UNIVERSITY OF CALIFORNIA BERKELEY
Department of Civil Engineering
Spring 2020

Structural Engineering,
Mechanics and Materials
Professor: S. Govindjee

Midterm Exam: CE132
Wednesday March 11
50 Minutes

Problem	Score
#1	/50
#2	/30
#3	/20
Total	/100

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Name

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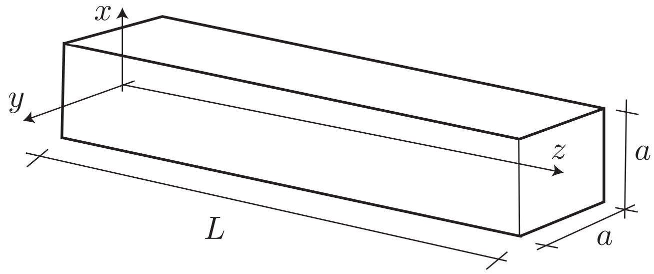
SID

Permitted Materials: 1 Crib Sheet.

No questions permitted during exam. If you have a concern about the wording of a question, explain your concern along with your answer.

1. Consider a prismatic body with square cross-section with area a^2 and length L.



The displacement field has been measured to be

$$
\begin{aligned}
u(x,y,z) &= k_1 + k_2 x \\
v(x,y,z) &= k_3 y + k_4 z \\
w(x,y,z) &= k_5 x + k_6 y + k_7 z\,,
\end{aligned}
$$

where the k_i are given constants with appropriate units and are such that the deformation can be considered to be small.

(a) (10pts) Find an expression for the corresponding strain field.

(b) (10pts) At the point $(x,y,z) = (0,0,L/2)$, what is the normal strain in the direction

$$
\boldsymbol{N} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{pmatrix} ?
$$

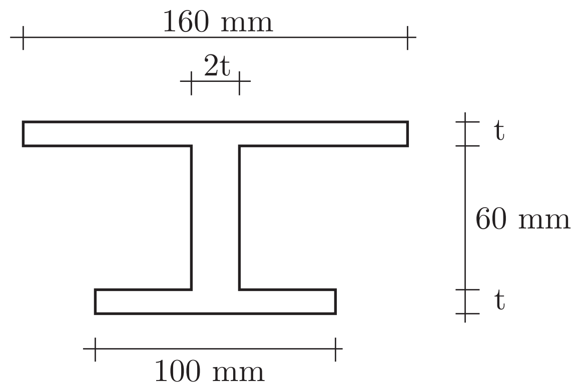
(c) (10pts) Assuming $k_2 = k_3 = 0$ and assuming that the material of the body is linear elastic with elastic constants λ and G. Find an expression for the stress field.

(d) (10pts) For the stress in part (c), at the point $(x,y,z) = (0,0,L/2)$, what is the the normal stress on the plane with normal

$$
\boldsymbol{N} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{pmatrix} ?
$$

(e) (10pts) Assuming $k_2 = k_3 = k_4 = k_6 = 0$, what is the condition on k_5 and k_7 to ensure that yield does not occur according to the Mises yield condition. Assume the yield in uniaxial tension is given by Y.

2. (30pts) Consider a torsion bar with a cross-section as shown below. If the shear modulus of the material is $G = 75\ \text{kN/mm}^2$, find the needed thickness t so that the torsional stiffness, GJ_{eff}, is at least $25 \times 10^3\ \text{kN} \cdot \text{mm}^2$. In your computation, you may assume that $t \ll 30$ mm; a point that you should verify after completing your computation.



3. Consider a linear elastic half-space. The stresses in the half-space subject to a vertical point force P at $(x, y) = (0, 0)$ are given by:

$$\sigma^{\mathrm{v}}_{xx} = \boldsymbol{e}_x \cdot \boldsymbol{T}\boldsymbol{e}_x = -\frac{2P}{\pi}\frac{x^2 y}{(x^2+y^2)^2},$$
$$\sigma^{\mathrm{v}}_{yy} = \boldsymbol{e}_y \cdot \boldsymbol{T}\boldsymbol{e}_y = -\frac{2P}{\pi}\frac{y^3}{(x^2+y^2)^2},$$
$$\sigma^{\mathrm{v}}_{xy} = \boldsymbol{e}_x \cdot \boldsymbol{T}\boldsymbol{e}_y = -\frac{2P}{\pi}\frac{xy^2}{(x^2+y^2)^2}.$$

The stresses in the half-space subject to a horizontal point force H at $(x, y) = (0, 0)$ are given by:

$$\sigma^{\mathrm{h}}_{xx} = \boldsymbol{e}_x \cdot \boldsymbol{T}\boldsymbol{e}_x = -\frac{2H}{\pi}\frac{y^3}{(x^2+y^2)^2},$$
$$\sigma^{\mathrm{h}}_{yy} = \boldsymbol{e}_y \cdot \boldsymbol{T}\boldsymbol{e}_y = -\frac{2H}{\pi}\frac{xy^2}{(x^2+y^2)^2},$$
$$\sigma^{\mathrm{h}}_{xy} = \boldsymbol{e}_x \cdot \boldsymbol{T}\boldsymbol{e}_y = -\frac{2H}{\pi}\frac{x^2 y}{(x^2+y^2)^2}.$$

Write an expression for the stresses to the problem shown below along the line $x = 0$.