ME C85 / CE C30 Midterm 2 Spring 2023

Name: Solutions

Please write your name at the top of each page as indicated. Write answers in the space provided. Show additional work on the back side if necessary. **Box your final answers** where applicable, or else you may not receive full credit for your work. **Do not remove or add any pages.**

In all questions, when drawing free body diagrams, do not simply draw over the provided images – instead, draw a new schematic of the free body and add the appropriate loading.

Good luck!

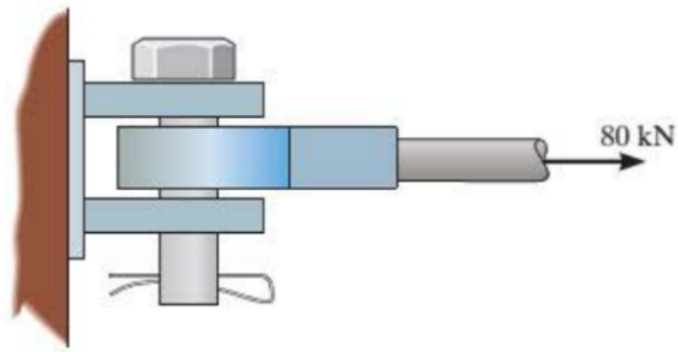
Question 1	Question 2	Question 3	Question 4	Total
/25	/25	/25	/25	/100

For grading purposes only: do not mark.

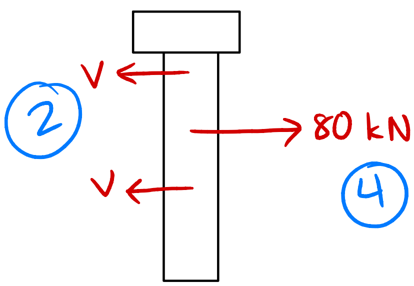
Spring 2023
ME C85/ CE C30 Midterm 2

1. Stress and Design (25 Points)

The vertical bolt is made of material having a failure shear stress of 160 MPa and supports the assembly as shown, under a horizontal force of 80 kN. In the following question, ignore the mass of any elements and assume that shear stress in the bolt is uniform within any transverse cross-section.



A. **[8 points]** Draw a free body diagram of the bolt.



$$\sum F_x = 0 = 80\text{ kN} - 2V$$

$$\rightarrow V = 40\text{ kN} \quad 2$$

B. **[9 point]** Determine the minimum required bolt diameter (to the nearest millimeter) so that it will not fail in shear under the loading shown.

$$\underbrace{\tau_{fail} = \frac{V}{A}}_{3} = \frac{V}{\frac{\pi}{4}d^2} \longrightarrow \underbrace{d = \sqrt{\frac{4V}{\pi \tau_{fail}}}}_{2}$$

$$\longrightarrow d = \sqrt{\frac{4 \times 40\text{ kN}}{\pi \times 160\text{ MPa}}} = 0.0178\text{ m} = \boxed{18\text{ mm}}$$

4

C. **[4 points]** What would be the minimum required bolt diameter if the design specifications required a factor of safety of 3.0?

1 $F.S. = \dfrac{\tau_{fail}}{\tau_{allow}} \longrightarrow \tau_{allow} = \dfrac{\tau_{fail}}{F.S.} = \dfrac{160\ MPa}{3} = 53.33\ MPa$ 1

From part B,

$$d = \sqrt{\frac{4V}{\pi \tau_{allow}}} = \sqrt{\frac{4 \times 40\,kN}{\pi \times 53.33\,MPa}} = 0.0309\,m = \boxed{31\,mm}$$

2

D. **[4 points]** Assume the material has a Young's modulus of 200 GPa and a Poisson's ratio of 0.3. For a bolt diameter of 15 mm, what is the average shear strain developed in the bolt for this loading?

$$G = \underbrace{\frac{E}{2(1+\nu)}}_{1} = \frac{200\ GPa}{2(1+0.3)} = \underline{76.92\ GPa}$$

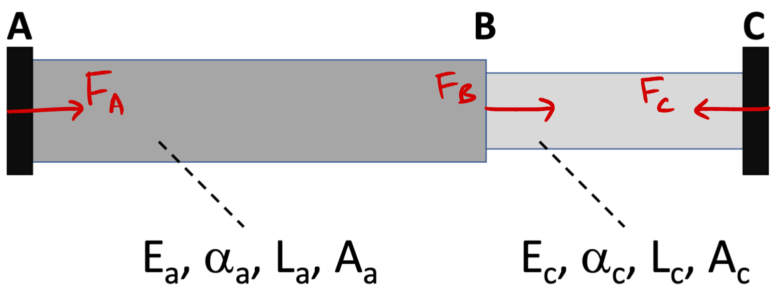
1

$$\tau = G\gamma = \frac{V}{A} \longrightarrow \gamma = \overbrace{\frac{V}{AG}}^{1} = \frac{40\ kN}{\frac{\pi}{4}(0.015\,m)^2 \times 76.9\ GPa}$$

$$\boxed{\gamma = 2.94 \times 10^{-3}\ rad}$$ 1

2. Thermal Expansion (25 Points)

Two rods (one aluminum, one copper) are joined at B and are fully supported by rigid walls at each end, as shown. If initially there is no stress in either rod, derive the equation for the reaction force at end A after the two rods are uniformly heated to a temperature change of ΔT. Your solution must be in terms of only the following: the Young's modulus (E_a, E_c), coefficient of thermal expansion (α_a, α_c), cross-sectional area (A_a, A_c), and length (L_a, L_c) of each rod and the change in temperature ΔT. As part of your derivation, explain all assumptions and your logic.



Rigid Walls $\longrightarrow \delta_{AB} + \delta_{BC} = 0$ 5

Static Equilibrium $\longrightarrow \sum F_B = 0 = F_A - F_c \longrightarrow F_A = F_c = P$ 5

$$\delta_{AB} = \alpha_a \Delta T L_a - \frac{PL_a}{A_a E_a}, \quad \delta_{BC} = \alpha_c \Delta T L_c - \frac{PL_c}{A_c E_c}$$ 5

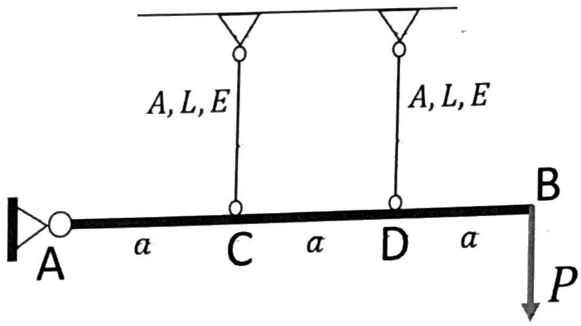
$$\delta_{AB} + \delta_{BC} = \alpha_a \Delta T L_a - \frac{PL_a}{A_a E_a} + \alpha_c \Delta T L_c - \frac{PL_c}{A_c E_c} = 0$$

$$(\alpha_a L_a + \alpha_c L_c)\Delta T = P\left(\frac{L_a}{A_a E_a} + \frac{L_c}{A_c E_c}\right)$$

$$\boxed{P = \frac{(\alpha_a L_a + \alpha_c L_c)\Delta T}{\left(\frac{L_a}{A_a E_a} + \frac{L_c}{A_c E_c}\right)}}$$ 10

3. Axial Deformation (25 Points)

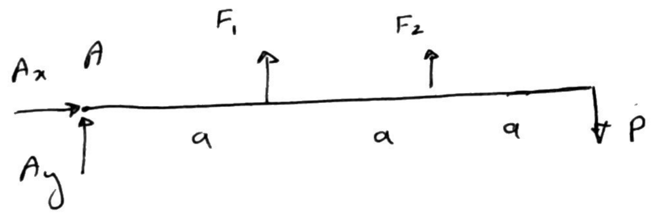
The rod ACDB is rigid and is attached to two equal vertically oriented elastic cables (cross sectional area A, length L, and Young's modulus E) at C and D. At end B, a known vertical force P is applied, as shown. Assume the support at A is a hinge joint, ignore the mass of all elements and the rod ACDB, and assume all deformations are small. For this statically indeterminate system:



A) **[10 points]** Draw a fully labeled free body diagram of rod ACDB and write out the corresponding three equations of static equilibrium. Identify and label all unknown forces.

FBD :

$y \uparrow \rightarrow x$

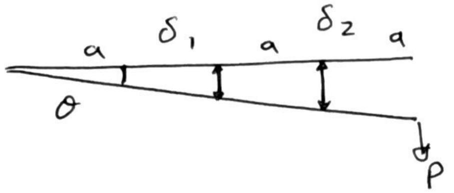


$\xrightarrow{+} \sum F_x = 0 \rightarrow A_x = 0$

$+\uparrow \sum F_y = 0 \rightarrow A_y + F_1 + F_2 - P = 0$

$\circlearrowleft_+ \sum M_A = 0 \rightarrow -3Pa + 2F_2 a + F_1 a = 0$

B) **[15 points]** Describe how you would solve for all the forces from part A. There is no need to solve the equations, but **write out any additional equations** that are needed and explain the principles and your logic. [Hint: number all of your independent equations, the total number of which needs to be the same as the number of unknowns].



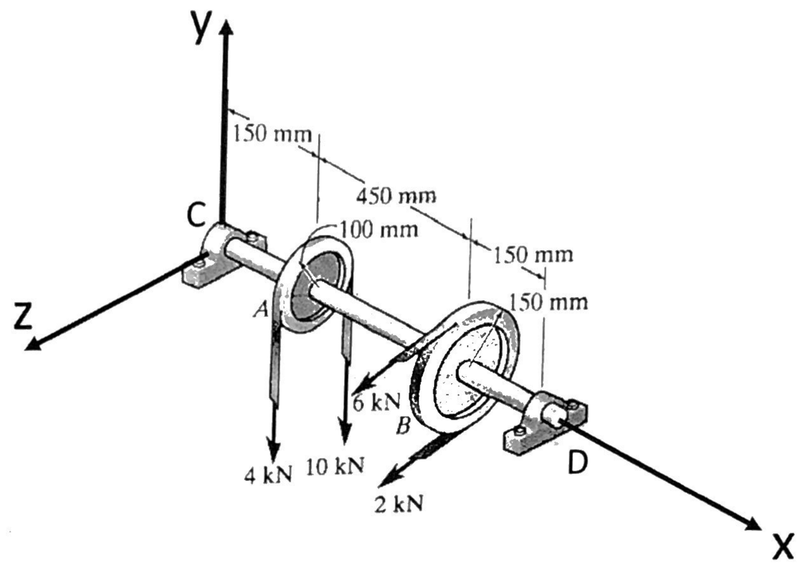
$$\frac{a}{\delta_1} = \frac{2a}{\delta_2} \rightarrow \delta_2 = 2\,\delta_1 \rightarrow \Delta l_2 = 2\Delta l_1$$

$$\Rightarrow \frac{F_2\, l}{AE} = 2\,\frac{F_1\, l}{AE} \rightarrow F_2 = 2F_1$$

to find F_1 and F_2 we can use $\Sigma M_A = 0$ and
to find A_y we can use $\Sigma F_y = 0$

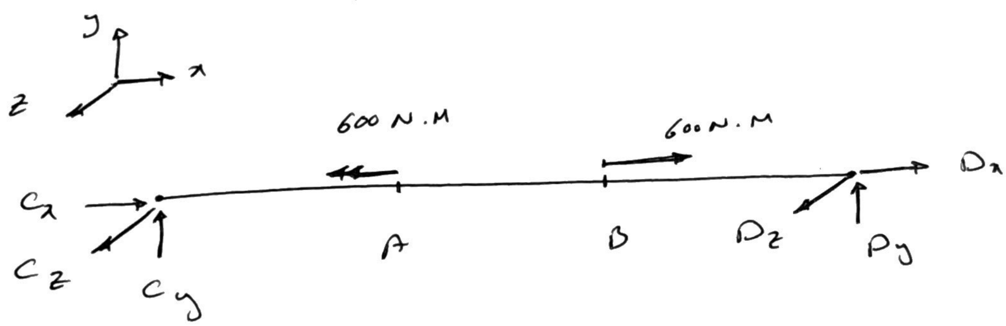
4. Torsion (25 Points)

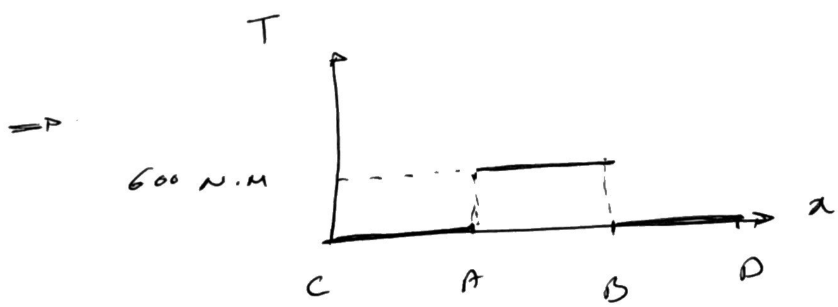
A uniform shaft CD has a diameter of 40 mm and shear modulus of elasticity G=100 GPa. The shaft is rotating at a constant angular speed, is loaded as shown by the belts at A and B, and is supported by frictionless bearings at C and D.



A. **[7 points]** Draw a graph of the internal torsion in the shaft along its length, x, between C and D.

FBD:



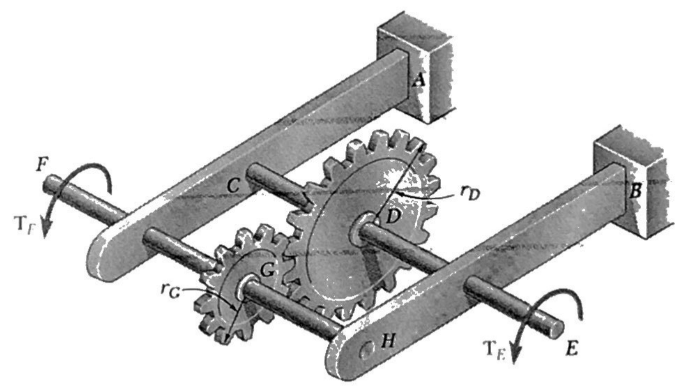


B. **[5 points]** What is the maximum shear stress (from torsion) in this shaft and where does it occur?

$$\tau = \frac{Tr}{J} \Rightarrow \tau = \frac{600^{N.m} * 20 * 10^{-3}\ m}{\frac{\pi}{2} * 2^4 * 10^{-8}\ m^4}$$

τ_{max} is at $\rho = r = 20mm$ and between A – B

C. **[7 points]** Gears G and D are rotating at constant angular speed, driven by the torque T_F applied to shaft F, as shown. Derive an equation that relates the torque in shaft E, T_E, in terms of T_F and the gear radii r_D and r_G.



$$\left.\begin{array}{l} T_F = F_G \cdot r_G \\ T_E = F_D \cdot r_D \\ \text{and } F_D = F_G \end{array}\right\} \rightsquigarrow \frac{T_F}{T_E} = \frac{r_G}{r_D}$$

or $P_G = P_D \rightsquigarrow T_F\, \omega_G = T_E\, \omega_D$ and $\omega_G = \frac{v_G}{r_G}$, $\omega_D = \frac{v_D}{r_D}$

and $v_D = v_G \Rightarrow \frac{T_F}{T_E} = \frac{r_G}{r_D} = \frac{\omega_D}{\omega_G}$

D. **[6 points]** For the gears of part C, derive a relation between the rotating speeds of each gear, ω_G and ω_D, in terms of the respective gear radii r_G and r_D.

$$v_G = v_D \rightarrow r_D \omega_D = r_G \omega_G$$

$$\rightarrow \frac{\omega_D}{\omega_G} = \frac{r_G}{r_D} = \frac{T_F}{T_E}$$