \text{ W/m}^2 = \sigma T_f^4 - \sigma T_0^4 = \sigma(T_f^4 - (15 + 273.15 \text{ K})^4)$$
>
> Plug in the Stefan-Boltzmann constant and calculate T_f:
>
> $$\frac{1.956 \text{ W/m}^2}{5.67 \times 10^{-8} \text{W/(m}^2\text{K}^4)} + (288.15 \text{ K})^4 = T_f^4$$
> $$T_f = 288.5$$
> $$\Delta T = 288.51 - 288.15 = 0.36°\text{C}$$
>
> (Remember, the scale of C and K are the same, so no unit conversion is needed here to report our answer in degrees Celsius.)

$\boxed{3}$ 6. **Global Warming Potential:** Why do we specify a timescale for global warming potential (e.g. 100-year GWP) of individual greenhouse gases, such as CH_4 and N_2O? Select only one.

a. Incoming solar radiation will change over time
b. Earth's albedo changes over time
c. Different greenhouse gases have different lifetimes in the atmosphere
d. Policy-makers may not care about warming long into the future

> **Solution:**
>
> The correct answer c: we specify the duration for GWP values because different greenhouse gases have different lifetimes in the atmosphere.

$\boxed{10}$ 7. **Methane:** You are going to burn methane as a fuel and you plan to calculate the greenhouse gas-intensity per MJ of thermal energy you get from the fuel.

(a) Suppose your methane (CH_4) is fossil-derived. What is the CO_2-intensity of burning it for fuel (g CO_2/MJ)? You may use 50.0 MJ/kg as the heating value.

Solution:

$$\frac{1 \text{ kg CH}_4}{50 \text{ MJ}} \cdot \frac{1 \text{ mol CH}_4}{0.016 \text{ kg}} \cdot \frac{1 \text{ mol CO}_2}{1 \text{ mol CH}_4} \cdot \frac{44 \text{ g CO}_2}{1 \text{ mol CO}_2} = 55 \text{ g CO}_2/\text{MJ}$$

(b) Suppose you still burn the fossil-derived methane, but for every kg of methane you burn as fuel, an amount equivalent to 2% of what your burned leaks out into the atmosphere. What is the greenhouse gas intensity (g CO_2-equivalent/MJ)? Use GWP of 25 for methane (GWP for CO_2 = 1).

Solution:

$$55 \text{ g CO}_2/\text{MJ} + 0.02 \cdot \frac{1 \text{ kg CH}_4}{50 \text{ MJ}} \cdot \frac{1000 \text{ g}}{1 \text{ kg}} \cdot \frac{25 \text{ g CO}_2e}{1 \text{ g CH}_4}$$

$$55 \text{ g CO}_2e/\text{MJ} + 10 \text{ g CO}_2e/\text{MJ} = 65 \text{ g CO}_2e/\text{MJ}$$

(c) Now you are burning methane sourced from an anaerobic digester that processes manure. As in part b, for every kg of methane you burn as fuel, an amount equivalent to 2% of what your burned leaks out into the atmosphere. What is the greenhouse gas intensity (g CO_2-equivalent/MJ), assuming you care only about gases that contribute to long-term net warming?

Solution:
Because it is a biogenic source (contemporary carbon), we can zero out the CO_2 emissions and just account for the methane. This is the same calculation we did for methane leakage in part b:

$$0.02 \cdot \frac{1 \text{ kg CH}_4}{50 \text{ MJ}} \cdot \frac{1000 \text{ g}}{1 \text{ kg}} \cdot \frac{25 \text{ g CO}_2e}{1 \text{ g CH}_4} = 10 \text{ g CO}_2e/\text{MJ}$$

$\boxed{5}$ 8. **Waste Management:** Which items are commonly recycled with today's collection, sorting, and recycling infrastructure? Select all that apply.

a. LDPE bag
b. PET bottle
c. Office paper
d. Cardboard box
e. Aluminum can
f. HDPE flat mailer envelope
g. PP clamshell container
h. Cylindrical HDPE container

i. PVC plastic shrink wrap

j. Cylindrical LDPE container

Solution:

The correct answers are b, c, d, e, h.

$\boxed{10}$ 9. **Oxygen Sag Curve:** You are discharging wastewater to a river. Assume deoxygenation and reaeration rate coefficients are $k_d = 0.4$/day and $k_r = 0.15$/day, respectively. Relevant data is below:

Parameter	Wastewater	River Water
Flow (m^3/s)	2.5	2.5
L_0 (mg/L)	6	2
DO (mg/L)	1	5
DO_{sat}	8.0	8.0

Downstream of the discharge point, the water in the river moves at a constant speed of 0.7 m/s. How far downstream of the discharge point will the dissolved oxygen deficit peak? Report your answer in meters. Round your answer to the nearest meter.

Solution:

To solve this problem, we will need to solve for t_c and then use the river's speed to determine the distance from the discharge point. As provided in homework 6, the equation for t_c is:

$$t_c = \frac{1}{k_r - k_d} \ln\left(\frac{k_r}{k_d}\left(1 - \frac{D_0(k_r - k_d)}{k_d L_0}\right)\right)$$

We are given k_r and k_d but we still need to calculate L_0 and D_0. Because the flows of the wastewater and river water are equal, we can just take the average of the two values in both cases:

$$L_0 = \frac{6 \text{ mg/L} + 2 \text{ mg/L}}{2} = 4 \text{ mg/L}$$

Remember, D_0 is the oxygen deficit, so $D_0 = DO_{sat} - DO$:

$$D_0 = \frac{(8 - 1 \text{ mg/L}) + (8 - 5 \text{ mg/L})}{2} = \frac{10 \text{ mg/L}}{2} = 5 \text{ mg/L}$$

Now we have D_0 and L_0 so we can plug those into the equation for t_c:

$$t_c = \frac{1}{0.15 \text{ day}^{-1} - 0.4 \text{ day}^{-1}} \ln\left(\frac{0.15 \text{ day}^{-1}}{0.4 \text{ day}^{-1}}\left(1 - \frac{5 \text{ mg/L}(0.15 \text{ day}^{-1} - 0.4 \text{ day}^{-1})}{0.4 \text{ day}^{-1}(4 \text{ mg/L})}\right)\right)$$

$$t_c = 1.61 \text{ days}$$

$$d = (1.61 \text{ days})(0.7 \text{ m/s})(3600 \text{ s/h})(24 \text{ h/day}) = 97,400 \text{ m}$$

$\boxed{10}$ 10. **Population Growth & Carrying Capacity:** The current global population (in 2021) is about 7.9 billion and it is growing at 1.05% annually. If you assume logistic growth with a carrying capacity of 15 billion people, what would the global population be in 2100? Round to the nearest 0.1 billion.

Solution:

Recall the logistic growth equation:

$$P(t) = \frac{K}{1 + e^{-rt}(\frac{K}{P_0} - 1)}$$

We need to use r_0 to compute r:

$$r_0 = r(1 - \frac{P_0}{K})$$

$$r = \frac{0.0105}{1 - \frac{7.9 \text{ billion people}}{15 \text{ billion people}}} = 0.02218 \text{ year}^{-1}$$

Now we can plug that into our logistic growth equation:

$$P(t) = \frac{15 \text{ billion}}{1 + e^{-0.02218 \text{ yr}^{-1} \cdot 79 \text{ yr}}(\frac{15 \text{ billion}}{7.9 \text{ billion}} - 1)}$$

$$P(t = 79) = 13.0 \text{ billion people}$$

$\boxed{10}$ 11. **Air Conditioner and Batteries:** You bought a fancy new "Powerwall" battery system to provide backup power to your home. It has 25 kWh of usable storage capacity. Its maximum charge and discharge rate is 5 kW. You want to keep your air conditioning unit running during summertime blackouts and your AC unit requires 3.5 kW. Assume your battery loses 7% of its storage energy during discharge.

(a) How many hours can you run your AC using your battery after the blackout starts if you discharge it fully?

Solution:
We have 25 kWh of storage capacity, and we will get slightly less than that because of the losses during discharge. Our air conditioner uses 2.5 kWh per hour of operation. So we can calculate the time we can run the AC unit as follows:

$$\text{t} = \frac{25 \text{ kWh} \cdot (1 - 0.07)}{3.5 \text{ kWh / h}} = 6.6 \text{ hours}$$

(b) How much electricity would your AC unit use each month (30 days) if you chose to run it for 12 hours per day? Report your answer in kWh.

Solution: We take the kW value and multiply it by the total number of hours:

$$E = 3.5 \text{ kW} \cdot 12 \text{ h/day} \cdot 30 \text{ days} = 1260 \text{ kWh}$$

$\boxed{10}$ 12. **Renewable Electricity Generation:** You are comparing 2 potential options for generating electricity: install 10 wind turbines or cover 2 km^2 with solar PV panels. Use an average windspeed of 10 m/s (assume this already adjusts for speeds outside the cut-in and cut-out limit). The rotor diameter for each wind turbine is 80 m. The irradiance (solar resource) at your location is 5.5 kWh/m^2/day. You may use 1.225 kg/m^3 as the density of air. Assume 10% loss for conversion of DC to AC power (where needed). Your wind turbines are 50% efficient (in an absolute sense, not relative to the Betz maximum) and your solar PV panels are 20% efficient.

(a) How much electricity will the wind turbines be able to sell to the grid in 1 year? Report your answer in kWh/year.

Solution:
We must first calculate the energy in the column of wind entering each turbine:

$$E = \frac{1}{2}mv^2$$

$$m = \rho\frac{vt\pi d^2}{4}$$

$$v = 10 \text{ m/s}$$

$$\rho = 1.225 \text{ kg/m}^3$$

$$d = 80 \text{ m}$$

$$t = (1 \text{ yr})(365 \text{ days/yr})(24 \text{ hrs/day})(60 \text{ min/hr})(60 \text{ s/min}) = 31,536,000 \text{ s}$$

$$m = 1.225 \text{ kg/m}^3\frac{(10 \text{ m/s})(31536000 \text{ s})(\pi)(80 \text{ m})^2}{4}$$

$$m = 1.942 \times 10^{12} \text{ kg/year}$$

$$E = \frac{1}{2}(1.942 \times 10^{12} \text{ kg/year})(10 \text{ m/s})^2$$

$$E = 9.71 \times 10^{13} \text{ J/year}$$

This is the energy entering each turbine in a single year. Because our turbine is 50% efficient, we will cut that in half.

$$E = 4.855 \times 10^{13} \text{ J/year}$$

Now convert J to kWh:

$$E = 4.855 \times 10^{13} \text{ J/year} \cdot 10 \text{ turbines} \cdot (\frac{1 \text{ kWh}}{3.6 \times 10^6 \text{ J}}) = 1.35 \times 10^8 \text{ kWh/year}$$

Note that we do not need to account for DC-AC conversion because wind turbines generate AC power.

(b) How much electricity will the solar PV panels be able to sell to the grid in 1 year? Report your answer in kWh/year.

Solution:

We know that our panels get 5.5 kWh/m^2/day of energy from the sun and we are covering 2 km^2. We also know they are 20% efficient. Because solar PV panels generate DC power, we will need to account for the DC-AC conversion losses of 10%.

$$E = (5.5\ \text{kWh/m}^2/\text{day})(2\ \text{km}^2)\frac{(1000\ \text{m})^2}{(1\ \text{km})^2}(0.2)(1-0.1)$$

$$E = 1.98 \times 10^6\ \text{kWh/day}(365\ \text{days/year})$$

$$E = 7.23 \times 10^8\ \text{kWh/year}$$

$\boxed{10}$ 13. **Bayes Theorem:** You have developed a robot that uses image recognition to inspect drinking water pipes and detect potential leaks. It never misses a leak (100% of real leaks are identified). However, when it runs through pipes with no leaks, it will return a false positive 10% of the time. You run the robot through 100 separate lengths of pipe, 30 of which *do* have leaks (70 are not leaky). What percentage of the leaks detected by your robot will be false positives?

Solution:

We have 100 total samples in this problem. 30 have leaks and we know the robot will catch all of them. We also know that, of the non-leaking pipe lengths, the robot will incorrectly flag 10% of them as leaking. So:

$$P(positive|leak) = 1$$

$$P(positive|no\ leak) = 0.1$$

$$P(leak) = \frac{30}{100} = 0.3$$

$$P(no\ leak) = \frac{70}{100} = 0.7$$

So the robot will catch all 30 of the leaking pipes. Of the 70 non-leaking pipes, it will incorrectly flag 10% of them as leaking.

$$\text{real leaks} = 30$$

$$\text{falsely identified leaks} = 0.1(70) = 7$$

$$\text{fraction of false positives in total identified leaks} = \frac{7}{30+7} = 19\%$$

As you can see, one can reason this out without memorizing any equations. The formula from Bayes Theorem would be:

$$P(no\ leak|positive) = \frac{P(positive|no\ leak) \cdot P(no\ leak)}{P(positive)}$$

$$P(no\ leak|positive) = \frac{0.1 \cdot 0.7}{0.3 + 0.7 \cdot 0.1}$$

The result is the same: 19%

$\boxed{4}$ 14. **Nuclear Energy:** What does a moderator do in a nuclear reactor? Select all that apply.

a. Absorbs neutrons

b. Slows neutrons down

c. Enables continuation of the chain reaction

d. Stops or slows the chain reaction

Solution:

The correct answers are: b, c.

Potentially useful constants and conversion factors:

Constants Other Useful Values:
Stefan-Boltzmann constant: $\sigma = 5.67 \times 10^{-8}$ W/(m^{2}K^4)
Radius of the Earth: $R_E = 6.3781 \times 10^6$ m
Heat capacity of water = 4.18 kJ kg^{-1}°C^{-1}
Enthalpy of vaporization for water = 2257 kJ kg^{-1}
Lower Heating Value of Coal = 29.0 MJ/kg
Lower Heating Value of Natural Gas = 47.1 MJ/kg
Lower Heating Value of Pure Methane = 50.0 MJ/kg
Approximate Molecular Weight of Natural Gas = 19 g/mol
R = 0.0821 $\frac{\text{L atm}}{\text{mol K}}$
1 barn = 1.0×10^{-28} m^2
Atomic masses (in g/mol) are:
C = 12.011
H = 1.008
O = 15.999
N = 14.007
Ca = 40.08
Useful unit conversion factors:
1 ton = 0.907185 metric tons
Remember: ton = short ton
Remember: tonne = metric ton
Remember: 1 m^3 = 1000 liters
Temperature in Kelvin = $°C + 273.15$