Grading rubric:

Correct	Sig figs	Unit error	Minor error	Error	Major error	Tried something	No submission
	-5%	-10%	-25%	-50%	-75%	-90%	-100%
	-0.1pts	-0.2pts	-0.5pts	-1pts	-1.5pts	-1.8pts	-2pts

Correct numerical answers provided without sufficient rationale or justification were graded as a minor error (partial justification) or error (no justification).

B1. ENERGY BALANCE FOR A FLAT PLANET

Suppose the Earth is really flat. Imagine an Earth that is shaped like a large penny, with one side that directly faces the sun at all times. Also suppose that this flat Earth is the same temperature everywhere, including the side that faces away from the sun. Neglect any radiation losses from the rim (i.e., very thin side edge) of the Earth, and assume there is no albedo or greenhouse effect. **Estimate the surface temperature T_E of this new, flat Earth, assuming an incoming solar radiation intensity $S_0 = 1360$ W/m^2.**

The energy entering the system is:

$E_{in} = S * \pi R^2$ (Surface of the disk - one side)

The energy leaving the system is:

$E_{out} = L * 2\pi R^2$ (Two sides of the disk)

With $L = \sigma T^4$

By resolving $E_{in} = E_{out}$, we get:

$S * \pi R^2 = \sigma T^4 * 2\pi R^2$

$S = \sigma T^4 * 2$

$T = \sqrt[4]{\frac{S}{2\sigma}} = 331\ K$

B2. WORLD SUPPLY OF COBALT

Worldwide reserves of cobalt are estimated to be 7 million metric tons, with a current production rate of 140,000 tons per year. How many years of supply remain assuming production grows exponentially at 3% per year?

We have $P(t) = P_0 e^{rt}$

We want to find the number of years after which the aggregate production will be of $Q = 7\ million\ tons$.

$$\int_0^T P(t)dt = \int_0^T P_0 e^{rt}$$

$$Q = \frac{P_0}{r}(e^{rT} - 1)$$

$$T = \frac{1}{r}ln\left(\frac{rQ}{P_0} + 1\right) = \frac{1}{0.03}ln\left(\frac{0.03*7x10^6}{140,000} + 1\right) = 31\ years$$

B3. HEATING VALUE FOR ACETYLENE

Calculate the **lower and higher heating values** for acetylene (C_2H_2). Answer in kJ/g units. The complete stoichiometric combustion of acetylene is as follows:

$C_2H_2 + 2.5\ O_2 + 10\ N_2 \rightarrow 2\ CO_2 + 1\ H_2O + 10\ N_2$
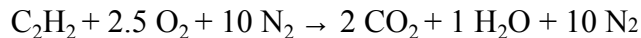
Heats of formation are C_2H_2, +227 kJ/mol; H_2O (g), –242 kJ/mol; CO_2, –394 kJ/mol. O_2 and N_2 have zero heat of formation. The heat of vaporization of water is 2.5 kJ/g.

We sum the different heats of formation accordingly to the stoichiometric coefficients:
$$\Delta h_c = 2\ \Delta h_{f,CO_2} + \Delta h_{f,H_2O} - \Delta h_{f,C_2H_2} = 2(- 394\ kJ/mol) + (- 242\ kJ/mol) - (227\ kJ/mol)$$
$$\Delta h_c = -1257\ kJ/mol$$

We then divide by the molecular weight of acetylene (2x12 + 2x1 = 26 g/mol)
$$LHV = \frac{-\Delta h_c}{M_{C_2H_2}} = 48.3\ kJ/g$$

To calculate the HHV, we simply add the heat needed to vapor the water produced during the combustion (here 1 mole of H_2O is produced for 1 mole of C_2H_2 burned):
$$HHV = LHV + \frac{MW_{H2O}}{MW_{C2H2}} * \Delta h_{v,H2O} = 50.0\ kJ/g$$

B4. WIND TURBINE EFFICIENCY

An offshore wind turbine with rotor diameter D = 220 m delivers its rated power output of 12 MW at wind speeds of 12 m/s or higher. For a wind speed of 15 m/s, calculate the fraction of the available kinetic energy of the air flow past the turbine per unit time that is being converted to electricity. Assume an air density of 1.2 kg/m^3

From the lecture, we know that the available kinetic energy for a 15m/s wind speed is:

$$P_0 = \frac{1}{2} * \pi * (\frac{D}{2})^2 * \rho_{air} * v^3 = \frac{1}{2} * \pi * (\frac{220}{2})^2 * (1.2\ kg/m^3) * (15\ m/s) = 77\ MW$$

Since the wind speed is higher than 12 m/s, the power output P = 12 MW. Therefore:

$$\eta = \frac{P}{P_0} = 16\%.$$

B5. DRIVING RANGE FOR AN ELECTRIC VEHICLE

Suppose an electric vehicle with a fully-charged 60 kWh battery is driving on a flat level highway at a steady speed of 110 km/h = 30.6 m/s. **Estimate the driving range** (answer in km) given relevant data listed below.

Drag Coefficient $C_D = 0.23$
Rolling Resistance Coefficient $C_R = 0.015$
Frontal Area $A_F = 2.2\ m^2$
Vehicle Mass m = 1800 kg
DC Electric Motor Efficiency $\eta = 90\%$
Transmission Efficiency $\varepsilon = 100\%$
Power to run Air Conditioning $P_{ACC} = 500$ W
Air Density $\rho_{air} = 1.2\ kg/m^3$

The resistive force for the vehicle is equal to:

$$F_{res} = C_d * A_f * \frac{\rho_{air} * v^2}{2} + C_r * m * g = 549\ N$$

The resulting electric power consumption for vehicle propulsion is:

$$P_{elec,motor} = \frac{F_{res} * v}{\eta * \varepsilon}$$

The total electric power consumption is therefore:

$$P_{consumed} = P_{elec,motor} + P_{AC} = 19\ kW$$

We then calculate the driving range based on the battery capacity:

$$Range = \frac{Battery\ capacity}{P_{consumed}} * v = 344\ km$$

B6. SHELTERING INDOORS TO AVOID FOREST FIRE SMOKE

Suppose outdoor air quality is poor due to smoke from nearby forest fires. Assume the outdoor level of airborne particulate matter (PM) is 150 μg/m³. Consider a home with V = 400 m³ and an air exchange rate $Q/V = 0.5$ hr⁻¹. There are no indoor sources of smoke, and the initial concentration of smoke indoors is zero. **Calculate the indoor PM concentration**, one hour after the outdoor air starts to be significantly polluted by forest fire smoke. Assume there are no air purifiers or filters indoors. Answer in μg/m³ units.

The variation of concentration is given by:

$$V\frac{dC}{dt} = QC_{out} - QC$$

$$\frac{dC}{dt} = \frac{Q}{V}(C_{out} - C)$$

$$\frac{dC}{C_{out}-C} = \frac{Q}{V}dt$$

$$- \ln(\frac{C-C_{out}}{-C_{out}}) = \frac{Q}{V}t$$

$$C - C_{out} =- C_{out}e^{-\frac{Q}{V}t}$$

We therefore have:

$$C = C_{out}(1 - e^{-\frac{Q}{V}*t}) = 59\,\mu g/m^3$$

B7. FILTERING INDOOR AIR

Suppose for the same house as in the previous question, there is an air filter operating with a flow rate of $Q_F = 2000$ m³/hr and a filtration efficiency of 80% that removes smoke particles from indoor air. **What will be the steady state concentration of PM indoors?** Give your answer in μg/m³ units.

The steady state equation in this situation is given by:

$$C_{out} * Q = C_{steady} * Q + C_{steady} * Q_F * \eta = C_{steady}(Q + Q_F * \eta)$$

We therefore have:

$$C_{steady} = \frac{C_{out}}{1+\frac{Q_F}{Q}\eta}$$

$$C_{steady} = \frac{C_{out}}{1+\frac{Q_F * V}{V * Q}*\eta} = 17\,\mu g/m^3$$

B8. MITIGATING HYDROGEN SULFIDE ODOR

Hydrogen sulfide (H_2S) is a foul-smelling gas that is soluble in water. One odor mitigation strategy involves altering the pH of water to reduce the amount of dissolved sulfur present in volatile (H_2S) form. Assume the only important dissolved forms are undissociated H_2S and the bisulfide ion, HS^-. The dissociation reaction is

$$H_2S \Leftrightarrow HS^- + H^+ \qquad \text{with } K_{A1} = \frac{[HS^-][H^+]}{[H_2S]} = 10^{-7.1}$$
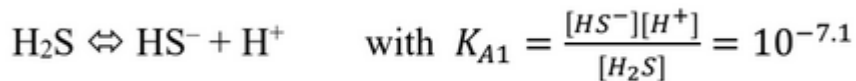
Calculate and compare the fraction of total dissolved sulfur present in H_2S form at pH=7 and pH=8.

The definition of pH is:
$$pH = -\log[H^+] \Leftrightarrow [H^+] = 10^{-pH}$$

The fraction of H2S is equal to:
$$c_{H_2S, pH} = \frac{[H_2S]}{[H_2S]+[HS^-]} = \frac{1}{1+\frac{[HS^-]}{[H_2S]}}$$

And we know from the dissociation reaction equation that:
$$\frac{[H^+][HS^-]}{[H_2S]} = K_{A1} \Leftrightarrow \frac{[HS^-]}{[H_2S]} = \frac{K_{A1}}{[H^+]} \text{ with } K_{A1} = 10^{-7.1} = 7.9 \times 10^{-8} \text{ mol/L}$$

By using the definition of pH, we therefore have:
$$\frac{[HS^-]}{[H_2S]} = \frac{K_{A1}}{10^{-pH}}$$
$$c_{H_2S, pH} = \frac{1}{1+\frac{K_{A1}}{10^{-pH}}}$$

We then calculate the values for pH=7 and pH=8.
$$c_{H_2S, pH=7} = 56\%$$
$$c_{H_2S, pH=8} = 11\%$$

We therefore prefer a pH of 8 as the fraction of total dissolved sulfur present as H_2S (the volatile form) is lower in this situation.

B9. DISSOLVED OXYGEN IN A RIVER

Suppose deoxygenation and reaeration rate coefficients in a river are equal, k = 0.3/day. Given an initial dissolved oxygen deficit of 3 mg/L, and an ultimate oxygen demand of 10 mg/L for water in the river just downstream of an effluent discharge, **calculate the maximum value for the dissolved oxygen deficit** downstream (answer in mg/L).

From the lecture, we know that the deficit is equal to $D(t) = (kL_0t + D_0)e^{-kt}$.

We can thus calculate the derivative of D:

$\frac{dD}{dt} = kL_0e^{-kt} - (kL_0t + D_0) * ke^{-kt} = ke^{-kt} * (L_0 - L_0kt + D_0)$

We then want to find the maximum of D which is reached for:

$\frac{dD(t)}{dt} = 0 \Leftrightarrow L_0(1 - kt_c) + D_0 = 0 \Leftrightarrow t_c = \frac{L_0 - D_0}{kL_0} = 2.3 \, days$

We finally simply have to plug this value in the initial D formula:

$D(t_c) = 5.0 \, mg/L$

B10. UNCERTAINTY ANALYSIS

Given a measured value $x = 4.0 \pm 0.2$, estimate the resulting uncertainty in $y = x^3$. In other words, **what is the appropriate value for σ_y** in the calculated result $y = 64 \pm \sigma_y$?

$y = f(x) = x^3$

$\frac{df}{dx} = 3x^2$

$\sigma_y^2 = (\frac{df}{dx} * \sigma_x)^2$

$\sigma_y = \left|(3x^2) * \sigma_x\right| = 48 \times 0.2 = 9.6$