Midterm Exam Template

Due March 17 at 10:00am on bCourses

Instructions

1. The exam duration is 50 minutes. 10 extra minutes are allotted for scanning and submission.
2. The exam consists of 3 questions, worth a total of 100 points.
3. Exam questions will be made available at 9:00am on March 17th under the Midterm assignment on the bCourses class page. The exam should be submitted to the same assignment no later than 10:00am the same day. Late submissions will not be accepted or graded.
4. It is your responsibility to make sure you can download from and upload to bCourses. For this reason, a practice submission assignment has been available.
5. Questions can be asked at the regular Zoom lecture link during the exam. Only language clarification questions will be answered.
6. Solve the questions neatly and methodically using the provided template. The template can be printed and scanned or used as a soft copy on a tablet device. If you do not have access to a printer or a tablet, you can also use your own paper for the solution, provided that your scanned solution is clear and legible.
7. If you run out of space for any problem, indicate that the solution continues on the extra page at the end.
8. This is an open notes examination. However, you are ONLY allowed to use the materials posted on bCourses as hard copies or as pdf files. No other material is allowed.
9. You are NOT allowed to use the Internet or any software on your computer during the exam.
   Exceptions: downloading the questions, uploading the answers, accessing files on CE191 bCourses page.
10. You are prohibited to consult with any individual (real or virtual) while solving this exam.
11. Clearly state assumptions you have made for any missing information.
12. **Important:** In the next page, provide the requested information in the designated spaces and carefully read and sign the Honor Pledge. If you cannot print this template, please copy and sign the pledge by hand.

Name: Solution

SID: _____

Honor Code: I have not given or received aid in this examination from any other person. I have taken an active part in seeing to it that others as well as myself uphold the spirit and letter of this Honor Code. I followed the above instructions list precisely.

Signature: _____

Leave the rest of this page blank!

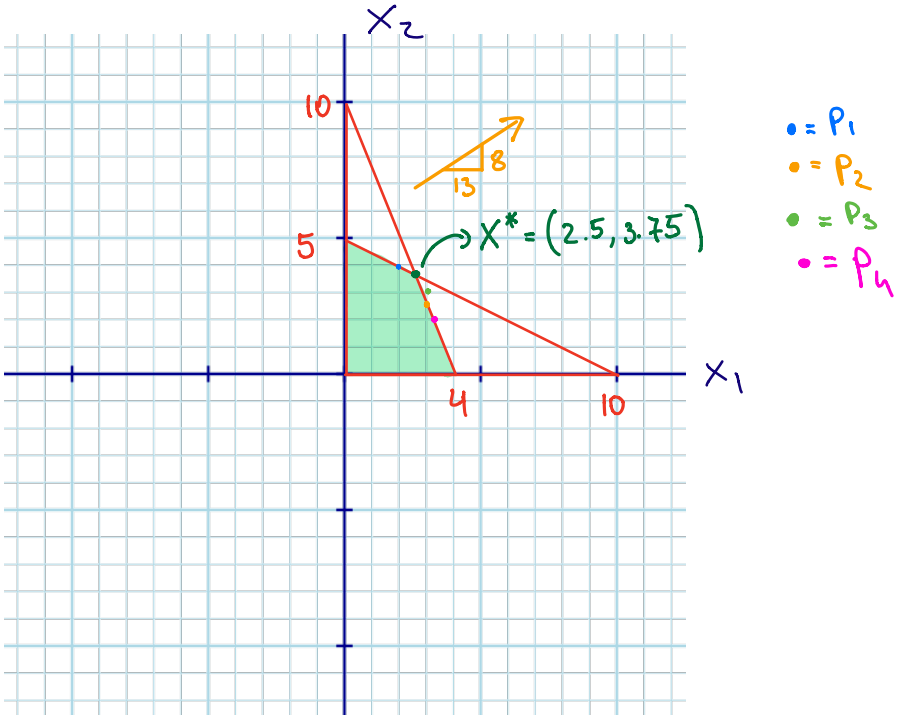
Problem 1. (40 points)

1.

$$C = [13 \quad 8]^T$$

$$A = \begin{bmatrix} 5 & 2 \\ 1 & 2 \\ -1 & 0 \\ 0 & -1 \end{bmatrix}, \quad b = \begin{bmatrix} 20 \\ 10 \\ 0 \\ 0 \end{bmatrix}$$

2.



3.

$$\left.\begin{aligned} x_1 + 2x_2 &= 10 \\ 5x_1 + 2x_2 &= 20 \end{aligned}\right\} \; 4x_1 = 10 \Rightarrow$$

$$x_1^* = 2.5$$

$$x_2^* = 3.75$$

$$\Rightarrow \quad \bar{c}^T \bar{x}^* = 62.5$$

4.

P1: $x_1 \geq 3$

From the graph $x_1^* = 3 \Rightarrow x_2^* = (20 - 5 \times 3)/2 = 2.5$

$\Rightarrow c^T x^* = 59$

P2: $x_1 \leq 2$

From the graph $x_1^* = 2 \Rightarrow x_2^* = (10 - 2)/2 = 4$

$\Rightarrow c^T x^* = 58$

5.

P2 is not active since we have an integer solution

Branch on P1:

P3: $x_2 \geq 3$ ($x_1 \geq 3$ from P1)

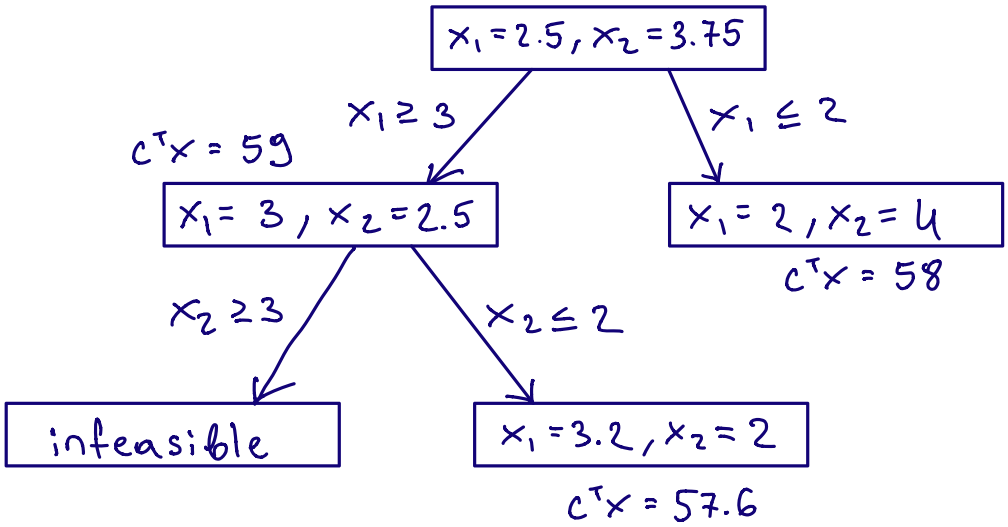
$5x_1 + 2x_2 \leq 20$ makes this problem infeasible → not active

P4: $x_2 \leq 2$

From the graph $x_2 = 2 \Rightarrow x_1 = 3.2 \Rightarrow c^T x^* = 57.6 \leq 58$.

$\Rightarrow x^* = (2, 4)$, $c^T x^* = 58$

6.



Problem 2. (20 points)

1a.

$$\nabla_{\bar{x}} f = \begin{bmatrix} 6x_1 \\ 1/x_2 \end{bmatrix}$$

1b.

$$\bar{x}^1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} - 0.1 \begin{bmatrix} 6(1) \\ 1/(1) \end{bmatrix} = \begin{bmatrix} 0.4 \\ 0.9 \end{bmatrix}$$

$$\bar{x}^2 = \begin{bmatrix} 0.4 \\ 0.9 \end{bmatrix} - 0.1 \begin{bmatrix} 6(0.4) \\ 1/(0.9) \end{bmatrix} = \begin{bmatrix} 0.160 \\ 0.789 \end{bmatrix}$$

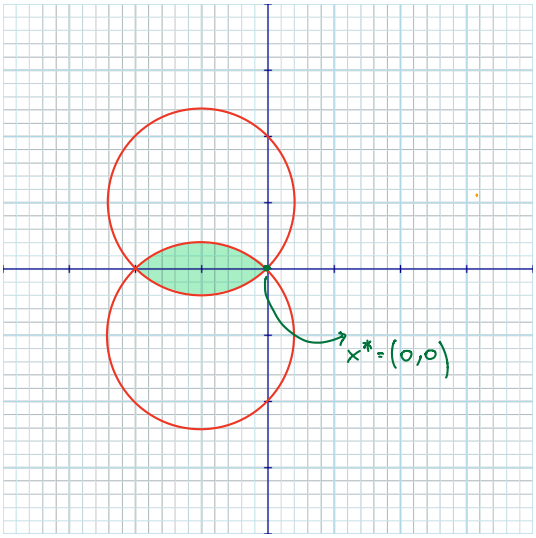
2.

$$\frac{df}{dx} = 2x, \quad x^1 = 0.5 - (1)(2*0.5) = -0.5$$

$$x^2 = -0.5 - (1)(2*(-0.5) = 0.5$$

GD will not converge since the step size is too big and causes the solution to oscillate between ± 0.5.

Problem 3. (40 points)



2.

$$\mathcal{L} = (x_1^2 + x_2^2) + \lambda_1\left((x_1+1)^2 + (x_2-1)^2 - 2\right) + \lambda_2\left((x_1+1)^2 + (x_2+1)^2 - 2\right)$$

3.

Stationarity:

$$x_1^* + (\lambda_1^* + \lambda_2^*)(x_1^* + 1) = 0$$
$$x_2^* + \lambda_1^*(x_2^* - 1) + \lambda_2^*(x_2^* + 1) = 0 \qquad (1)$$

Primal feasibility:

$$(x_1^* + 1)^2 + (x_2^* - 1)^2 - 2 \le 0$$
$$(x_1^* + 1)^2 + (x_2^* + 1)^2 - 2 \le 0 \qquad (2)$$

Non-negativity: $\lambda_1^* \ge 0, \ \lambda_2^* \ge 0$ (3)

Complementary slackness:

$$\lambda_1^*\left[(x_1^* + 1)^2 + (x_2^* - 1)^2 - 2\right] = 0$$
$$\lambda_2^*\left[(x_1^* + 1)^2 + (x_2^* + 1)^2 - 2\right] = 0 \qquad (4)$$

4.

From (1) and (3), we can conclude:

$x_1^* = 0, x_2^* = 0 \Rightarrow$

$\left| \begin{array}{l} \lambda_1 = -\lambda_2 \\ \lambda_1 = \lambda_2 \end{array} \right. \Rightarrow \lambda_1^* = 0, \lambda_2^* = 0$

and $\bar{x}^*$ is optimal.

5.

For maximization, multiply objective by -1

From 1. $x^* = (-2, 0)$, and the stationarity conditions are:

$$\left. \begin{array}{l} -x_1^* + (\lambda_1^* + \lambda_2^*)(x_1^* + 1) = 0 \\ -x_2^* + \lambda_1^*(x_2^* - 1) + \lambda_2^*(x_2^* + 1) = 0 \end{array} \right\} \begin{array}{l} 2 - \lambda_1^* - \lambda_2^* = 0 \\ -\lambda_1^* + \lambda_2^* = 0 \end{array}$$

$\Rightarrow \lambda_1^* = \lambda_2^* = 1$

6.

KKT conditions are always necessary for an optimum. Since the constraints and objective function are convex, they are also sufficient

Extra space

Make sure to label which problem you are continuing.