Spring 2023
ME C85/ CE C30 Midterm 1

Name: Solutions (AF)

Please write your name at the top of the page as indicated; write answers in the space provided; Show additional work on the back side if necessary. ***Do not remove or add any pages.***

Good luck!

Problem 1 (25 Points)

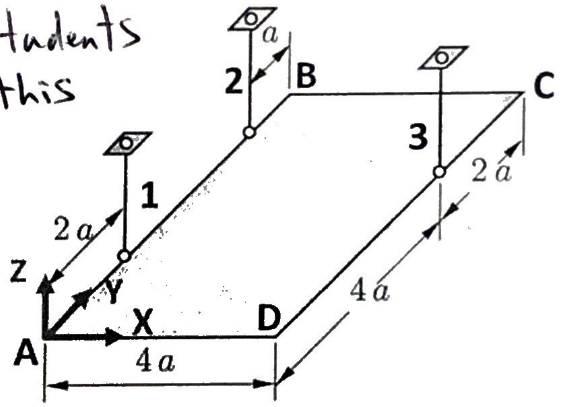
A rectangular plate of negligible weight is suspended by three vertical wires 1, 2, 3 as shown. Assume the origin is at point A, as shown.

a. In vector form, what is the resultant of the moments created by the forces in all three wires about point A? Assume $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ are the unit vectors along the X, Y, and Z directions, respectively. **[10 points]**

many students missed this

Assume

$T_1 = T_2 = T_3 = T$



$$\underline{M} = \sum \underline{r} \times \underline{F} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 2a & 0 \\ 0 & 0 & T \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 5a & 0 \\ 0 & 0 & T \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 4a & 4a & 0 \\ 0 & 0 & T \end{vmatrix}$$

5 writing the equation

$\cancel{\underline{M} = Ta(11\underline{i} - 4\underline{j})}$

3 for execution of equation

$$\boxed{\underline{M} = Ta(11\,\underline{i} - 4\,\underline{j} + 0\,\underline{k})}$$

2 final answer/clear train of thought

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b. If the plate is subjected to a single concentrated downwards vertical force Q (not shown), determine the (x,y) location of the force so that the plate is in static equilibrium. **[8 points]**

② $\sum M_x = 0 = 2aT + 5aT + 4aT - Qy$

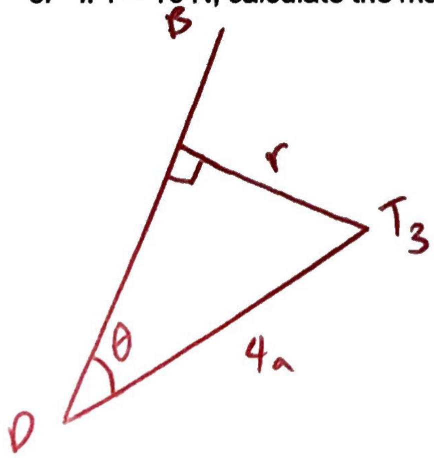
$T = \frac{Q}{3} \rightarrow \boxed{y = \frac{11}{3}a}$ ②

② $\sum M_y = 0 = 4aT - Qy = \frac{4}{3}aQ - Qx$

$\rightarrow \boxed{x = \frac{4}{3}a}$ ②

-1 for each wrong sign

c. If T = 10 N, calculate the magnitude of moment created by wire 3 about line BD. **[7 points]**



$\theta = \tan^{-1}\left(\frac{4a}{6a}\right)$

$r = 4a\sin\theta = \frac{8}{\sqrt{13}}a$ ②

$\boxed{\underline{M} = \underline{r} \times \underline{F} = \frac{80}{\sqrt{13}}a \text{ N}}$ $\left(\frac{80}{\sqrt{13}} = 22.2\right)$

② for moment equation

③ for answer

~~for units~~

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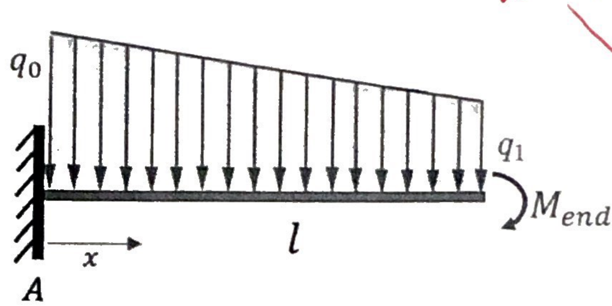
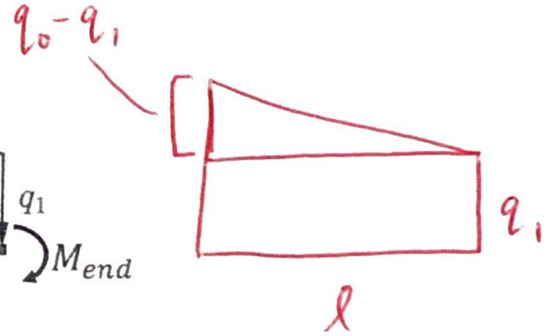
Problem 2 (25 Points)

A beam of length L is loaded by a couple moment M_{end} at its end and by a distributed force per unit length, ranging from q_0 to q_1 along its length as shown, and is fully supported at A. Ignore the mass of the beam.

a. Determine the magnitude of an equivalent single force for the distributed loading and its location. **[9 points]**





$$\boxed{F = \frac{l}{2}(q_0 + q_1)} \quad 3$$

$$3 \quad \bar{x} = \frac{\sum xA}{\sum A} = \frac{l\frac{l}{2}q_1 + \frac{l}{3}(q_0 - q_1)\frac{l}{2}}{q_1 l + \frac{q_0 - q_1}{2} l} = \frac{\frac{l^2}{6}(q_0 + 2q_1)}{\frac{l}{2}(q_0 + q_1)}$$

$$\boxed{\bar{x} = \frac{l}{3}\frac{(q_0 + 2q_1)}{(q_0 + q_1)}} \quad 3$$

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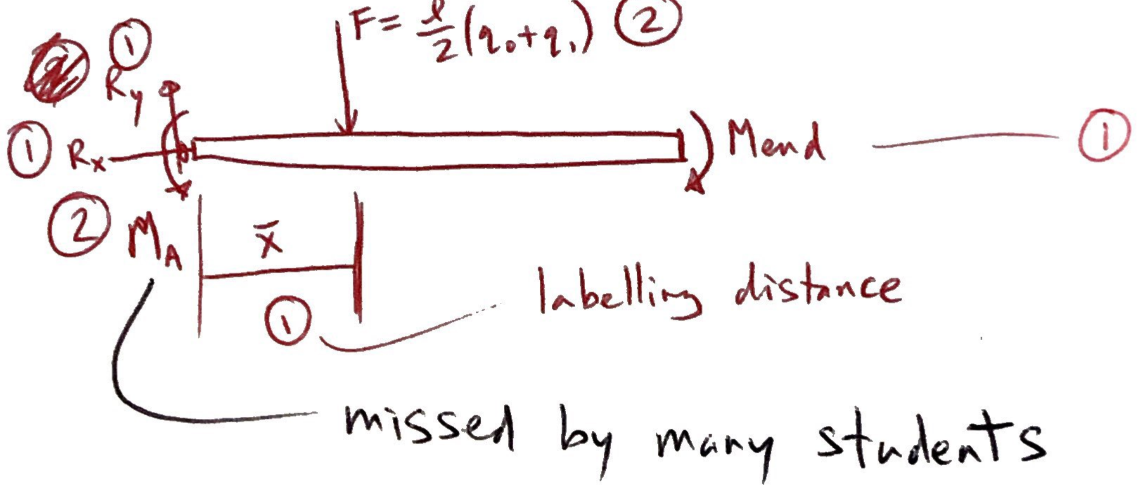
b. Using this equivalent force, draw a fully labeled free-body diagram of the beam. **[8 points]**



c. Solve for the external reaction loads (forces and/or ~~moments)~~ at ~~A required to maintain~~ static equilibrium of the beam. Clearly denote the directions of all loads. **[8 points]**

2 $\sum F_y = 0 = R_y - F \longrightarrow \boxed{R_y = \frac{\ell}{2}(q_0+q_1)}$ 2

2 $\sum M_A = 0 = M_A - F\bar{x} - M_{end}$

$$M_A = \frac{\ell}{2}(q_0+q_1)\,\frac{\ell}{3}\,\frac{(q_0+2q_1)}{(q_0+q_1)} + M_{end}$$

$$\boxed{M_A = \frac{\ell^2}{6}(q_0+2q_1) + M_{end}}$$ 2

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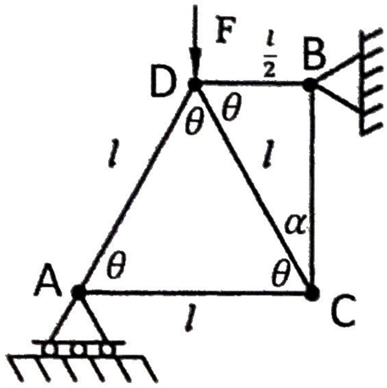
Problem 3 (25 Points)

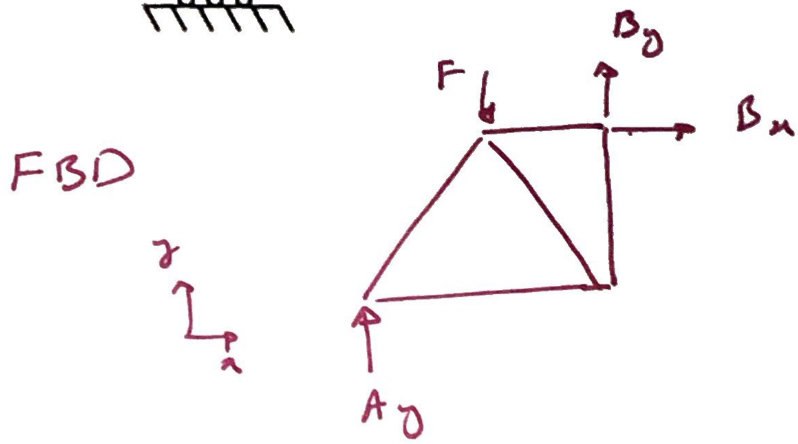
ACBD is a truss structure with a pin joint at B, rollers at A, and external force F at D.

a. Find the external reaction forces at A and B. **[5 points]**



$\alpha = 30^\circ, \theta = 60^\circ$

FBD



2.5

$+\rightarrow \sum F_x = 0 \rightarrow B_x = 0$

$+\uparrow \sum F_y = 0 \rightarrow B_y + A_y - F = 0$

$\circlearrowleft + \sum M_B = 0 \rightarrow F\frac{l}{2} - A_y l = 0$

1

$$\Rightarrow \begin{cases} B_x = 0 & 0.5 \\ B_y = \frac{F}{2} & 0.5 \\ A_y = \frac{F}{2} & 0.5 \end{cases}$$

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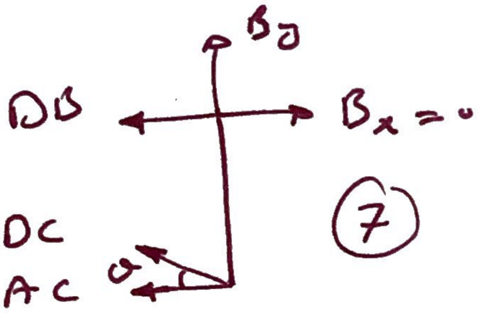
b. Using the method of sections, find the force in truss member DC. Denote if the member is in compression or tension. **[10 points]**



7

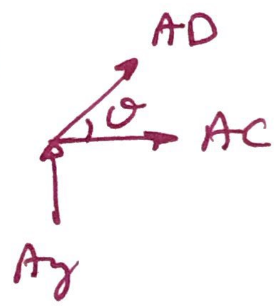
1 $\Sigma F_y = 0 \rightarrow B_y + DC \sin\theta = 0$

$\rightarrow DC = -\frac{F}{\sqrt{3}}$ Comp.

1 1

c. Using the method of joints, find the force in the truss member AC. Denote if the member is in compression or tension. **[10 points]**

7



$\Sigma F_y = 0 \rightarrow A_y + AD \sin\theta = 0$

1 $\Sigma F_x = 0 \rightarrow AC + AD \cos\theta = 0$

$\rightarrow AC = A_y \cot\theta$

$\rightarrow AC = \frac{F}{2} * \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{F}{2\sqrt{3}}$ 1

Tension 1

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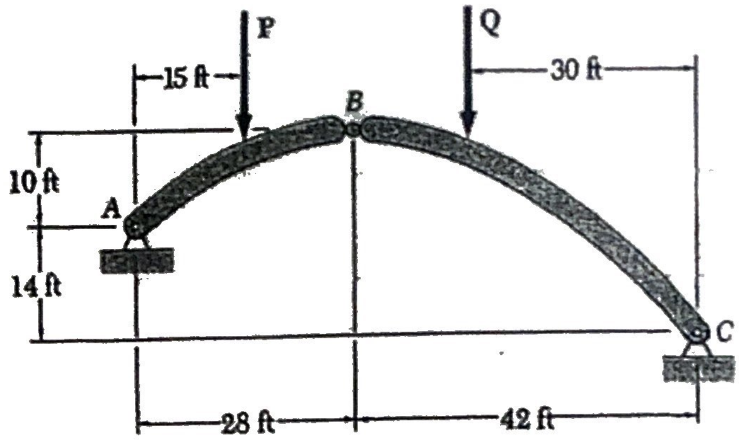
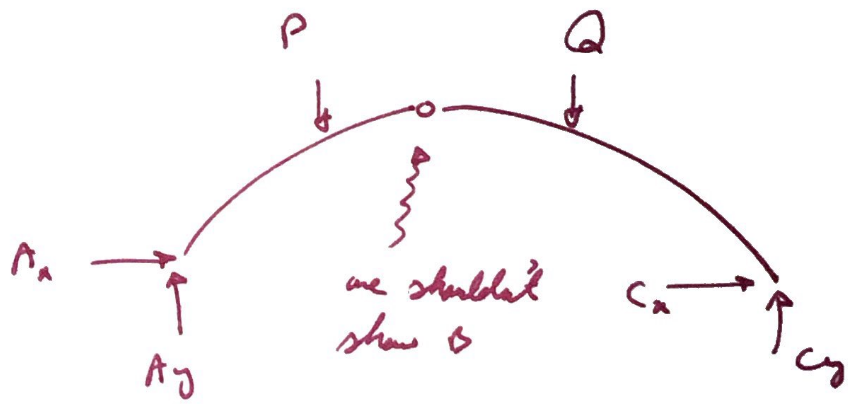
Problem 4 (25 Points)

The curved rod AB is attached to the curved rod BC by using a pin (hinge joint) at point B. (Note: these rods are rigid bodies) The supports at A and C are pin supports (hinge joints). For this system, show how you would go about finding the external reaction forces at A and C and the forces going through the pin joint at B (there is no need to actually solve the equations for this problem). Specifically:



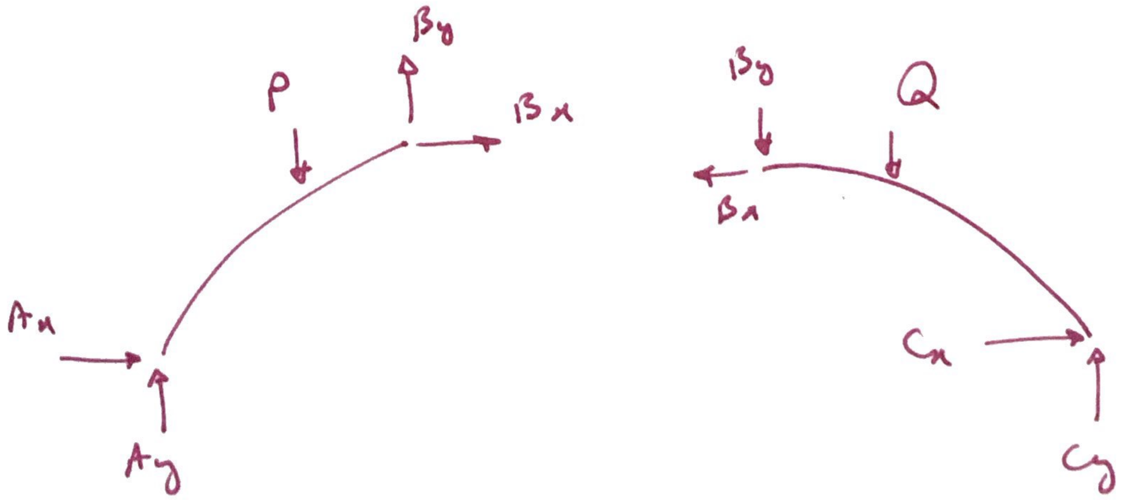
a. Draw all required free body diagrams, fully labeled. **[10 points]**

1st FBD



or

2nd FBD



b. Identify all unknowns. **[3 points]**

$A_x, A_y, C_x, C_y, B_x, B_y \longrightarrow 1^{st}$ FBD

$A_x, A_y, C_x, C_y \longrightarrow 2^{nd}$ FBD

c. Write out all required equations [hint: number of unknowns and equations should be equal]. **[12 points]** we need to use 2^{nd} FBD only.

AB $\rightarrow$ $\sum F_x = 0 \rightarrow A_x + B_x = 0$

$\sum F_y = 0 \rightarrow A_y + B_y - P = 0$

$\circlearrowleft_+ \sum M_A = 0 \rightarrow 28 B_y - 10 B_x - 15 P = 0$

BC $\rightarrow$ $\sum F_x = 0 \rightarrow C_x - B_y = 0$

$\sum F_y = 0 \rightarrow C_y - B_y - Q = 0$

$\circlearrowleft_+ \sum M_C = 0 \rightarrow 24 B_x + 42 B_y + 30 Q = 0$