CE100 Fall 2017 Exam 2

Name:

Hints for success:

- Do your own work
  - If your work is not your own, your entire exam will be given zero credit and you will be referred to Student Judicial Affairs
- Express all components of vector quantities
- Keep units throughout your solution
- Make assumptions when needed, and state them clearly.
- Explain your work clearly enough that a student who took CE100 two years ago could easily understand what you are doing.
- Breathe

Reference information(Moody, minor loss, and drag charts included as a separate packet)

TABLE B.2

Physical Properties of Water (SI Units)[a]

Temperature (°C)	Density, ρ (kg/m^3)	Specific Weight[b], γ (kN/m^3)	Dynamic Viscosity, μ (N·s/m^2)	Kinematic Viscosity, ν (m^2/s)	Surface Tension[c], σ (N/m)	Vapor Pressure, p_v [N/m^2(abs)]
0	999.9	9.806	1.787 E − 3	1.787 E − 6	7.56 E − 2	6.105 E + 2
5	1000.0	9.807	1.519 E − 3	1.519 E − 6	7.49 E − 2	8.722 E + 2
10	999.7	9.804	1.307 E − 3	1.307 E − 6	7.42 E − 2	1.228 E + 3
20	998.2	9.789	1.002 E − 3	1.004 E − 6	7.28 E − 2	2.338 E + 3
30	995.7	9.765	7.975 E − 4	8.009 E − 7	7.12 E − 2	4.243 E + 3
40	992.2	9.731	6.529 E − 4	6.580 E − 7	6.96 E − 2	7.376 E + 3

TABLE B.4

Physical Properties of Air at Standard Atmospheric Pressure (SI Units)[a]

Temperature (°C)	Density, ρ (kg/m^3)	Specific Weight[b], γ (N/m^3)	Dynamic Viscosity, μ (N·s/m^2)	Kinematic Viscosity, ν (m^2/s)	Specific Heat Ratio, k (—)	Speed of Sound, c (m/s)
−40	1.514	14.85	1.57 E − 5	1.04 E − 5	1.401	306.2
−20	1.395	13.68	1.63 E − 5	1.17 E − 5	1.401	319.1
0	1.292	12.67	1.71 E − 5	1.32 E − 5	1.401	331.4
5	1.269	12.45	1.73 E − 5	1.36 E − 5	1.401	334.4
10	1.247	12.23	1.76 E − 5	1.41 E − 5	1.401	337.4
15	1.225	12.01	1.80 E − 5	1.47 E − 5	1.401	340.4
20	1.204	11.81	1.82 E − 5	1.51 E − 5	1.401	343.3
25	1.184	11.61	1.85 E − 5	1.56 E − 5	1.401	346.3
30	1.165	11.43	1.86 E − 5	1.60 E − 5	1.400	349.1

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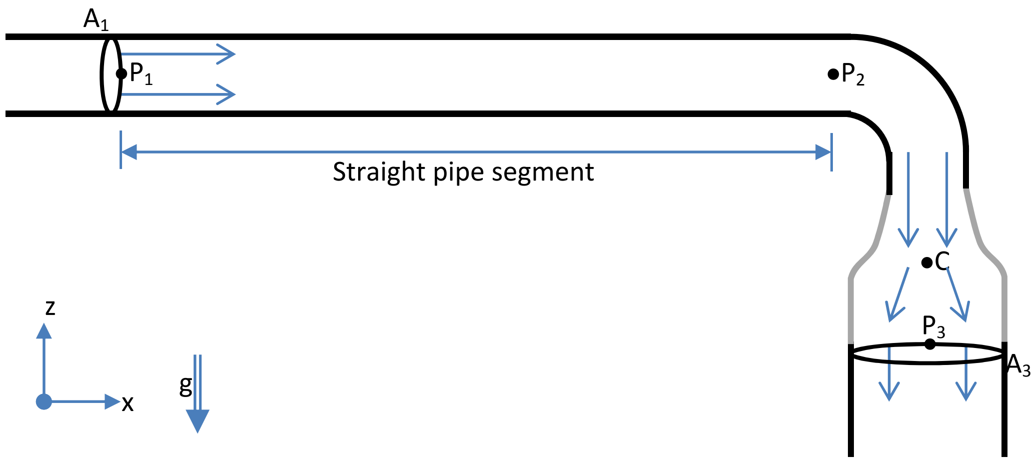
1 Consider the following pipe system:



- Water flows steadily within the pipe at constant density ρ=1000 kg/m³. → this is at 5°C (see chart on exam cover) use ν and γ at 5°C
- The straight pipe segment has L=11.3 cm, diameter D=1.13 cm, and is the end of a 5 m pipe run.
- The straight pipe segment has smooth walls.
- Velocities are approximately uniform over A_1, A_2 and A_3.
- P_1=1000 Pa; $A_1 = 1\text{x}10^{-4}\ \text{m}^2$; $\overline{V}_1 = 1$ m/s
- P_2=? $A_2 = 1\text{x}10^{-4}\ \text{m}^2$; $\overline{V}_2 = 1$ m/s
- Information given here are only relevant for this question, not the other questions in this exam.

A) Calculate P_2.

Energy equation from 1 to 2. Check: steady flow – given, constant-density – given, streamline from 1 to 2 – assume.

$$\underbrace{\frac{P_1-P_2}{\gamma}}+\underbrace{\frac{V_1^2-V_2^2}{2g}}_{0}+\underbrace{z_1-z_2}_{0}-h_L\underbrace{-h_T+h_P}_{\substack{\text{no pump or}\\ \text{turbine between}\\ 1\text{ and }2}}=0 \Rightarrow P_2 = P_1-\gamma h_L$$

$z_1 = z_2$

1) $V_1 = \overline{V}_1$
2) $V_2 = \overline{V}_2$
3) $\overline{V}_1 A_1 = \overline{V}_2 A_2$
4) $A_1 = A_2$
1,2,3,4 $\Rightarrow V_1 = V_2$

find h_L from straight-pipe (major) loss formula: $h_L = \frac{\overline{V}^2}{2g}\frac{L}{D} f\left(Re, \frac{\varepsilon}{D}\right) K_D\left(Re, \frac{L}{D}\right)$

$$Re = \frac{D\overline{V}}{\nu} = \frac{(1.13\times10^{-2}\,m)(1\,m/s)}{1.519\times10^{-6}\,m^2/s} = 7439$$

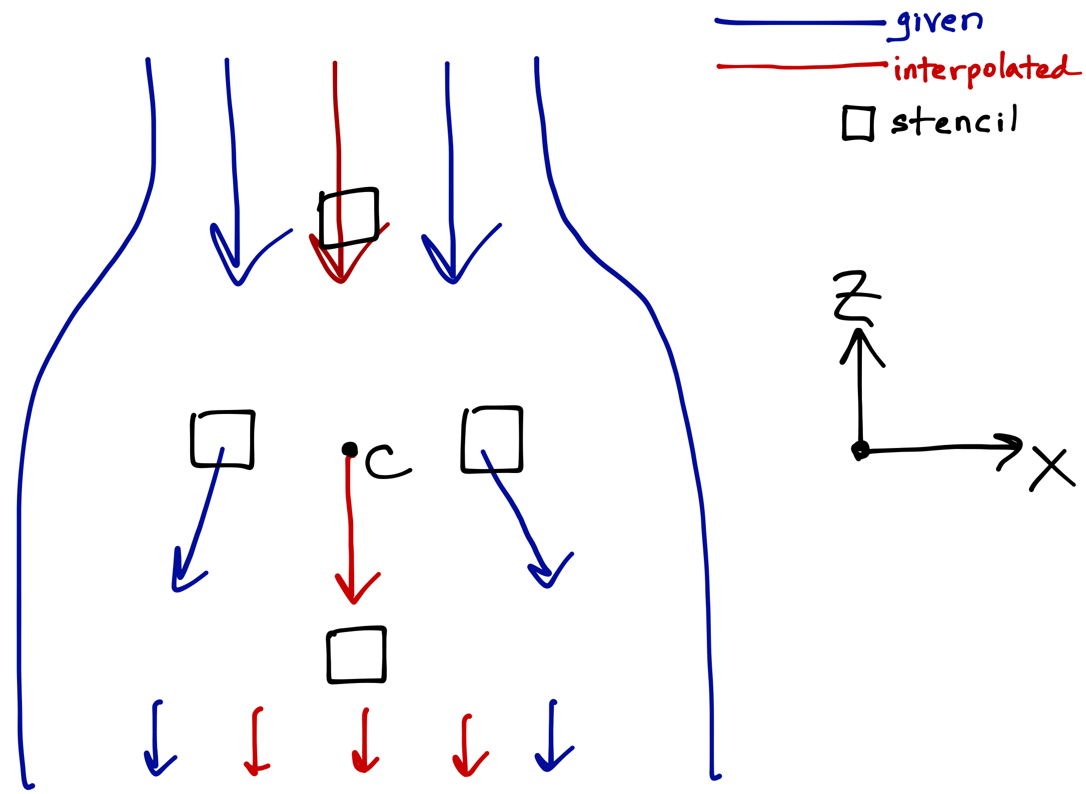
$\frac{\varepsilon}{D} = 0$ (smooth) – use smooth curve on Moody chart

$f(7439, 0) \approx 0.034$ from Moody chart

$K_D \approx 1$ because development occurs in the 5m before location 1. Observe: $K_D(L=5.11\,m) - K_D(L=5.00\,m) \approx 0$.

$$h_L = \frac{(1\,m/s)^2}{2g}\left(\frac{11.3\,cm}{1.13\,cm}\right)(0.034) = 0.017\ m$$

$$P_2 = 1000\ Pa - (9807\ N/m^3)(0.017\,m) = \boxed{833\ Pa}$$



B) At location C, state whether the following are >0, <0, =0, or not possible to evaluate:

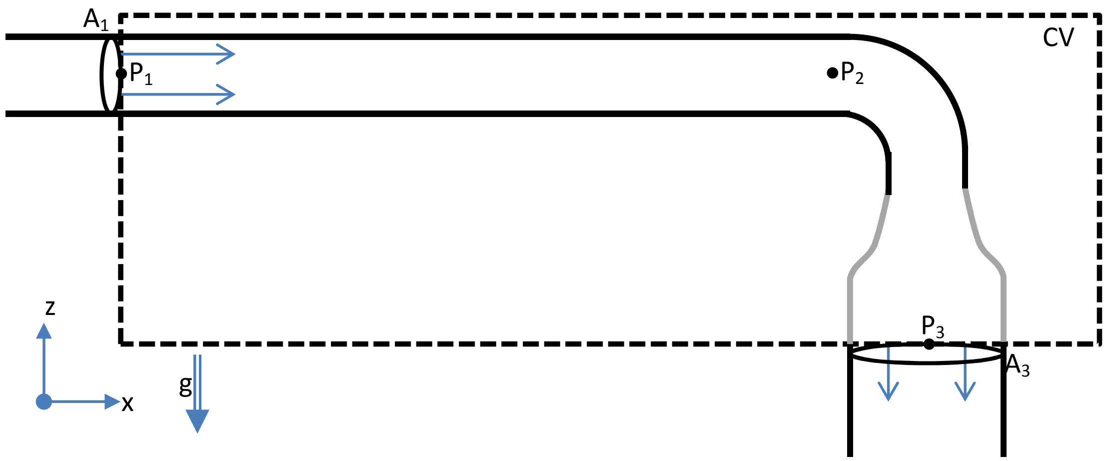
$\frac{\partial u}{\partial x} > 0$ As x increases (moving rightwards), u goes from negative (left stencil) to zero (near C) to positive (right stencil). u increases as x increases.

$\frac{\partial w}{\partial x} = 0$ As x increases (moving rightwards), w goes from negative (left stencil) to the same negative value (right stencil). w doesn't change as x increases.

$\frac{\partial u}{\partial z} = 0$ As z increases (moving upwards), u goes from zero (bottom stencil) to zero (top stencil). u doesn't change as z increases.

$\frac{\partial w}{\partial z} < 0$ As z increases (moving upwards), w goes from weakly negative (bottom stencil) to strongly negative (top stencil). w decreases as z increases.

2 Calculate the anchoring force acting on this control volume:



- Water flows steadily within the pipe at constant density ρ=1000 kg/m^3.
- Air outside the pipe contributes negligible pressure, shear, and weight to this analysis.
- Mass within the control volume = 200 g of metal and 500 g of water.
- Velocities are approximately uniform over A_1, A_2 and A_3.
- P_1=12 kPa; $A_1 = 1\times10^{-4}$ m^2; $\overline{V}_1$ = 1 m/s.
- P_3=15 kPa; $A_3 = 2\times10^{-4}$ m^2; $\overline{V}_3$ = 0.5 m/s.
- Information given here are only relevant for this question, not the other questions in this exam.

Use force balance (momentum conservation with steady flow and non-deforming non-accelerating CV)

$\underline{F}_G + \underline{F}_P + \underline{F}_S + \underline{F}_A + \underline{F}_T = 0$ where F_T = thrust = −momentum flowrate integral $= -\int_{CS} \rho \underline{u}(\underline{u}\cdot\underline{n})\,dA$

All terms are vectors

$\underline{F}_G = mg(-\hat{z}) = -(0.7\text{ kg})(9.8\text{ m/s}^2) = -6.86\text{ N }\hat{z}$

pressure acts into CV even though flow is coming out.

$\underline{F}_P = P_1A_1(+\hat{x}) + P_3A_3(+\hat{z})$ + negligible pressure on the other parts of the control surface (given)

$= (\underbrace{12\times10^3\text{ N/m}^2}_{\text{kPa}})(1\times10^{-4}\text{ m}^2)\,\hat{x} + (15\times10^3\text{ N/m}^2)(2\times10^{-4}\text{ m}^2)\,\hat{z} = 1.2\text{ N }\hat{x} + 3\text{ N }\hat{z}$

$\underline{F}_S = 0$ At inlet & outlet, no evident gradient in flow components parallel to A_1 or A_3. At other parts of control surface, no shear is given.

$\underline{F}_A = ?$

$$\underline{F}_T = -\int_{in}\rho\underbrace{(\bar{V}_1(+\hat{x}))}_{\substack{\text{vector } u \text{ is } \bar{V}_1 \text{ and}\\ \text{is in } +\hat{x}\\ \text{direction}}}\underbrace{(-\bar{V}_1)}_{(\underline{u}\cdot\underline{n})}dA - \int_{out}\rho\underbrace{(\bar{V}_3(-\hat{z}))}_{\substack{\text{vector } u\\ \text{is } \bar{V}_3 \text{ and}\\ \text{is in } -\hat{z}\\ \text{direction}}}\underbrace{(\bar{V}_3)}_{(\underline{u}\cdot\underline{n})}dA - \int_{side}\rho(0)(0)\,dA = \underbrace{\rho\bar{V}_1^2A_1(\hat{x})}_{\substack{\text{thrust pushes}\\ \text{CV contents}\\ \text{rightward like}\\ \text{a "pocket"}}} + \underbrace{\rho\bar{V}_3^2A_3(\hat{z})}_{\substack{\text{thrust pushes}\\ \text{CV contents}\\ \text{upward like}\\ \text{a "rocket"}}}$$

$\underline{F}_T = (1000\text{ kg/m}^3)(1\text{ m/s})^2(1\times10^{-4}\text{ m}^2)\,\hat{x} + (1000\text{ kg/m}^3)(0.5\text{ m/s})^2(2\times10^{-4}\text{ m}^2)\,\hat{z} = 0.1\text{ N }\hat{x} + 0.05\text{ N }\hat{z}$

Summing all components in the force balance:

$-6.86\text{ N }\hat{z} + 1.2\text{ N }\hat{x} + 3\text{ N }\hat{z} + 0 + \underline{F}_A + 0.1\text{ N }\hat{x} + 0.05\text{ N }\hat{z} = 0$

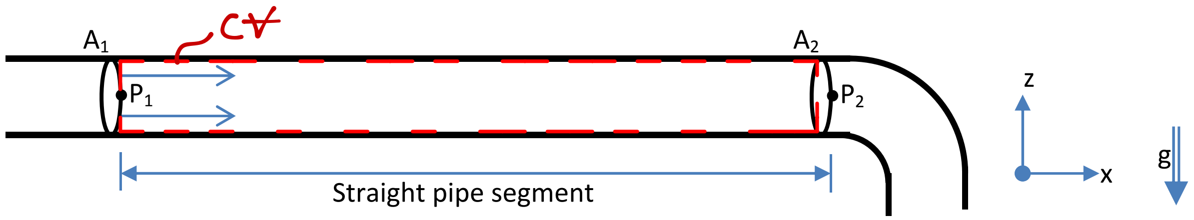
Solve for $\underline{F}_A$: $(1.2\text{ N }\hat{x} + 0.1\text{ N }\hat{x}) + (-6.86\text{ N }\hat{z} + 3\text{ N }\hat{z} + 0.05\text{ N }\hat{z}) = -\underline{F}_A$

$1.3\text{ N }\hat{x} - 3.81\text{ N }\hat{z} = -\underline{F}_A$

$\boxed{\underline{F}_A = -1.3\text{ N }\hat{x} + 3.81\text{ N }\hat{z}}$ (anchor pushes left and up)

resisting weight is its biggest job.

3 Consider the following pipe system:



- Water flows steadily within the pipe at constant density ρ=1000 kg/m^3.
- The straight pipe segment has L=11.3 cm, diameter D=1.13 cm, and is the end of a 5 m pipe run.
- The only information given about the velocity profiles is that the velocity profile at A_1 is the same as the velocity profile at A_2.
- P_1=2000 Pa; $A_1 = 1\text{x}10^{-4}$ m^2; $\overline{V_1}$ = 1 m/s
- P_2=1200 Pa; $A_2 = 1\text{x}10^{-4}$ m^2; $\overline{V_2}$ = 1 m/s
- Information given here are only relevant for this question, not the other questions in this exam.

A) Calculate the shear stress on the wall of the straight pipe section.

We want shear stress τ, but velocity profile is unknown, so we cannot use Newton's shear model $\|\tau_{xr}\| = \mu \left\|\frac{\partial u}{\partial r}\right\|$

We could run energy equation, but we would need to figure out how h_L is quantitatively related to τ.

Force balance $\overset{?}{F_S}, \overset{\checkmark}{F_G}, \overset{\checkmark}{F_T}, \overset{0}{\cancel{F_A}}, \overset{\checkmark}{F_P}$ allows us to solve for F_S, and the definition of stress gives $\tau = \frac{F_S}{A}$.

Force balance is vector equation, and we only need to solve the x-direction because τ is only in x (by observation).

Use C∀ labeled on picture, just inside the pipe walls.

$F_{S_x} = ?$

$F_{G_x} = 0$ (gravity is in $\hat{z}$)

$F_T = -\underbrace{\int_{in} \rho V^2 dA + \int_{out} \rho V^2 dA}_{\text{same (given)}} = 0$ cancel

$F_A = 0$ (C∀ doesn't cross wall)

$F_P = P_1A_1 - P_2A_2 = (2000\text{ Pa} - 1200\text{ Pa})(1\times10^{-4}\text{ m}^2) = 0.08\text{ N}$

$F_{S_x} + 0.08\text{ N} = 0$

Shear force on contents of C∀ is $F_S = -0.08$ N (wall pushes fluid in $-\hat{x}$)

equal and opposite force exerted on wall $F_{S_W} = 0.08$ N (fluid pushes wall in $+\hat{x}$)

Convert from shear force F_S to shear stress τ using the area over which shear acts.

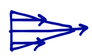
This area is the surface area of the wall inside the pipe (A_w).

A_w = Circumference $*$ length $= \pi D L = \pi(1.13\times10^{-2}\text{ m})(1.13\times10^{-1}\text{ m}) = 4\times10^{-3}\text{ m}^2$

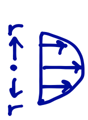
$\tau_w = \frac{F_{SW}}{A_w} = \frac{0.08\text{ N}}{4\times10^{-3}\text{ m}^2} = \boxed{20\text{ N/m}^2}$

B) Calculate the velocity gradient $\left\|\frac{\partial u}{\partial r}\right\|$ at the pipe wall, where u is the velocity in the x-direction and r is the radial coordinate in a cylindrical coordinate system, with $r = 0$ at the pipe center.

Newton's shear model $\|\tau_{xr}\| = \mu\left\|\frac{\partial u}{\partial r}\right\|$ so we can use result from part A, $\left\|\frac{\partial u}{\partial r}\right\| = \frac{\|\tau_{xr}\|}{\mu} = \frac{20\ N/m^2}{1.519\times10^{-3}\ \frac{N\cdot s}{m^2}} = \boxed{13{,}167\ s^{-1}}$

$\underbrace{\text{If you assume a linear velocity profile,}}_{\text{This assumption leads to an error of about } 10^3.}$  make sure that $\bar{V}$ is the average velocity, not the maximum

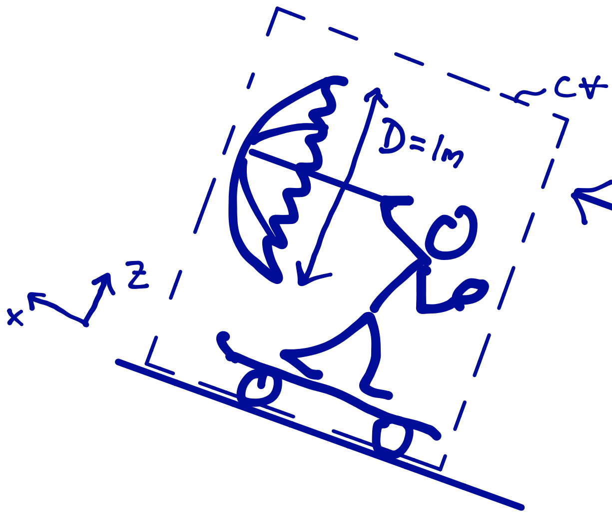
1-point bonus: state whether $\frac{\partial u}{\partial r}$ is positive, negative, or zero. No justification needed.

origin  u decreases outward from origin

r increases outward from origin

$\Big\}\ \frac{\partial u}{\partial r} < 0$ regardless of where we evaluate it

4 A person is riding a skateboard down an incline. They use an umbrella to control their speed. By holding the umbrella behind them, they achieve a steady velocity of 2 m/s. Estimate the drag force provided by the umbrella, and include a picture that helps to explain your assumptions.



V_{fs} (relative to C∀ including person and umbrella)

$V_{fs} = -2 \text{ m/s } \hat{x}$

$F_D = \frac{1}{2}\rho V_{fs}^2 \, a \, C_D(Re, \text{geometry})$

→ for hollow hemisphere at $Re > 10^4$ $C_D \approx 1.4$ (reference packet p.13-14)

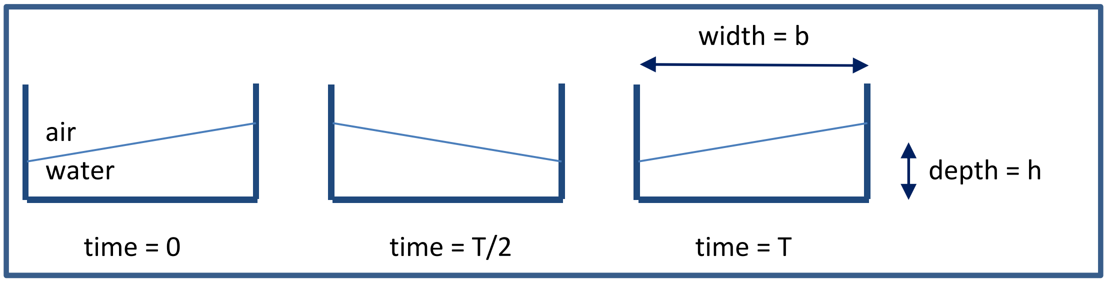
$Re \approx \frac{V_{fs} D}{\nu} = \frac{(2 \text{ m/s})(1 \text{ m})}{1.5\times10^{-5} \text{ m}^2/\text{s}} > 10^4$ ✓

→ use projected area $\pi\left(\frac{D}{2}\right)^2 = 0.785 \text{ m}^2$

$F_D = \frac{1}{2}(1 \text{ kg/m}^3)(2 \text{ m/s})^2(0.785 \text{ m}^2)(1.4) = \boxed{2.2 \text{ N}}$

in −x direction

5 In the laboratory, you measure the oscillation period T for a full-basin wave in a square basin:



Data Table	h	b	g	T
Model 1	0.1 m	1 m	9.8 m/s^2	7 s
Model 2	0.1 m	0.1 m	9.8 m/s^2	3 s
Model 3	0.001 m	0.01 m	9.8 m/s^2	0.7 s

Predict T for a basin that has b=100m and h=10m. You are told that T=f(h,b,g) for a basin with b=100m.

s m m m/s^2 $\rightarrow$ r=2 reference dimensions in MLT (no FLT possible)

Buckingham's theorem suggests 4 variables − 2 reference dimensions = 2 nondimensional Π-terms.

Choose T & h as non-repeating variables, leaving b & g as repeaters.

Construct Π-terms $\Pi_1 = \frac{T^2 g}{b}$ $\left[\frac{s^2\, m/s^2}{m}\right]$ = nondimensional

$\Pi_2 = \frac{h}{b}$ $\left[\frac{m}{m}\right]$ = nondimensional

other options $\left(\frac{T^2 g}{h}, \frac{b}{h}\right)$ $\left(T\sqrt{\frac{g}{h}}, \frac{b}{h}\right)$ $\left(T\sqrt{\frac{g}{b}}, \frac{h}{b}\right)$ and more

Buckingham's theorem $\Pi_1 = F(\Pi_2)$ or $\frac{T^2 g}{b} = F\left(\frac{h}{b}\right)$

Model 1, model 3, and the prototype (full-size thing) all have $\Pi_2 = \frac{h}{b} = 0.1$

The principle of similitude states that they will all have the same value for $F(\Pi_2) = F(0.1)$

Because $\Pi_1 = F(\Pi_2)$, Model 1, Model 3, and the prototype will have the same value for Π_1

Model 1 has $\Pi_1 = 480$
Model 3 has $\Pi_1 = 480$
> checks out (data collapses)

Prototype must also have $\Pi_1 = 480$ by similitude $\frac{T_p^2 g}{b_p} = 480$ or $T_p = \sqrt{\frac{b_p 480}{g}} = \boxed{70 \text{ seconds}}$

If we had used other Π-groups we would have: $\left(\frac{T^2 g}{h}, \frac{b}{h}\right) = (4800, 10)$

or $\left(T\sqrt{\frac{g}{h}}, \frac{b}{h}\right) = (69.3, 10)$

or $\left(T\sqrt{\frac{g}{b}}, \frac{h}{b}\right) = (21.9, 0.1)$

etc.

1-point bonus: would T for the 100m-basin be smaller, larger, or the same on Mars, where g=3.7 m/s^2?

$\boxed{\text{Larger.}}$ For any choice of Π-terms, when g$\downarrow$ T$\uparrow$.

1A	1B	2	3A	3B	4	5	Total (28)