SID: ____________________ Name: ____________________

Midterm Exam

CE 191 | Fall 2014
UC Berkeley

Oct 16, 2014, 9:40 – 10:30am

Exam Rules

- Exam ends at 10:30:00 am
- Open textbook. Closed class notes, closed lab assignments, closed handouts.
- One sheet of hand-written prepared notes (front & back, letter size).
- Hand-held calculators are permitted, but not necessary.
- No phones, tablets, laptops, smart watches, or other internet-connected devices.

HONOR PLEDGE: Copy (NOW) and SIGN (after the exam is completed): "I have neither given nor received aid on this exam, nor have I observed a violation of the *Berkeley Campus Code of Student Conduct*."

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SIGNATURE
(Sign **after** the exam is completed.)

The maximum possible score is **20 points**. To maximize your score, read the directions carefully and write legibly. Where appropriate show your work. This is an opportunity to show off how much you understand. Let your knowledge pour forth!

True-False / Multiple Choice Section

Please clearly mark your answer. You are not asked to show work. Each problem is worth **1 point**.

Problem 1

Consider the optimization problem:

$$\begin{aligned} \min_{x_1,x_2} \quad & x_1 + x_2 && (1) \\ \text{s. to:} \quad & 2x_1 + x_2 \leq 10 && (2) \\ & x_1 \geq 0 && (3) \end{aligned}$$

___________ **(T or F)** Constraints (2) and (3) are active at the minimum.

Problem 2

___________ **(T or F)** An equivalent formulation of equality constraint $h(x) = 0$ is

$$\begin{aligned} h(x) &\leq 0, \\ -h(x) &\leq 0 \end{aligned}$$

Problem 3

___________ **(T or F)** Consider a linear integer program (IP). The relaxed IP problem solution, i.e. the fractional solution, provides a bound for the optimal solution to the original IP.

Problem 4

___________ **(T or F)** The following is NOT an integer programming problem

$$\begin{aligned} \min: \quad & f(x) = monthlyPayment(x) + gasCost(x) + maintenanceCost(x) \\ \text{s. to:} \quad & x \in \{\text{Tesla Model S, BMW 5 series, MB E-class, Cadillac CTS, Audi A6}\} \end{aligned}$$

Problem 5

Quadratic objective function $f(x_1, x_2) = -4x_1 + 2x_2 + 4x_1^2 - 4x_1x_2 + x_2^2$ can be written as $f(x) = \frac{1}{2}x^TQx + R^Tx$, where $x = [x_1, x_2]^T$ and

(a) $Q = \begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}$, $R = \begin{bmatrix} -4 \\ 2 \end{bmatrix}$, (b) $Q = \begin{bmatrix} 4 & -4 \\ -4 & 1 \end{bmatrix}$, $R = \begin{bmatrix} -4 \\ 2 \end{bmatrix}$

(c) $Q = \begin{bmatrix} 8 & -4 \\ -4 & 2 \end{bmatrix}$, $R = \begin{bmatrix} -4 \\ 2 \end{bmatrix}$, (d) $Q = \begin{bmatrix} 8 & -4 \\ -4 & 2 \end{bmatrix}$, $R = \begin{bmatrix} -8 \\ 4 \end{bmatrix}$

(e) none of the above

Problem 6

Optimization theory finds practical use in:
(a) structural engineering
(b) transportation engineering
(c) energy and environment
(d) construction & project management
(e) geoengineering
(f) all the above and more

Short Answer Section

For this section, partial credit is awarded. You MUST show your work. Unsupported answers, even if correct, receive zero credit. In other words, a right answer with wrong or no reasoning could lead to zero credit. I will not answer the question: "Is this enough writing to receive full credit?" If you show enough steps to (i) systematically solve the problem, and (ii) demonstrate application of the methods learned in class, then you will be fine. Please box your final answer.

Problem 7 : Different Flavors of LP (6 pts)

$$\max_{x_1,x_2} \quad J = x_1 + x_2 \tag{4}$$

$$\text{s. to:} \quad x_1 + 2x_2 \leq 14 \tag{5}$$

$$2x_1 + x_2 \leq 14 \tag{6}$$

$$x_1, x_2 \geq 0 \tag{7}$$

Part (a) : Formulation for Computer

Formulate the linear program (LP) into the form: $\min_x \; c^T x$ subject to $Ax \leq b$, where $x = [x_1, x_2]^T$. What are the appropriate matrices `c, A, b`?

Part (b) : Graphical Solutions by Hand

Draw the feasible set. Is the feasible set bounded? Draw isolines for the cost function on your graph. Use graphical arguments to determine the optimal solution.

Part (c) : Integer Programming and Branch & Bound

Suppose we add the additional constraints that $x_1, x_2 \in \mathbb{Z}$. That is, we are interested in integer solutions. Use the branch & bound method to solve this integer programming problem. Draw a tree of subproblems, as seen in class and the notes.

Problem 8 : Students Planning Classes (4 pts)

The junior-year CEE students are planning their course schedules for next semester. Each student can select among 10 core elective courses. The relevant problem data is given as follows:

- Each student is assigned three courses.
- Each course has enrollment limit b_j, where $j = 1, 2, \cdots, 10$ denotes the course number.
- Sixty-four students are seeking to get their top three choices.

To facilitate this, UC Berkeley decides to replace the antiquated/obnoxious Telebears system and adopt a preference rating system. That is, each student i ranks each elective course on a scale of 1 to 10, where 10 = first choice (greatest preference) and 1 = last choice (lowest preference). The students can only rate one course a 10, one course a 9, and so forth. Assume all students follow this rule. Thus, in general, student i gives course j a preference of p_{ij}, where $1 \leq p_{ij} \leq 10$.

The Vice Chair of Academics has asked the Civil Systems faculty to design an online tool to register students while maximizing everyone's collective preferences. We shall employ optimization!

(a) Define your mathematical notation. Be precise.
(b) Using this notation, formulate (i) the objective function and (ii) all the constraints. Use summations.
(c) Add a constraint (or constraints) so that no student is assigned to any course below their fifth choice.

Problem 9 – Optimal Water Channel Design (4 pts)

This problem investigates optimal water channel cross-section design. Consider the water channel cross-section shown in Fig. 1, with circular corners of radii r, flat bottom of width w, and cross-sectional area a. The goal is to determine the optimal channel dimensions that maximize the cross-sectional area a, subject to a maximum "wetted perimeter" $P_{\max}$ (denoted in the color blue in Fig. 1).

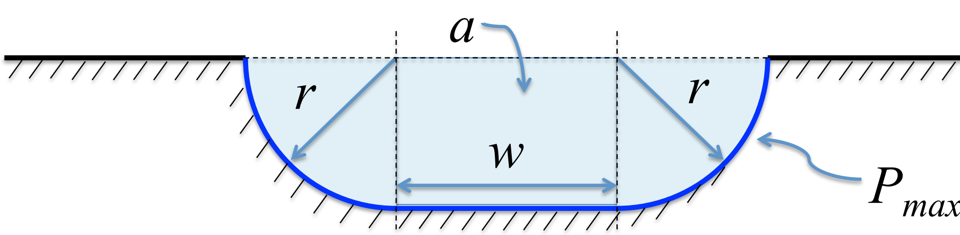

Figure 1: Water channel with circular corners.

Part (a) – Formulation

(i) Formulate the objective function and constraint(s).
(ii) What are the decision variables?
(iii) Is the objective function linear or quadratic in the decision variables?
(iv) Is (Are) the constraint(s) linear or quadratic in the decision variables?
(v) Formulate the matrices `c, A, b` for LP solver: $\min_x c^T x$ subject to $Ax \leq b$,
or the matrices `Q, R, A, b` for QP solver: $\min_x \frac{1}{2}x^T Qx + R^T x$ subject to $Ax \leq b$.