Midterm Exam #1
CEE 11 – Engineered Systems and Sustainability
University of California at Berkeley
Fall Semester 2022

NAME: Solutions

Instructions: answer the questions that follow directly on these pages in the spaces provided. Use the back of the page if you need more room for your answer. If you believe there is insufficient information provided to answer a question completely, state reasonable additional assumptions and proceed from there.

This quiz is closed-book/closed-notes, but see last page for aid sheet. You may use a calculator.

Please write your name in the space provided above!

Time: 50 minutes

Question	SCORE:	OUT OF:
1	_____	5
2	_____	5
3	_____	5
4	_____	5
Total	_____	20

1. SOLAR HOT WATER HEATER

A rooftop solar collector with surface area A = 4 m^2 has water circulating through it at a rate of 4 liters per minute while exposed to sunlight with intensity of 800 W m^{-2}. Assume 50% of the sunlight energy is captured by the collector and heats the water flowing through it. What would be the temperature increase of the water passing through the collector? Answer in ˚C.

Solar energy absorbed by water

$$\dot{Q} = 50\% \cdot 800\,\frac{W}{m^2} \times 4\,m^2$$

$$= 1600\,W = 1600\,\frac{J}{s}$$

Temperature increase of water

$$\Delta T = \frac{\dot{Q}}{\dot{m}\,C_{p,H_2O}} = \frac{1600\ J/s \times \frac{1\,kJ}{1000\,J}}{4\,\frac{L}{min} \times \frac{1\,kg}{L} \times \frac{1\,min}{60\,s} \times 4.2\,\frac{kJ}{kg\,^\circ C}}$$

$$= \frac{1.6\ kJ/s}{0.28\,\frac{kJ}{s \cdot {}^\circ C}}$$

$$= +5.7^\circ C$$

2. ENERGY BALANCE FOR A FLAT PLANET

Suppose the Earth really is flat, shaped like a large coin with one side that faces the sun at all times. Assume that this flat Earth is the same temperature everywhere, including the side that faces away from the sun, that the thickness of the flat Earth "coin" is negligible, and that planetary albedo is zero. Find the equilibrium surface temperature (answer in ˚C) given incoming solar shortwave radiation S = 1360 W m^{-2}.

$\alpha = 0$

Incoming: (one side) $S \cdot \pi R_E^2 (1-\alpha) = S \cdot \pi R_E^2$

Outgoing (both sides) $L \cdot 2\pi R_E^2 = \sigma T_E^4 \cdot 2\pi R_E^2$

Incoming = Outgoing at balance so

$$S \cdot \pi R_E^2 = \sigma T_E^4 \cdot 2\pi R_E^2$$

$$T_E^4 = \frac{S}{2\sigma}$$

$$T_E = \left(\frac{S}{2\sigma}\right)^{1/4}$$

$$= \left[\frac{1360 \ W/m^2}{2 \times 5.67 \times 10^{-8} \frac{W}{m^2 K^4}}\right]^{1/4}$$

$$= 331 K = 58^\circ C$$

3. LOGISTIC GROWTH

Suppose human population grows from 7.5 billion in 2020 to an ultimate population of 12 billion following the logistic growth curve. Assuming a growth rate of 1.25% per year in 2020, in what year will the world population reach 10 billion?

In 2020, $r_{2020} = 1.25\%/\text{yr.} = r\left(1 - \frac{P_{2020}}{K}\right)$

so $r = \dfrac{1.25\%\,\text{yr}}{1 - \frac{7.5}{12}} = 3.33\%/\text{yr}.$

when does future population reach
$P(t) = 10$ billion?

$$P(t) = \frac{K}{1 + e^{-rt}\left(\frac{K}{P_{2020}} - 1\right)} = 10 \text{ billion}$$

$$\frac{12 \text{ billion}}{10 \text{ billion}} - 1 = e^{-rt}\left(\frac{12 \text{ billion}}{7.5 \text{ billion}} - 1\right)$$

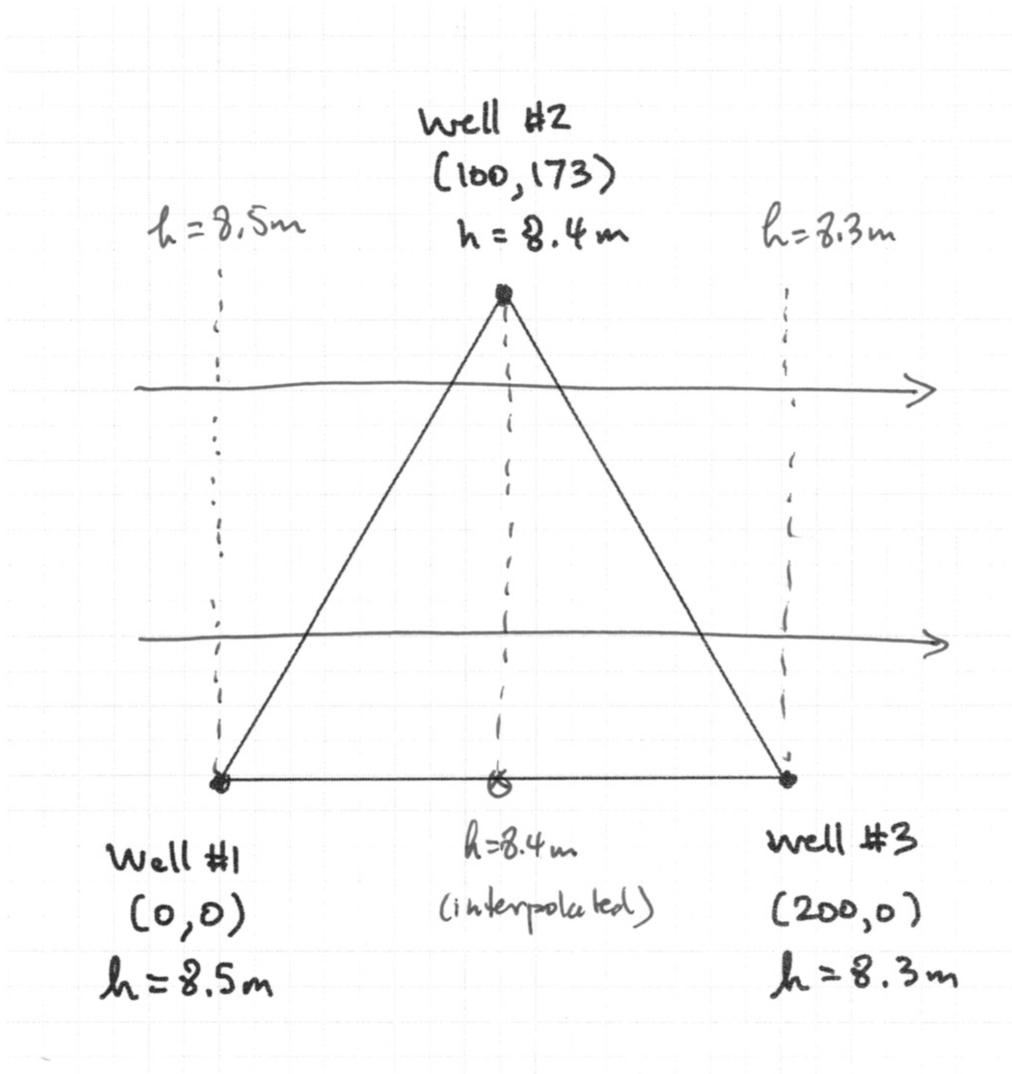
$$-\frac{0.033}{\text{yr}} \times t = \ln\left(\frac{1.2 - 1}{1.6 - 1}\right) = -1.1$$

$t = 33$ yrs so year 2053

is when we get to 10 billion.

4. THREE-WELL TECHNIQUE

Three observation wells located at the vertices of an equilateral triangle that measures 200 meters on each side have the following heads: well 1, 8.5 m; well 2, 8.4 m; well 3, 8.3 m. See diagram below for plan view layout of the wells. Calculate the hydraulic gradient and draw and label equipotential lines and streamlines on the diagram.



$$\frac{dh}{dL} = \frac{h_1 - h_3}{200\text{m}} = \frac{0.2\text{m}}{200\text{m}} = 0.001 \frac{\text{m}}{\text{m}}$$