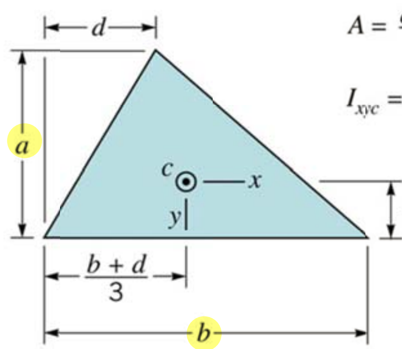
CE100 Fall 2017 Exam 1

Name: Solution

Hints for success:

- Do your own work
  - If your work is not your own, your entire exam will be given zero credit and you will be referred to Student Judicial Affairs
- Express all components of vector quantities
  - You can do this by giving 3 components or by giving resultant magnitude and direction.
- Keep units throughout your solution
- Make assumptions when needed, and state them clearly.
- Explain your work clearly enough that a student who took CE100 two years ago could easily understand what you are doing.
- Breathe

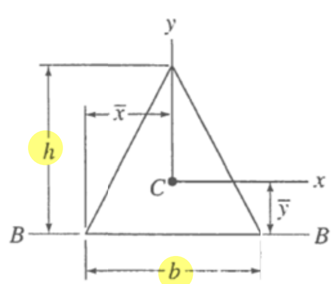
Reference information



$A = \frac{ab}{2}$ $\quad I_{xc} = \frac{ba^3}{36}$

$I_{xyc} = \frac{ba^2}{72}(b - 2d)$ = 0 when b=2d

:



Isosceles triangle (Origin of axes at centroid)

$A = \frac{bh}{2} \quad \bar{x} = \frac{b}{2} \quad \bar{y} = \frac{h}{3}$

$I_x = \frac{bh^3}{36} \quad I_y = \frac{hb^3}{48} \quad I_{xy} = 0$

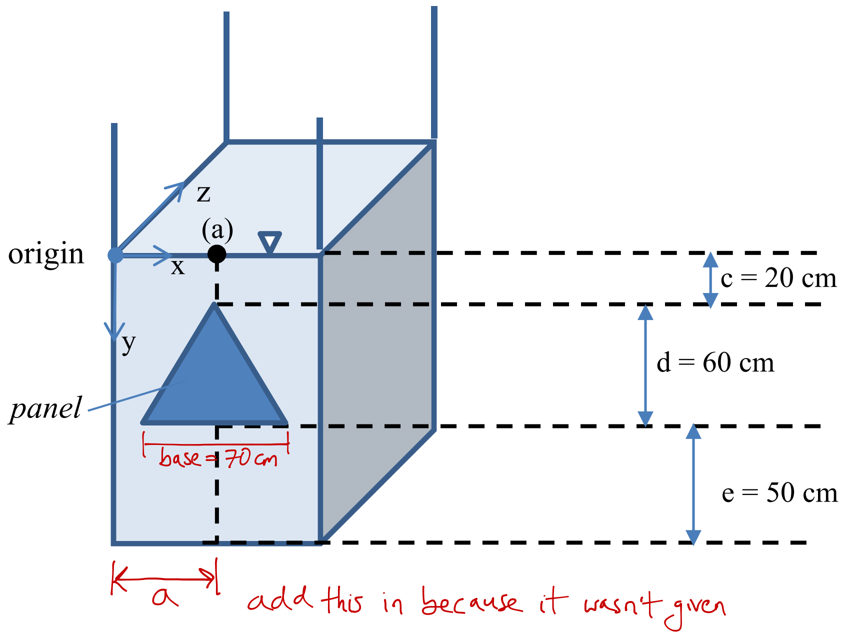
$I_p = \frac{bh}{144}(4h^2 + 3b^2) \quad I_{BB} = \frac{bh^3}{12}$

(*Note:* For an equilateral triangle, $h = \sqrt{3}\,b/2$.) ← not true

Physical Properties of Water (SI Units)[a]

Temperature (°C)	Density, ρ (kg/m³)	Specific Weight[b], γ (kN/m³)	Dynamic Viscosity, μ (N·s/m²)	Kinematic Viscosity, ν (m²/s)	Surface Tension[c], σ (N/m)	Vapor Pressure, p_v [N/m²(abs)]	Speed of Sound[d], c (m/s)
0	999.9	9.806	1.787 E − 3	1.787 E − 6	7.56 E − 2	6.105 E + 2	1403
5	1000.0	9.807	1.519 E − 3	1.519 E − 6	7.49 E − 2	8.722 E + 2	1427
10	999.7	9.804	1.307 E − 3	1.307 E − 6	7.42 E − 2	1.228 E + 3	1447
20	998.2	9.789	1.002 E − 3	1.004 E − 6	7.28 E − 2	2.338 E + 3	1481
30	995.7	9.765	7.975 E − 4	8.009 E − 7	7.12 E − 2	4.243 E + 3	1507

1) There is a panel shaped like an equilateral triangle (base b = 70 cm, height d = 60 cm) in a tank wall. The tank is full of static and constant-density freshwater at 20 Celsius.



A) Calculate the force due to water on the panel.

Centroid of triangle is $\frac{2}{3}$ down from the top (see info on cover page)

y-location of centroid is $y_c = c + \frac{2}{3}d = 20\,cm + \frac{2}{3}(60\,cm) = 60\,cm$.

x-location of centroid is $x_c = a$ (which you need to define in the problem because it wasn't given).

Force on panel $\|\underline{F}_R\| = \bar{P}A$ = average pressure * panel area

$\bar{P} = P\big|_{(x_c, y_c)}$ = pressure at centroid = $P_{atm} + \gamma y_c$ working in gage, $P_{atm} = 0$, $\bar{P} = \gamma y_c$

$$\|\underline{F}_R\| = \gamma y_c A = (9{,}789\ N/m^3)(0.6\ m)\left(\tfrac{1}{2} * 0.6\ m * 0.7\ m\right) = 1{,}233.4\ N$$

Force acts from water onto the panel, which is the -z direction

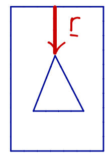
$$\boxed{\underline{F}_R = 1{,}233.4\ N \text{ in } -z \text{ direction or } \underline{F}_R = (0, 0, -1233.4)\ N}$$

B) Calculate the torque about point (a) due to water on the triangular panel.

Resultant Force $\underline{F}_R$ acts at center of pressure (x_R, y_R)

$$y_R = y_c + \frac{I_{xc}}{y_c A} = 60\text{ cm} + \frac{\frac{1}{36}(70\text{cm})(60\text{ cm})^3}{(60\text{ cm})(\frac{1}{2}*70\text{cm}*60\text{cm})} = 60\text{cm} + 3\tfrac{1}{3}\text{ cm} = 0.633\text{ m}$$

$$x_R = x_c + \frac{I_{xyc}}{y_c A} = a + \frac{0}{y_c A} = a$$



moment arm with respect to (a) goes from (a, 0, 0) to $(x_R, y_R, 0)$

$\underline{r} = (0, 0.633, 0)$ m

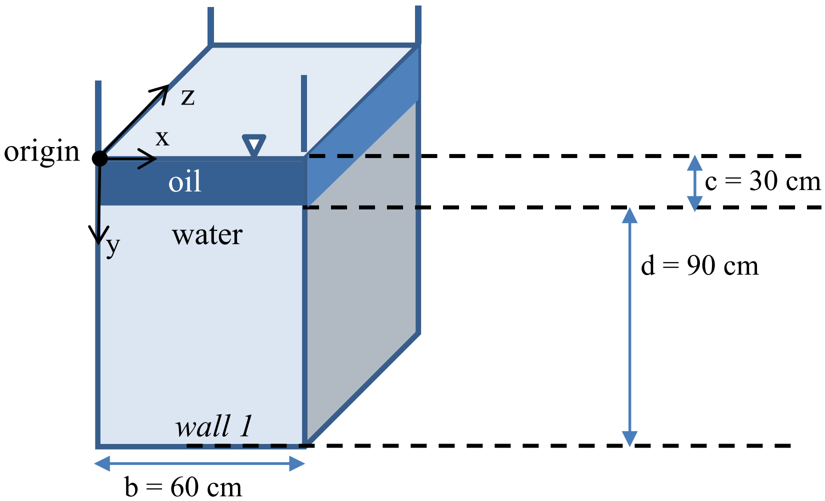
Force at center of pressure was found in part A

$\underline{F}_R = (0, 0, -1233.4)$ N

$$\text{Torque} = \underline{r} \times \underline{F}_R = \begin{vmatrix} i & j & k \\ 0 & 0.633 & 0 \\ 0 & 0 & -1233.4 \end{vmatrix} = (-780.75, 0, 0)\ \text{N}\cdot\text{m}$$

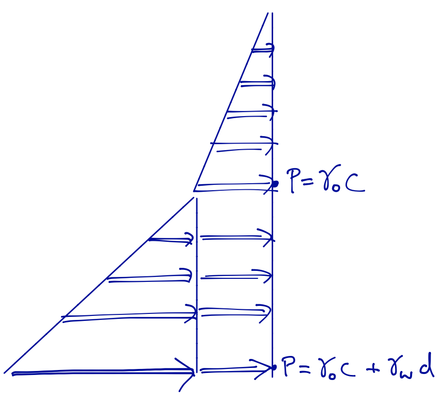
$$\boxed{\underline{T} = 780.75\ \text{N}\cdot\text{m in } -x \text{ direction, or } (-780.75, 0, 0)\ \text{N}\cdot\text{m}}$$

2) Calculate the force due to oil and water on wall 1. The oil is static and constant-density with $\rho_o = 800\ kg/m^3$. The water is static and constant-density with $\rho_w = 1000\ kg/m^3$. Use any method.

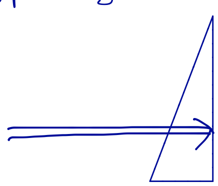


pressure prism decomposition

forces on Wall 1:

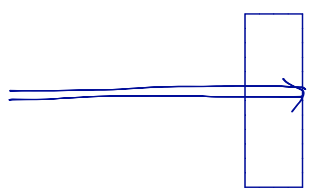


express many infinitesimal forces as 3 resultant forces:



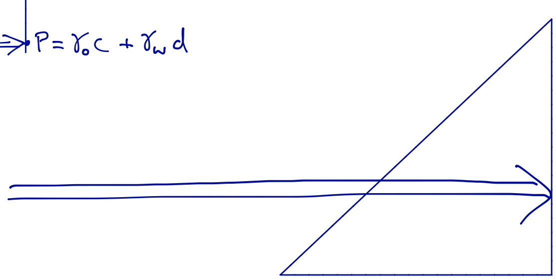
$F_{R_1} = \frac{1}{2}\gamma_o c(bc) = \frac{1}{2}(800\ kg/m^3)(9.8\ m/s^2)(0.3\ m)(0.6\ m)(0.3\ m) = 212\ N$ in -z direction

acts at $x_R = 30\ cm$ $y_R = 20\ cm$ $z_R = 0$



$F_{R_2} = \gamma_o c(bd) = (800\ kg/m^3)(9.8\ m/s^2)(0.3\ m)(0.6\ m)(0.9\ m) = 1270\ N$ in -z direction

acts at $x_R = 30\ cm$ $y_R = 75\ cm$ $z_R = 0$



$F_{R_3} = \frac{1}{2}\gamma_w d(bd) = \frac{1}{2}(1000\ kg/m^3)(9.8\ m/s^2)(0.9\ m)(0.6\ m)(0.9\ m) = 2381\ N$ in -z direction

acts at $x_R = 30\ cm$ $y_R = 90\ cm$ $z_R = 0$

$F_{R_T} = F_{R_1} + F_{R_2} + F_{R_3} = 212 + 1270 + 2381 = \boxed{3863\ N \text{ in -z direction}}$

Direct integration method: $dF = P\,dA \quad P = \begin{cases} \gamma_o y & \text{for } y < c \\ \gamma_o c + \gamma_w (y-c) & \text{for } y > c \end{cases}$

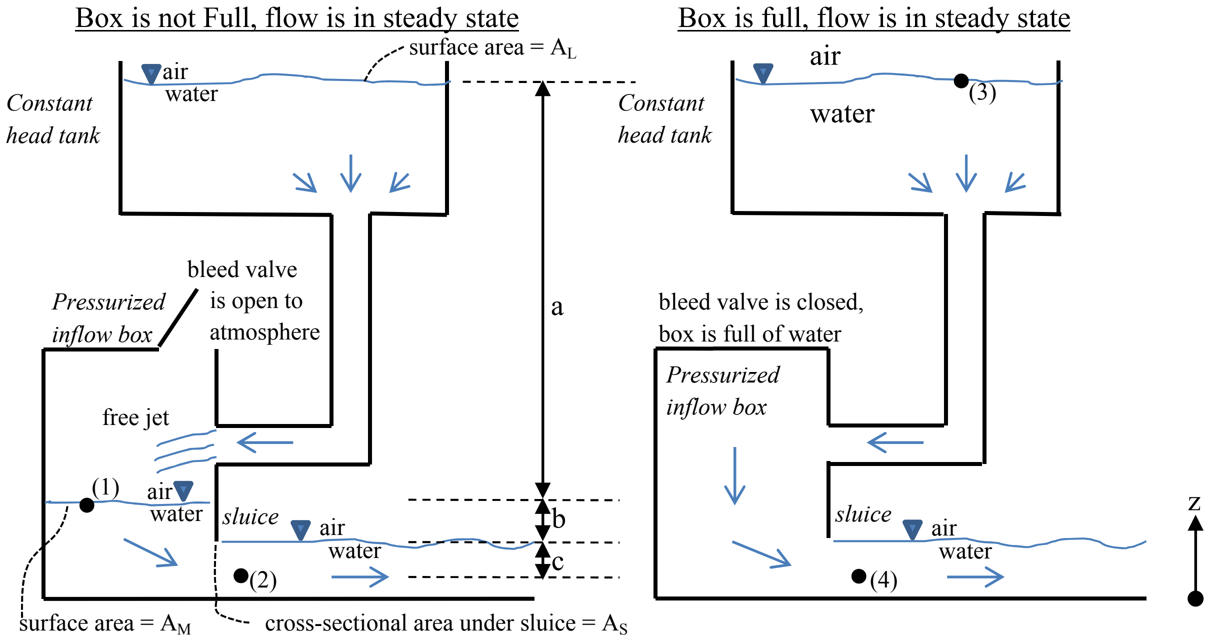
$$F = \int_{\text{wall 1}} dF = \int_{\text{wall 1}} P\,dA = \int_{\text{wall 1}} P\,dx\,dy = \int_{x=0}^{60\text{ cm}} \int_{y=0}^{120\text{ cm}} P\,dx\,dy = \int_{x=0}^{60\text{ cm}} \int_{y=0}^{30\text{ cm}} P\,dx\,dy + \int_{x=0}^{60\text{ cm}} \int_{y=30\text{cm}}^{120\text{ cm}} P\,dx\,dy = \int_{x=0}^{60\text{ cm}} \int_{y=0}^{30\text{ cm}} [\gamma_o y]\,dx\,dy + \int_{x=0}^{60\text{ cm}} \int_{y=30\text{cm}}^{120\text{ cm}} [\gamma_o c + \gamma_w (y-c)]\,dx\,dy$$

$$F = (60\text{cm})\left[\gamma_o \frac{y^2}{2}\right]_0^{30\text{ cm}} + (60\text{cm})\left[\gamma_o c y + \gamma_w \frac{y^2}{2} - \gamma_w c y\right]_{30\text{ cm}}^{120\text{ cm}} = (60\text{ cm})\left(800\,\tfrac{\text{kg}}{\text{m}^3}\right)\left(980\,\text{cm/s}^2\right)\tfrac{1}{2}(30\text{ cm})^2 + (60\text{ cm})\left(800\,\tfrac{\text{kg}}{\text{m}^3}\right)\left(980\,\text{cm/s}^2\right)(30\text{ cm})(120\text{cm} - 30\text{ cm})$$

$$(60\text{ cm})\left(1000\,\tfrac{\text{kg}}{\text{m}^3}\right)\left(980\,\text{cm/s}^2\right)\tfrac{1}{2}\left[(120\text{cm})^2 - (30\text{ cm})^2\right] - (60\text{ cm})\left(1000\,\tfrac{\text{kg}}{\text{m}^3}\right)\left(980\,\text{cm/s}^2\right)(30\text{cm})(120\text{ cm} - 30\text{ cm})$$

$$= 3.863 \times 10^{11}\ \text{cm}^4\,\frac{\text{kg}}{\text{m}^3\text{s}^2} = 3863\,\frac{\text{kg m}}{\text{s}^2} = \boxed{3863\text{ N}}$$ in the $-z$ direction.

3) During Lab 1 we filled the "pressurized inflow box." This question explains why. Consider:



- a = 2.0m; b = 0.2m; c = 0.1m
- head loss between 1 and 2 is h_{L12} = 0.01m; head loss between 3 and 4 is h_{L34} = 0.3m
- surface area of constant-head tank $A_L = 1\text{m}^2$; surface area of water in box $A_M = 0.16\text{m}^2$
- cross-sectional area under sluice gate $A_S = 0.0014\text{m}^2$
- uniform velocity over areas A_L, A_M, and A_S.
- constant-density water with γ_w=9800 N/m^3 and constant-density air with γ_a = 10 N/m^3
- the constant head tanks are refilled at the necessary Q_{in} to achieve steady state

Given the above information, calculate the water velocities V_2 and V_4.

Assume steady flow along a streamline from 1 to 2. Constant water density is given, so we can use energy equation

$$\frac{P_1 - P_2}{\gamma_w} + \frac{V_1^2 - V_2^2}{2g} + z_1 - z_2 - h_L - h_T + h_p = 0$$

Mass conservation with constant density from 1 inlet to 1 outlet gives $\bar{V}_M A_M = \bar{V}_S A_S$

Given uniform velocity over A_M and A_S, $V_1 = \bar{V}_M$ and $V_2 = \bar{V}_S$

$\Big\}$ $V_1 A_M = V_2 A_S$ or $V_1 = V_2 \frac{A_S}{A_M}$

Observe: $P_1 = P_{atmospheric} = 0$ (in gage pressure)

Extended Pascal's Principle from river surface across straight parallel streamlines to location (2) gives $P_2 = \gamma_w c$

z_1 has higher elevation than z_2, so $z_1 - z_2 = (b+c) > 0$

Combining all equations,

$$\frac{0 - \gamma_w c}{\gamma_w} + \frac{V_2^2}{2g}\left(\frac{A_S}{A_M}\right)^2 - \frac{V_2^2}{2g} + b + c - h_{L12} \overbrace{- h_T + h_p}^{\text{No pump or turbine between 1 \& 2}} = 0$$

$$\underbrace{-c - \frac{V_2^2}{2g}\left(1 - \left(\frac{A_S}{A_M}\right)^2\right) + b + c}_{\text{cancel}} - h_{L12} = 0 \Rightarrow \frac{V_2^2}{2g} = \frac{b - h_{L12}}{\left(1 - \left(A_S/A_M\right)^2\right)} \Rightarrow V_2 = \sqrt{\frac{2g(b - h_{L12})}{\left(1 - \left(A_S/A_M\right)^2\right)}}$$

$$\bar{V}_2 = \sqrt{\frac{2g(b - h_{L12})}{\left(1 - \left(A_S/A_M\right)^2\right)}} = \sqrt{\frac{2(9.81\ m/s^2)(0.2\,m - 0.01\,m)}{1 - \left(\frac{0.0014\ m^2}{0.16\ m^2}\right)^2}} = \boxed{1.93\ m/s}$$

Assume steady flow along a streamline from 3 to 4. Constant water density is given, so we can use energy equation

$$\frac{P_3 - P_4}{\gamma_w} + \frac{\bar{V}_3^2 - \bar{V}_4^2}{2g} + z_3 - z_4 - h_L - h_T + h_p = 0$$

Mass conservation with constant density from 1 inlet to 1 outlet gives $\bar{V}_L A = \bar{V}_S A_S$

Given uniform velocity over A_L and A_S, $\bar{V}_3 = \bar{V}_L$ and $\bar{V}_2 = \bar{V}_S$

$\Big\}$ $\bar{V}_3 A_L = \bar{V}_4 A_S$ or $\bar{V}_3 = \bar{V}_4 \frac{A_S}{A_L}$

Observe: $P_3 = P_{atmospheric} = 0$ (in gage pressure)

Extended Pascal's Principle from river surface across straight parallel streamlines to location (2) gives $P_4 = \gamma_w C$

z_3 has higher elevation than z_4, so $z_3 - z_4 = (a+b+c) > 0$

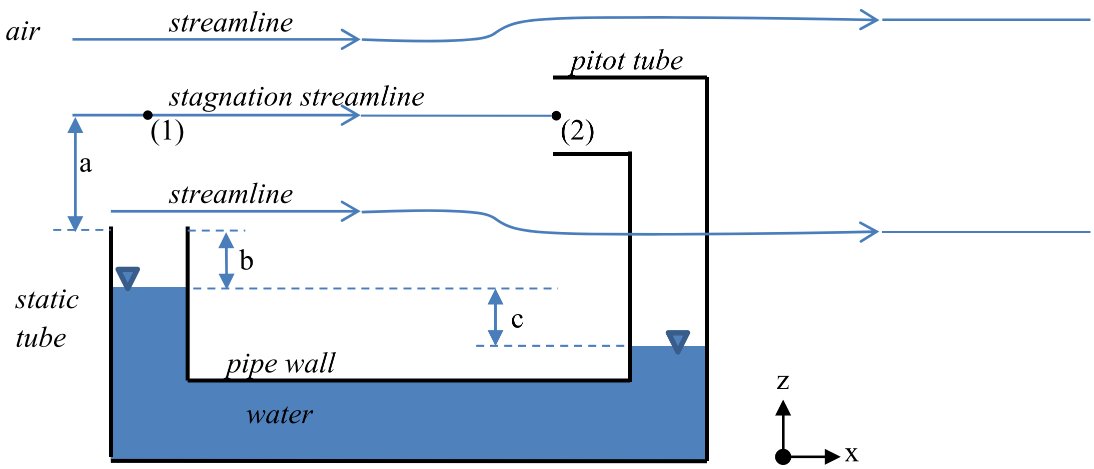
Combining all equations,

$$\frac{0 - \gamma_w C}{\gamma_w} + \frac{\bar{V}_4^2}{2g}\left(\frac{A_S}{A_L}\right)^2 - \frac{\bar{V}_4^2}{2g} + a + b + c - h_{L34} \overbrace{- h_T + h_p}^{\text{No pump or turbine between 3 \& 4}} = 0$$

$$\underbrace{-C - \frac{\bar{V}_4^2}{2g}\left(1 - \left(\frac{A_S}{A_L}\right)^2\right) + a + b + c}_{\text{cancel}} - h_{L34} = 0$$

$$\bar{V}_4 = \sqrt{\frac{2g(a + b - h_{L34})}{\left(1 - \left(A_S/A_L\right)^2\right)}} = \sqrt{\frac{2(9.81\ m/s^2)(2.0\,m + 0.2\,m - 0.3\,m)}{1 - \left(\frac{0.0014\ m^2}{1\ m^2}\right)^2}} = \boxed{6.11\ m/s}$$

4) During class, we saw a demonstration of a pitot-static tube that measured air velocity. A simplified version of this setup is as follows:



- a = 0.40m; b = 0.20m; c = 0.05m
- head loss between 1 and 2 is $h_{L12} = 0.01$m
- constant-density water with $\gamma_w = 9800$ N/m^3; constant-density air with $\gamma_a = 10$ N/m^3

Given the above information, calculate the air velocity V_1.

Constant-density air (given) is an important consideration, because it allows us to use extended Pascal's Principle and also the energy equation.

extended Pascal's Principle and regular Pascal's Principle together give: $P_1 + \gamma_a a + \gamma_a b + \gamma_w c = P_2 + \gamma_a a + \gamma_a b + \gamma_a c$

$\Rightarrow P_1 - P_2 = (\gamma_a - \gamma_w)c$

assuming that the streamline from (1) to (2) is steady, energy equation can be used: $\frac{P_1 - P_2}{\gamma_a} + \frac{V_1^2 - V_2^2}{2g} + z_1 - z_2 - h_L - h_T + h_p = 0$

$V_2 = 0$ because stagnation streamline stops at the pitot tube

$z_1 = z_2$ (no elevation change from 1 to 2)

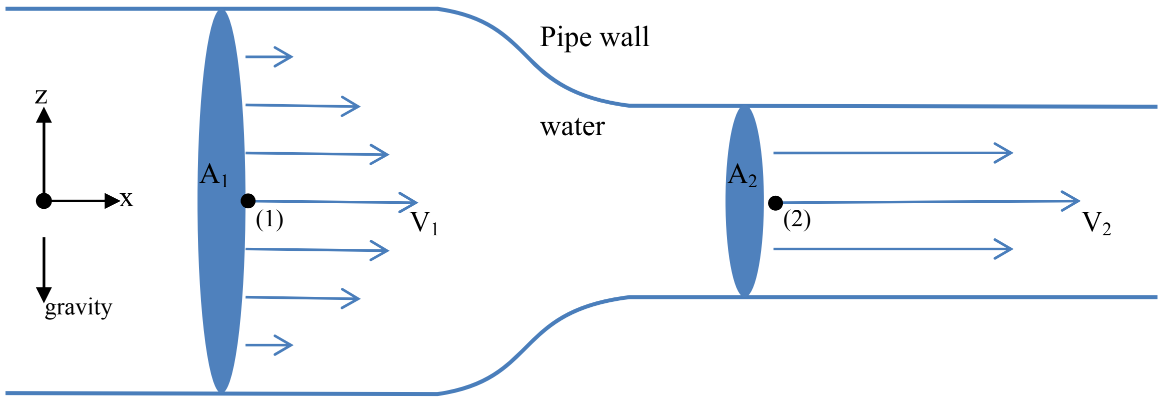
$h_T = 0$ and $h_p = 0$ (no pump or turbine)

$\Rightarrow \frac{P_1 - P_2}{\gamma_a} + \frac{1}{2g} V_1^2 - h_L = 0$

Combining, $\frac{V_1^2}{2g} = h_L - \frac{(\gamma_a - \gamma_w)c}{\gamma_a} \Rightarrow V_1 = \sqrt{2g\left(h_L + \frac{\gamma_w}{\gamma_a}c - c\right)} = \sqrt{(2 * 9.8 \text{ m/s}^2)\left(0.01\text{m} + \frac{9800 \text{ N/m}^3}{10 \text{ N/m}^3}(0.05\text{m}) - 0.05\text{m}\right)}$

$\boxed{V_1 = 30.98 \text{ m/s}}$

5) In this laminar flow, the velocity profile has a parabolic shape at cross-sections 1 and 2.



- $P_1 = 2$ kPa; $V_1 = 2$ m/s;
- $A_1 = 0.09\text{m}^2$; $A_2 = 0.03\text{m}^2$
- head loss between 1 and 2 is $h_{L12} = 0.10\text{m}$
- constant-density water with $\gamma_w = 9800\ \text{N/m}^3$

A) Calculate P_2

given: Corresponding velocity pattern at A_1 & A_2, thus $\frac{V_1}{\bar{V}_1} = \frac{V_2}{\bar{V}_2}$

flow from one inlet (A_1) to one inlet (A_2) with constant density (given) and flow perpendicular to inlet & outlet, thus $\bar{V}_1 A_1 = \bar{V}_2 A_2$

Combining these, $V_1 A_1 = V_2 A_2$

assume steady flow on streamline from 1 to 2, thus we can use energy equation: $\frac{P_1 - P_2}{\gamma} + \frac{V_1^2 - V_2^2}{2g} + \underbrace{\cancel{z_1 - z_2}}_{0\ \text{no elevation change}} - h_L \underbrace{\cancel{- h_T} + h_p}_{0\ \text{no turbine or pump between 1 \& 2}} = 0$

Combining all equations: $\frac{P_2}{\gamma} = \frac{P_1}{\gamma} + \frac{V_1^2}{2g} - \frac{V_1^2\left(\frac{A_1}{A_2}\right)^2}{2g} - h_L$

$$P_2 = P_1 + \frac{\gamma}{2g} V_1^2\left(1 - \left(\frac{A_1}{A_2}\right)^2\right) - \gamma h_L$$

$$P_2 = 2\ \text{kPa} + \frac{(9.8\ \text{kN/m}^3)(2\ \text{m/s})^2}{2(9.8\ \text{m/s}^2)}\left(1 - \left(\frac{0.09\ \text{m}^2}{0.03\ \text{m}^2}\right)^2\right) - (9.8\ \text{kN/m}^3)(0.10\ \text{m})$$

$$P_2 = 2\ \text{kPa} + 2\ \text{kPa}(1-9) - 0.98\ \text{kN}$$

$$\boxed{P_2 = -15.02\ \text{kPa}}$$

B) Is P_2 greater or less than atmospheric pressure?

Engineering convention is to work in gage pressure, and this question doesn't seem to be an exception (no mention of absolute, ideal gas, or vapor pressure).

In gage, negative pressures are allowed (in absolute they are not) and they indicate pressures $\boxed{\text{less than atmospheric.}}$

1A	1B	2	3	4	5A	5B	Total (28)