Midterm Exam

CE 191 | Spring 2019
UC Berkeley

Mar. 12, 2018, 1:10pm – 2:00pm

Exam Rules

- Exam ends at 2:00pm
- Closed class notes, closed lab assignments, closed handouts.
- One sheet of hand-written prepared notes (front & back, letter size).
- Hand-held calculators are permitted, but not necessary.
- No phones, tablets, laptops, or other internet-connected devices.
- Scratch paper is allowed.

HONOR PLEDGE: Copy (NOW) and SIGN (after the exam is completed): "I have neither given nor received aid on this exam, nor have I observed a violation of the *Berkeley Campus Code of Student Conduct*."

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SIGNATURE
(Sign **after** the exam is completed.)

The maximum possible score is **20 points**. To maximize your score, read the directions carefully and write legibly. Where appropriate show your work. This is an opportunity to show off how much you understand. Good luck!

True-False / Multiple Choice Section

Please clearly mark your answer. You are not asked to show work. Each problem is worth **1 point**.

Problem 1

__________ **(T or F)** The objective function $f(x_1, x_2) = 10x_1^2 + 5x_2^2$ has a unique minimum.

Problem 2

__________ **(T or F)** The optimization problem

$$\begin{array}{ll} \text{min :} & f(x) = midtermScore(x) + finalScore(x) + labScore(x) \\ \text{s. to:} & x \in \{\text{Jorge, Brian, Adrienne, Rachel, Paul}\} \end{array}$$

is an integer programming problem.

Problem 3

Recall the airplane landing example in class lecture. The logical disjunction $|t_1 - t_2| \geq \Delta$ can be transformed into the two AND conditions

$$\begin{array}{rcl} t_1 - t_2 & \geq & \Delta - Md \\ t_1 - t_2 & \leq & -\Delta + M(1-d) \end{array}$$

__________ **(T or F)** Setting d = 0 implies t_1 occurs before t_2

Problem 4

Let $x = [x_1, x_2, x_3]^T$ and consider the optimization problem:

$$\begin{array}{ll} \text{min:} & c^T x \quad (1) \\ \text{s. to:} & Ax \leq b \quad (2) \\ & x_2 \in \mathbb{Z} \quad (3) \\ & \quad (4) \end{array}$$

__________ **(T or F)** The solution x^* must lie on the boundary of the set defined by $Ax \leq b$.

Problem 5

The second-order Taylor series approximation of $f(x) = \sin(x)$ around $x = 0$ is:
(a) 1
(b) $\cos(x)$
(c) x
(d) $1 - \sin(x) - \frac{1}{2}x^2$
(e) none of the above

Short Answer Section

For this section, partial credit is awarded. You MUST show your work. Unsupported answers, even if correct, receive zero credit. In other words, a right answer with wrong or no reasoning could lead to zero credit. I will not answer the question: "Is this enough writing to receive full credit?" If you show enough steps to (i) systematically solve the problem, and (ii) demonstrate application of the methods learned in class, then you will be fine. Please box your final answer.

Problem 6 : The Knapsack Problem (5 pts)

Chris McCandless is planning a trip into the wilderness[1]. He can take along one knapsack and must decide how much food and equipment to bring along. The knapsack has a finite (i.e. limited) volume. However, he wishes to maximize the total "value" of goods in the knapsack.

Table 1: Knapsack Problem Notation

Symbol	Description	Value
x_1	Units of food	–
x_2	Units of equipment	–
c_1	Value per unit food	2
c_2	Value per unit equipment	1
v_1	Volume per unit food	2
v_2	Volume per unit equipment	3
n_1	Maximum units of food	4
K	Maximum volume of knapsack	9

Part (a) : Formulation

Formulate a linear program (LP) to solve the knapsack problem. Use the notation from Table 1. Include the cost function and constraints. Label the meaning of each constraint.

[1] Inspired by the 1996 non-fictional book "Into the Wild" written by Jon Krakauer. It's a great book!

Part (b) : Graphical Solutions

Draw the feasible set. Is the feasible set bounded? Draw isolines for the cost function on your graph. Use graphical arguments to determine the optimal solution for the relaxed problem.

Part (c) : Integer Programming and Branch & Bound

Suppose we add the additional constraints that $x_1, x_2 \in \mathbb{Z}$. That is, we are interested in integer solutions since the units of food and equipment are not divisible. Use the branch & bound method to solve this integer programming problem.

Problem 7 : Modified Traveling Salesman Problem (5 pts)

Another classical problem often studied in optimization is the traveling salesman. In this problem, a salesman, departing from city A is required to visit n cities and then return to city A. The salesman seeks to conduct his business as efficiently as possible via minimizing the total cost of travel. To cast this as a shortest path problem, we add an artificial terminal node (node K) and structure the path according to Fig. 1.

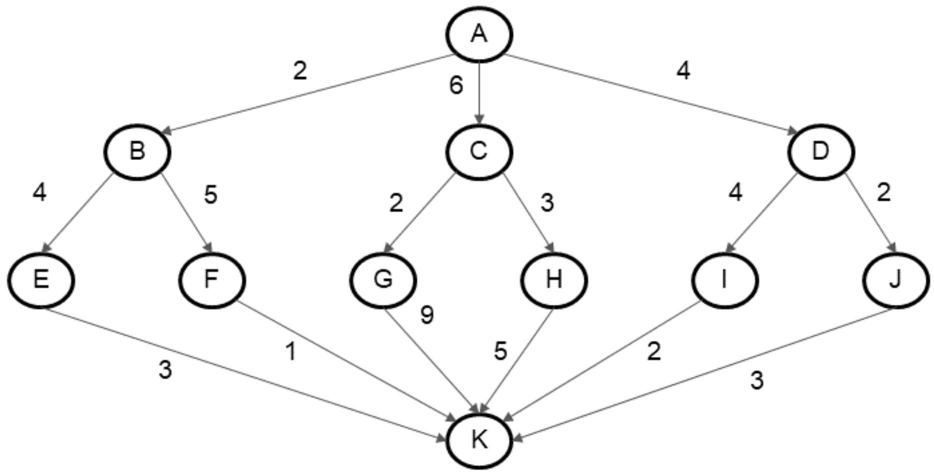

Figure 1: Network Interpretation of Modified Traveling Salesman Problem.

Using Dijkstra's algorithm, find the optimal path from node A to node K, as well as the cost of that path.

Problem 8 : Energy Portfolio Revisited (5 pts)

In Lab 2, you designed the optimal energy portfolio that minimizes risk subject to a maximum cost. The risk was computed as

$$J_{Lab2} = \sum_{i=1}^{8} \sigma_i^2 x_i^2 \tag{5}$$

which assumes no cross-correlation of prices between different energy sources. We relax that assumption now. For simplicity consider two energy sources $x = [x_1, x_2]^T$. The standard deviation in price is encoded into covariance matrix Q which now has non-zero off-diagonal elements

$$Q = \begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}. \tag{6}$$

The goal is to minimize risk where, for the purposes of this exam, we consider the unconstrained problem

$$\min \; x^T Q x. \tag{7}$$

Part (a)

Calculate the stationary point(s) $x^\dagger$ for this cost function.

Part (b)

Determine the nature of $x^\dagger$. Is $x^\dagger$ a minimum, maximum, or saddle point? Is it unique?

Part (c)

Is it possible to minimize risk by procuring a non-zero amount of energy x_1 and x_2? Why or why not?