UNIVERSITY OF CALIFORNIA BERKELEY
Department of Civil Engineering
Spring 2020

Structural Engineering,
Mechanics and Materials
Professor: S. Govindjee

Final Exam: CE132
Tuesday May 12
8:10am-11am

Problem	Score
#0	$\{0, -2\}$
#1	/25
#2	/25
#3	/15
#4	/15
#5	/10
#6	/10
Total	/100

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Name

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SID

Permitted Materials: Open book, open notes, open Matlab and Mathematica.
Prohibited: Use of the internet and any and all communication devices (except to access bCourses and gradescope)

Honor Code: I have not given or received aid in this examination. I have taken an **active part** in seeing to it that others as well as myself uphold the spirit and letter of this Honor Code.

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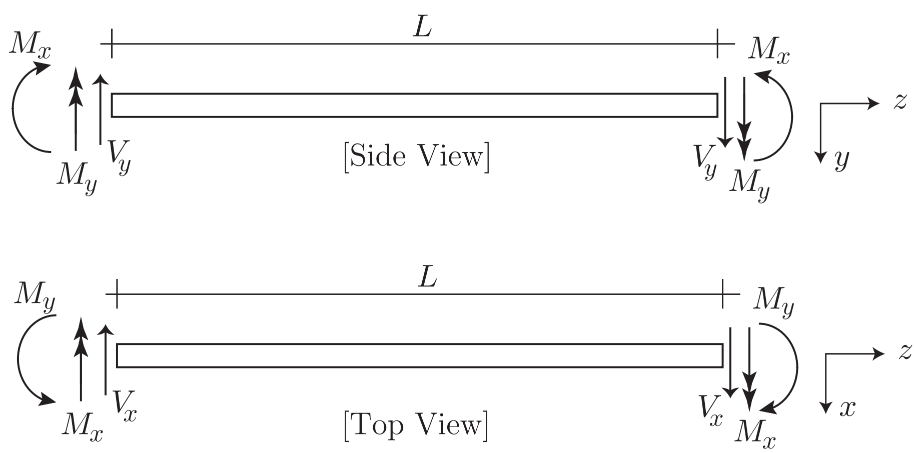
(Signature)

No questions permitted during exam. If you have a concern about the wording of a question, explain your concern along with your answer.

0. (0 pts) *Copy* the honor code at the top of your first answer page and *sign* your name to it, failure to do *both* (-2 pts).

1. (25 pts) An $L = 5$ m long beam with Young's modulus $E = 200$ GPa has been loaded at its ends with possibly moments and shear forces.



The curvatures of the beam κ_x and κ_y have been measured as a function of z to be constants:

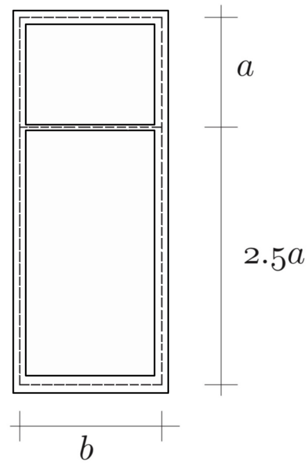
$$\kappa_x = 0.001 \text{ m}^{-1} \quad (1)$$
$$\kappa_y = 0.003 \text{ m}^{-1} \quad (2)$$

The beam has the following moments of inertia

$$I_x = 8.0 \times 10^{-3} \text{ m}^4 \qquad I_y = 5.0 \times 10^{-3} \text{ m}^4 \qquad I_{xy} = 3.0 \times 10^{-3} \text{ m}^4 . \quad (3)$$

Determine the end moments and shear forces that have been applied to the beam.

2. (25 pts) Shown is the cross-section of a thin-walled torsional member. The outer walls have thickness t and the inner wall has a thickness of $t/4$.



Assume $t = 1$ mm, $a = 300$ mm, $b = 400$ mm, and $G = 100$ kN/mm^2.

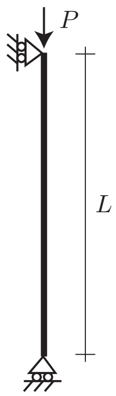
(a) Find the effective torsional stiffness of the structural member; i.e., find $T/\bar{\theta} = (GJ)_{\text{eff}}$.

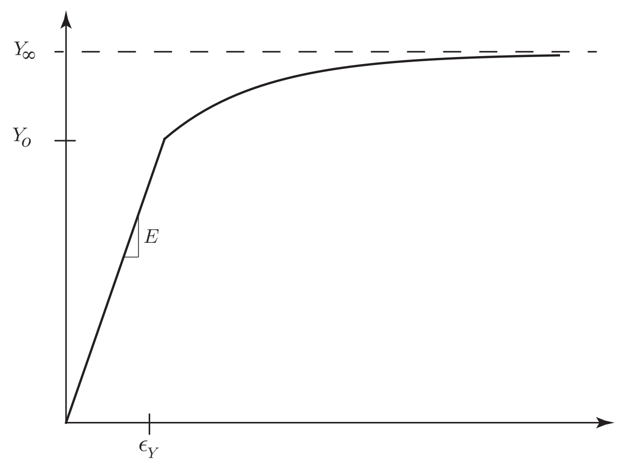
(b) Find the max shear stress and the location where it occurs.

3. (15 pts) Consider a pin-pin column subjected to an axial load P of length L and radius of gyration r. Assume the column is made of an elastic-plastic material with the following stress-strain relation

$$\sigma = \begin{cases} E\epsilon & \epsilon < \epsilon_Y \\ (Y_\infty - Y_o)\left(1 - e^{-(\epsilon-\epsilon_Y)}\right) + Y_o & \epsilon > \epsilon_Y \,, \end{cases} \tag{4}$$

where E is the Young's modulus, Y_o is the initial yield stress, Y_∞ is the yield stress at large strain, and ϵ_Y is the initial yield strain.



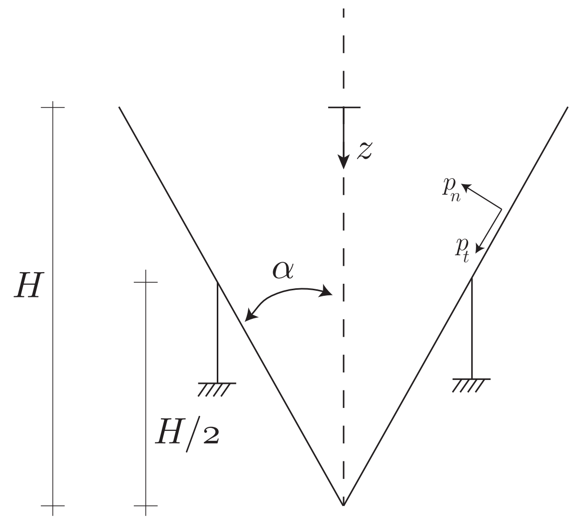


Assume that the column slenderness L/r is such that the critical buckling stress

$$\sigma_{\rm cr} > Y_o \equiv E\epsilon_Y \,. \tag{5}$$

Determine the (critical) buckling stress for the column in terms of the material parameters and the slenderness ratio, L/r.

4. (15 pts) Consider an axisymmetric conical hopper which is supported by a ring support at half-height. The shell wall is uniform of thickness t.



The hopper is filled with a granular material that generates normal and tangential wall pressures

$$p_n(z) = -k\left(1 - e^{-z/\zeta}\right) \tag{6}$$

$$p_t(z) = \quad k\left(1 - e^{-z/\zeta}\right) , \tag{7}$$

where k and ζ are given constants.

Find the hoop and meridional membrane resultants, $N_\theta(z)$ and $N_\varphi(z)$.

Express your answer in terms of k, ζ, z, H, and α.

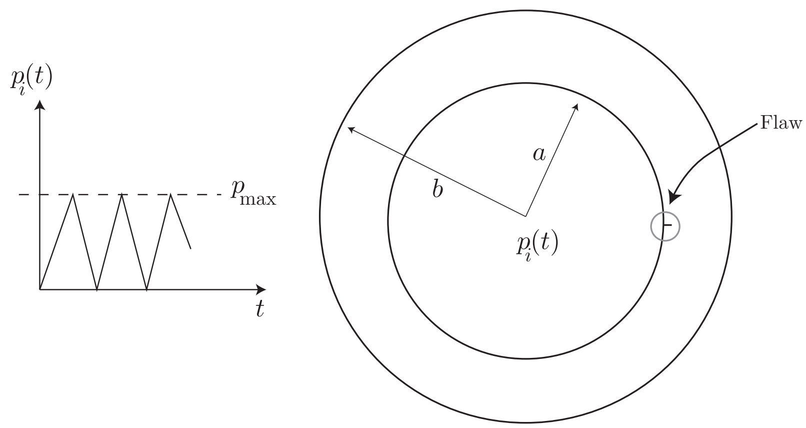
Useful facts:

(a) $\varphi = \frac{\pi}{2} + \alpha$;

(b) $\sin(\varphi) = \cos(\alpha)$;

(c) $\cos(\varphi) = -\sin(\alpha)$.

5. (10pts) An open-ended thick-walled cylinder of inner radius $a = 40$ mm and outer radius $b = 60$ mm is subjected to a time varying internal sawtooth pressure $p_i(t)$ with maximum pressure p_{max}.



The cylinder also has a detected crack-like flaw of size $a_i = 1$ mm oriented orthogonal to the hoop direction. Determine

(a) the magnitude of p_{max} such that the cylinder will not fail before $N = 10^6$ pressurization/de-pressuriztion cycles;

(b) the critical crack size for the pressure that you computed in part (a).

Assume

(i) the configuration correction factor for the crack is $Q = 1.8$, i.e., $K_{\text{I}} = Q\sigma_{\text{hoop}}\sqrt{\pi a}$;

(ii) the Young's modulus $E = 200$ kN/mm^2;

(iii) the critical stress intensity factor is $K_{\text{Ic}} = 10$ MPa$\sqrt{\text{m}}$;

(iv) the Paris crack growth exponent $m = 4$;

(v) the Paris crack growth coefficient $C = 5.0 \times 10^{-13}$(m/cycle)/(MPa$\sqrt{\text{m}}$)^4.

6. (10 pts) Consider a *clamped* circular plate of radius R with plate modulus D, both given. The plate is subjected to a pressure

$$p(r) = p_o\left(1 - \frac{r}{R}\right) , \tag{8}$$

where p_o is a given constant.

(a) Determine the expression $w(r)$ for the deflection of the plate, specialized for the given pressure loading; write your answer in a format that still includes the 4 (four) integration constants.

(b) State why you may *a priori* assume one of the integration constants to be zero.

(c) State the 3 (three) boundary conditions that you would use to solve for the remaining integration constants. Be reasonably specific but there is no need to actually solve for the constants – its just messy algebra.