#1

(a)

$$\boxed{\underset{\sim}{\epsilon} = \begin{bmatrix} k_2 & 0 & k_5/2 \\ 0 & k_3 & \frac{1}{2}(k_4+k_6) \\ k_5/2 & \frac{1}{2}(k_4+k_6) & k_7 \end{bmatrix}}$$

where $\epsilon_{xx} = \frac{\partial u}{\partial x}$

$\epsilon_{yy} = \frac{\partial v}{\partial y}$

$\epsilon_{zz} = \frac{\partial w}{\partial z}$

$\epsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$

etc.

(b)

$$\epsilon_n = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{pmatrix} \begin{bmatrix} k_2 & 0 & k_5/2 \\ 0 & k_3 & 1/2\,(k_4+k_6) \\ k_5/2 & \frac{1}{2}(k_4+k_6) & k_7 \end{bmatrix} \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{pmatrix} \begin{pmatrix} k_2/\sqrt{2} \\ k_3/\sqrt{2} \\ \frac{k_5+k_4+k_6}{2\sqrt{2}} \end{pmatrix} = \boxed{\frac{k_2+k_3}{2}}$$

(c) let $k_2 = k_3 = 0$

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{zx} \end{pmatrix} = \begin{bmatrix} 2G+\lambda & \lambda & \lambda & & & \\ \lambda & 2G+\lambda & \lambda & & & \\ \lambda & \lambda & 2G+\lambda & & & \\ & & & G & & \\ & & & & G & \\ & & & & & G \end{bmatrix} \begin{pmatrix} 0 \\ 0 \\ k_7 \\ 0 \\ k_4+k_6 \\ k_5 \end{pmatrix}$$

$$= \boxed{\begin{pmatrix} \lambda k_7 \\ \lambda k_7 \\ (2G+\lambda)\,k_7 \\ 0 \\ G(k_4+k_6) \\ G\,k_5 \end{pmatrix}}$$

(Berkeley ordering used, other oderings are fine too)

(d) Let $k_2 = k_3 = 0$

$$\sigma_n = \left(\frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}} \quad 0\right) \begin{bmatrix} \lambda k_7 & 0 & G k_5 \\ 0 & \lambda k_7 & G(k_4+k_6) \\ G k_5 & G(k_4+k_6) & (2G+\lambda)k_7 \end{bmatrix} \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{pmatrix}$$

$$= \left(\frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}} \quad 0\right) \begin{pmatrix} \lambda k_7/\sqrt{2} \\ \lambda k_7/\sqrt{2} \\ G(k_5+k_4+k_6)/\sqrt{2} \end{pmatrix} = \boxed{\lambda k_7}$$

(e) Assuming $k_2 = k_3 = k_4 = k_6 = 0$

$$\underline{T} = \begin{bmatrix} \lambda k_7 & 0 & G k_5 \\ 0 & \lambda k_7 & 0 \\ G k_5 & 0 & (2G+\lambda)k_7 \end{bmatrix}$$

$$\sqrt{\frac{1}{2}\left[0^2 + (2G k_7)^2 + (2G k_7)^2\right] + 3(G k_5)^2} \leq Y$$

$$\boxed{\sqrt{4G^2 k_7^2 + 3G^2 k_5^2} \leq Y} \quad \text{or} \quad \boxed{4k_7^2 + 3k_5^2 \leq (Y/G)^2}$$

#2 $G = 75 \ \text{kN}\cdot\text{mm}^2$ $\qquad GJ_{eff} \geq 25\cdot 10^3 \ \text{kN}\cdot\text{mm}^2$

$$GJ_{eff} = G\left[\frac{1}{3}160\cdot(t^3) + \frac{1}{3}100t^3 + \frac{1}{3}60\cdot 8t^3\right] \geq 25\cdot 10^3$$

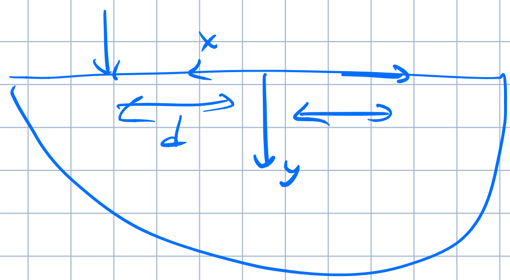
$$= 25 \ \text{kN/mm}^2 \left[740\right] t^3 \geq 25\cdot 10^3 \ \text{kN}\cdot\text{mm}^2$$

$$t^3 \geq \frac{1000}{740}$$

$$\boxed{t \geq 1.11 \ \text{mm}}$$

#3

Use superposition



$$\Sigma^{total} = \Sigma^{v}(x-d, y) + \Sigma^{h}(x+d, y)$$

at $x=0$

$$\sigma_{xx} = \sigma_{xx}^{v}(-d, y) + \sigma_{xx}^{h}(d, y)$$

same for σ_{yy} and σ_{xy}

$$\Rightarrow \quad \sigma_{xx} = -\frac{2P}{\pi}\frac{d^2 y}{(d^2+y^2)^2} - \frac{2H}{\pi}\frac{y^3}{(d^2+y^2)^2} = \boxed{\frac{-2}{\pi(d^2+y^2)^2}\left[Pd^2y + Hy^3\right]}$$

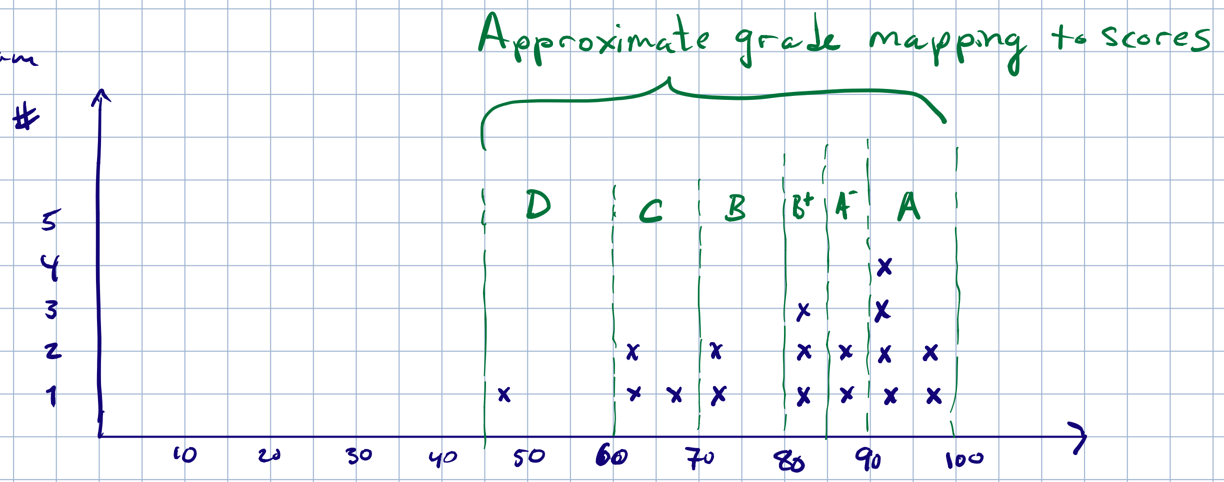
$$\sigma_{yy} = -\frac{2P}{\pi}\frac{y^3}{(d^2+y^2)^2} - \frac{2H}{\pi}\frac{d\,y^2}{(d^2+y^2)^2} = \boxed{\frac{-2}{\pi(d^2+y^2)^2}\left[Py^3 + Hdy^2\right]}$$

$$\sigma_{xy} = \frac{2P}{\pi}\frac{dy^2}{(d^2+y^2)^2} - \frac{2H}{\pi}\frac{d^2y}{(d^2+y^2)^2} = \boxed{\frac{2}{\pi(d^2+y^2)^2}\left[Pdy^2 - Hd^2y\right]}$$

Or

$$\boxed{\begin{aligned} \frac{-2y}{\pi(d^2+y^2)^2}\left[Pd^2 + Hy^2\right] &= \sigma_{xx} \\ \frac{-2y^2}{\pi(d^2+y^2)^2}\left[Py + Hd\right] &= \sigma_{yy} \\ \frac{2dy}{\pi(d^2+y^2)^2}\left[Py - Hd\right] &= \sigma_{xy} \end{aligned}}$$

Histogram



Binned $[5x, 5(x+1))$ $x = 0, 1, \ldots, 19$